Currency dependence of corporate credit spreads

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Abstract

Many pricing and risk management models need credit spread curves as an input. In the corporate bond market the estimation of credit spread curves is not trivial. Most issuers have only too few bonds outstanding and frequently these bonds are denominated in different currencies. To ensure a sufficient number of bonds for the estimation procedure in many cases bonds in different currencies have to be used which implies that the estimation procedure has to take into account potential currency effects. Under the hypothesis of zero correlation between the default variables and the exchange rates deflated by the relevant money market accounts we show using a rather general pricing framework that credit spreads are expected to be equal across different currencies. This paper analyses these effects and presents a new model which allows to estimate a credit spread curve for a single issuer with bonds in different currencies. This new model is based on the multi-curve estimation approach which allows a parsimonious joint estimation of a risk free term structure and the credit spread curve of the issuer. We reject the hypothesis of zero correlation between credit and exchange rate risk and present empirical evidence that there are significant differences of issuer specific credit spreads across different currencies in a representative sample of international corporate bonds. Moreover, this implies that dollar related credit spreads curves cannot be used without special care for pricing defaultable claims denominated in other currencies.

JEL: G12, G13, G15; C13

1 Introduction

Corporate credit spreads are defined as the differences between the zero-coupon yields related to an individual debtor or to a rating class and the risk-free zero-coupon yields for corresponding maturities. The knowledge of these corporate credit spreads is essential for many financial models and applications. Credit spreads are important inputs for many risk management and portfolio management models dealing with corporate bonds. Most notably, observed credit spreads are used to calibrate pricing models for defaultable claims to market price information in order to obtain implied estimates of the market price(s) of risk.

There is a growing academic literature devoted to various topics concerning the estimation of credit spread curves (see, e.g., Diaz and Skinner (2001)) but little attention has been paid to the currency dependence of corporate credit spreads (Kercheval et al. (2003) is one of the first studies explicitly addressing this question). We show in section 2 of this paper that in markets free of arbitrage the credit spread for an individual issuer has to be equal across different currencies if the factor processes driving the credit risk of the issuer are uncorrelated with the exchange rates deflated by the continuously compounded money market account in the reference currency. Under the hypothesis of zero correlation between credit risk and exchange rate risk credit spreads have to be equal across different currencies on the single issuer level. It is one important aim of this paper to provide empirical evidence regarding the currency dependence of corporate credit spreads and thus the existence of such a correlation.

There are two important cases where a potential currency dependence of corporate credit spreads would play an important role in practical applications. The first case, which is related to the estimation of credit spread curves, arises when spread curves are estimated on a single issuer basis. Usually, credit spread curves are estimated using bonds out of a pool of issuers in a specific rating class. Unfortunately, this method does not reveal the true credit spread for the individual issuer but only an average credit spread for the rating bucket. Elton et al. (2000) present evidence that pricing errors are huge for all rating classes and depend on several exogeneous factors. However, when attempting to estimate single issuer credit spread curves market prices of a minimum number of straight bonds of comparable seniority have to be available. It turns out that for many issuers this goal cannot be achieved unless bonds denominated in different currencies are jointly used by the estimation procedure. It is the second important objective of this paper to analyse whether bonds denominated in different currencies can be used for the estimation of credit spread curves, and, additionally, how possible currency effects can be taken into account. For this purpose we suggest an extended version of the multi-curve estimation approach originally presented by Houweling et al. (2001).

The second case where the question of currency dependence in the context of credit spreads arises is related to the application of observed credit spread curves which are usually estimated for entire rating classes rather than on a single issuer level. Corporate credit spreads are an essential input for many pricing models of defaultable claims as well as for credit risk management models. Some of these models are based on empirically observable quantities like probabilities of default, rating transition or generator matrices, and recovery or loss rates (see, e.g., Das and Tufano (1996) and Jarrow et al. (1997)). Since any model parameters fitted to these quantities are with respect to the empirical measure credit spread curves are required to obtain the relevant parameters under the equivalent pricing measure (see, e.g., Jarrow (2001) and Janosi et al. (2002)). This is also true for most credit risk management models used by practitioners, like, e.g. CreditMetrics. Other models are solely calibrated to obervable credit spread curves where the model parameters are obtained under the pricing measure only (see, e.g., Duffie and Singleton (1997), Duffee (1999)). This approach is unproblematic as long as the defaultable claims that have to be priced are denominated in the same currency as the bonds used to estimate the credit spread curves. Since most publicly available estimates of credit spread curves (e.g. via data vendors like Reuters and Bloomberg or published by rating agencies like Moody's and Standard&Poors) are constructed using dollar denominated bonds care must be taken when applying these spread curves for the pricing of claims denominated in different currencies like euro. Only under the assumption of zero correlation between credit and exchange rate risk these spread curves can be applied regardless of the currency. Otherwise corporate credit spreads have to be estimated for different currencies which may cause empirical problems for currencies with only a few number of corporate bonds outstanding. It is the third important aim of this paper to analyse whether dollar related credit spread curves can be used for pricing defaultable claims denominated in different currencies, and how possible currency effects can be taken into account.

We do not analyse the dynamics of credit spreads nor currency related differences between these spreads. Because of the lack of useful corporate credit spread data on the single issuer level this task is beyond the scope of this paper and left for future research.

In section 2 of this paper we show the elementary importance of the correlation between credit and exchange rate risk for the level of credit spreads in different currencies. In a first step we derive the basic result using a simple one-period model, and in a second step we extend our analysis to a rather general affine intensity based pricing framework. Chapter 3 summarizes the main problems related to the estimation of credit spread curves and introduces the methodology used in this paper. In addition, in this chapter we extend the multi-curve framework previously suggested by Houweling et al. (2001) to capture potential currency effects. This extended framework is also used to conduct formal tests of currency effects presented in chapter 4. This empirical analysis is based on a representative sample of international corporate bond data. Section 5 concludes the paper.

2 Theoretical framework

In this section we show the elementary importance of the correlation between credit and exchange rate risk for the level of credit spreads in different currencies. In a first step we derive the basic result using a simple one-period model, and in a second step we extend our analysis to a rather general affine intensity based pricing framework.

Consider a financial market where a corporation subject to default risk issues two zero-coupon bonds with identical maturity, e.g. T years, but denominated in two different currencies. The time t prices of these corporate bonds are denoted as C_t^1 and C_t^2, respectively. In addition, there exists a riskless money market account in both currencies with riskless interest rates r^1 and r^2. Finally, assume that the market is frictionless and free of arbitrage. In particular there are no transaction costs and short sales are allowed. At time zero the prices of both corporate bonds are fixed at one unit of the relevant currency and the final payoff of the corporate bonds are given by

$$C_T^1 = e^{(r^1+s^1)\cdot T} \cdot I_{survive}, \quad C_T^2 = e^{(r^2+s^2)\cdot T} \cdot I_{survive}$$

where s^c denotes the corporate credit spread in currency c and $I_{survive}$ denotes a indicator variable with value of one if the issuer meets all its obligations and zero if the issuer defaults. Note, that for the sake of simplicity we implicitly assume a zero recovery rate.

Let Q denote the equivalent martingale measure where the riskless money market account in the first currency is used as the numeraire. Since under this measure all relative price processes with respect to the numeraire asset are martingales we derive the following condition using the corporate bond traded in the first currency:

$$1 = e^{-r^1 \cdot T} \cdot E^Q[e^{(r^1+s^1)\cdot T} \cdot I_{survive}] = e^{s^1 \cdot T} \cdot E^Q[I_{survive}]$$

This condition implies that the survival probability under Q is equal to e^{-s1}. Let X_t denote the time t price of one unit of currency two expressed in units of currency one. Then the time zero price of the corporate bond issued in currency two equals X_0. Using the fact that the martingale probability must also hold for the prices of the second corporate bond expressed in currency one we obtain a second condition:

$$X_0 = e^{-r^1 \cdot T} \cdot E^Q[X_T \cdot e^{(r^2+s^2)\cdot T} \cdot I_{survive}] = e^{(r^2+s^2-r^1)\cdot T} \cdot E^Q[X_T \cdot I_{survive}]$$

Since the exchange rate X_T and the default variable $I_{survive}$ are both random variables the covariances between these variables have to be taken into account. Dependent on this covariance a general relationship between the currency related spreads s^1 and s^2 can be derived. In contrast to the arguments presented in Kercheval et al. (2003) these spreads are not equal in the general case. The "arbitrage" condition derived in that paper obviously does only hold for riskless bonds because the default state is not taken into account. However, it can be shown that in a special case when the exchange rate and the default variable are uncorrelated the spreads have in fact to be equal. If the correlation is zero then the condition derived above can be rewritten as follows:

$$X_0 = e^{(r^2+s^2-r^1)\cdot T} \cdot E^Q[X_T] \cdot E^Q[I_{survive}]$$

Making use of the knowledge of the survival probability under Q and the fact that the expected exchange rate under Q equals the arbitrage-free forward exchange rate we obtain

$$X_0 = e^{(r^2+s^2-r^1)\cdot T} \cdot X_0 \cdot e^{(r^1-r^2)\cdot T} \cdot e^{(-s^1)\cdot T} = X_0 \cdot e^{(s^2-s^1)\cdot T}$$

which in turn implies that $s^1 = s^2$.

To show the effect of the currency on the credit spread curve in fairly general framework we again assume one corporation with two zero-coupon bonds outstanding with maturity T in different currencies c (c =1,2) with their time zero prices denoted by $C_{0,T}^{c}$. For each currency we have a riskless money-market account and a riskless zero-coupon bond with maturity T. The time zero price of the money market accounts equals one unit of currency and the prices of the zero-coupon bonds are denoted by $P_{0,T}^{c}$. Finally, the terminal value of the money market accounts are given by

$$A_T^c = e^{\int_0^T r_u^c du}$$

Using a single factor affine term structure model yields the following functional forms for the riskless and the risky zero-coupon bonds in currency 1 (see, e.g. Duffie and Singleton (1999)) under the assumption $COV(I_{survive}; A_T^1) = 0$:

Riskless zero-coupon bond: $P_{0,T}^1 = a^1(0,T) \cdot e^{-b^1(0,T) \cdot r^1}$

Risky zero-coupon bond: $C_{0,T}^1 = a^1(0,T) \cdot e^{-b^1(0,T) \cdot r^1} \cdot q$ with $q = \mathrm{E}^Q\left[I_{surive}\right]$

The functions a^1 and b^1 depend on the model parameters and the specific choice of the short rate dynamics. Note, that the survival probability q can also be expressed in a similar functional form where the hazard rate, i.e. the product of stochastic default intensity and stochastic loss rate, replaces the short rate. For the purpose of this argument we do not need this explicit representation. The important point is that the survival probability under Q equals the ratio of the risky and the riskless zero-coupon bond prices in currency 1.

Using the martingale property for the second risky zero-coupon bond we derive the following condition:

$$C_{0,T}^2 \cdot X_0 = \mathrm{E}^Q\left[\frac{C_{T,T}^2 \cdot X_T}{A_T^1}\right] = \mathrm{E}^Q\left[\frac{X_T}{A_T^1} \cdot I_{surive}\right]$$

Now we assume that $COV(I_{survive};\frac{X_T}{A_T^1})=0$ and obtain the following simplified expression for the expectation

$$\mathrm{E}^Q\left[\frac{X_T}{A_T^1}\cdot I_{surive}\right]=\mathrm{E}^Q\left[\frac{X_T}{A_T^1}\right]\cdot \mathrm{E}^Q\left[I_{surive}\right]=\mathrm{E}^Q\left[\frac{X_T}{A_T^1}\right]\cdot q$$

In the next step we switch from the risk neutral measure Q to the foreign risk neutral measure Q^* (see Björk (1998)) obtaining

$$\mathrm{E}^Q\left[\frac{X_T}{A_T^1}\right]\cdot q=\mathrm{E}^Q\left[\frac{X_T}{X_0}\cdot\frac{1}{A_T^1}\right]\cdot q\cdot X_0=\mathrm{E}^{Q^*}\left[\frac{1}{A_T^2}\right]\cdot q\cdot X_0=a^2(0,T)\cdot e^{-b^2(0,T)\cdot r^2}\cdot q\cdot X_0$$

and finally we end up with the following condition for the second zero-coupon bond price

$$C_{0,T}^2\cdot X_0=a^2(0,T)\cdot e^{-b^2(0,T)\cdot r^2}\cdot q\cdot X_0\Leftrightarrow C_{0,T}^2=a^2(0,T)\cdot e^{-b^2(0,T)\cdot r^2}\cdot q$$

In this general framework the crucial assumption is somewhat different since the covariance between the default variable and the evolution of the exchange rate deflated by the relevant money market account, rather than the exchange rate is assumed to be zero.

In order to derive expressions for the corporate yield spreads we calculate the risky continuous rates for the issuer in the two currencies using the corporate zero-coupon bond prices:

Currency 1:

$$R_{risky}^1=-\frac{1}{T}\cdot\ln(C_{0,T}^1)=-\frac{1}{T}\cdot\ln(a^1(0,T)\cdot e^{-b^1(0,T)\cdot r^1})+\frac{1}{T}\cdot\ln(\frac{1}{q})=R_{riskless}^1+\frac{1}{T}\cdot\ln(\frac{1}{q})$$

Currency 2:

$$R_{risky}^2=-\frac{1}{T}\cdot\ln(C_{0,T}^2)=-\frac{1}{T}\cdot\ln(a^2(0,T)\cdot e^{-b^2(0,T)\cdot r^2})+\frac{1}{T}\cdot\ln(\frac{1}{q})=R_{riskless}^2+\frac{1}{T}\cdot\ln(\frac{1}{q})$$

This implies that we obtain an identical credit spread $\frac{1}{T} \cdot \ln(\frac{1}{q})$ for both currencies.

Using a fairly general framework we have shown that a zero covariance between the default variable and the evolution of the exchange rates deflated by the relevant money market accounts implies equal credit spreads across currencies. From a theoretical point of view the assumption of no currency dependence of credit spreads translates to the assumption of zero covariance between these variables. But is this assumption reasonable? There seems to be no obvious answer to this question. In theory the pure existence of corporate bonds denominated in a specific currency implies a dependence of the default variable on the exchange rate because a rising exchange rate increases the market value of the company's debt and thus increases the default probability. On the other hand a rising exchange rate might improve the company's position in the export market and thus offset the other effect. Over all it is an empirical question whether this assumption is reasonable or not.

In the empirical analysis conducted in this paper we test for differences between observed credit spreads in different currencies. The results of these tests are directly applicable to the questions related to the pooling of bonds denominated in different currencies for the estimation of credit spread curves and to the use of dollar denominated credit spread curves as an input for pricing defaultable claims in a different currency. In addition, based on the prior analysis we interpret our results indirectly as results of testing the hypothesis of zero covariance between the default variable and the evolution of the exchange rates deflated by the relevant money market accounts.

3 Credit spread models

In this section we briefly explain the basic technique to estimate credit spread curves with prices of coupon bonds in an incomplete bond market. Then we describe a more advanced multi-curve model to estimate credit spread curves. This multi-curve model is thereafter extended to be able to produce stable credit spread curves on the single issuer level in the situation when the corporation has issued bonds in different currencies. We use the cubic splines method introduced by McCulloch (1971,1975). The choice of this model may seem arbitrary but Jankowitsch and Pichler (2002) show that in the multi-curve framework this model performs well compared to more complex estimation procedures. Also Elton et al. (2000) and Diaz and Skinner (2001) find no significant difference between splines models and non-linear models like Nelson and Siegel (1987) or Svensson (1994). The main conclusions of the paper are very unlikely to be affected by the choice of the specific functional form. Thus, we do not refer to more advanced splines models, like B-splines or exponential splines.

3.1 Basic estimation problem

First of all we describe the estimation problem in a single market setting. Then we turn to the multi-curve estimation problem when credit spread curves are to be estimated. Consider a bond market with N coupon bonds. A bond i is characterised by its market price P_i (quoted price plus accrued interest), its cash flow vector Z_i, and its vector of cash flow dates T_i. A bond market is said to be complete if the total number of distinct cash flow dates in the market is equal to the number of linearly independent cash flow vectors, i.e. if arbitrary cash flow structures can be replicated using portfolios of existing bonds. The absence of arbitrage implies the existence of an unique set of discount factors $D(t)$, where t denotes the time to any cash flow date in the market, for which

$$P_i = \sum_j Z_{ij} \cdot D(T_{ij})$$

holds for all $i = 1, ..., N$. The zero-coupon yields $r(t)$ are related to the discount factors via

$$r(t) = \log\left(\left(\frac{1}{D(t)}\right)^{1/t}\right)$$

In incomplete markets like the corporate bond markets the set of arbitrage-free discount factors is not unique. Given an arbitrary set of discount factors

$$\mathrm{P_i} = \sum_{\mathrm{j}} \mathrm{Z_{ij}} \cdot \mathrm{D(T_{ij})} + \varepsilon_{\mathrm{i}}$$

applies, where ε_i denotes the pricing error of bond *i*. The basic estimation problem is aimed to find a set of discount factors which has optimal explanatory power, i.e. which minimizes the pricing errors with respect to a given norm, and is represented by a continuous function depending on a parsimonious number of free parameters. Let *a* denote the set of parameters and *f* denote the specified function then we have

$$\mathrm{D(t) = f(t;a)}$$

and

$$\mathrm{P_i} = \sum_{\mathrm{j}} \mathrm{Z_{ij}} \cdot \mathrm{f(T_{ij};a)} + \varepsilon_{\mathrm{i}}$$

There are numerous models suggested to solve this estimation problem. These models mainly differ in the functional specification, the number of free parameters, and the quantity for which the functional form is specified, i.e. for the discount factors, the zero-coupon yields or the forward rates (see e.g. Schaefer (1981), Vasicek and Fong (1982) and Svensson (1994)).

3.2 Splines model

In the *splines model* introduced by McCulloch (1975) cubic splines are used to model the discount function. In this approach the maturity spectrum is divided into (not necessarily equally spaced) intervals. If the maturity spectrum is divided by *k-1* knots there are *k* free parameters to describe the entire discount function which is modelled as a linear combination of k prespecified component functions. Let f_k denote the component functions then the estimation problem reads

$$P_i = \sum_j Z_{ij} \cdot \sum_k a_k \cdot f_k(T_{ij}) + \varepsilon_i = \sum_k a_k \cdot \sum_j Z_{ij} \cdot f_k(T_{ij}) + \varepsilon_i$$

Employing the linear structure of this model the optimal parameters can easily be obtained by performing an OLS regression.

Finally, the credit spread curve of a specific issuer for a specific currency with respect to a reference curve for the same currency is calculated by subtracting the reference zero-coupon yield curve from the zero-coupon yield curve of the issuer using all bonds with the specific currency.

$$s_c(t) = r_c(t) - r_{c\text{-ref}}(t)$$

$s_c(t)$	credit spread between the issuer curve and the reference curve for currency c for maturity t
$r_c(t)$	zero-coupon yield for the issuer for currency c for maturity t
$r_{c\text{-}ref}(t)$	reference zero-coupon yield curve for currency c for maturity t

3.3 Multi-curve splines model

In the *multi-curve splines model* introduced by Houweling et al. (2001) the reference zero-coupon yield curves and the credit spread curves of a specific issuer for all currencies are estimated jointly. All the curves are estimated with cubic splines as introduced in section 3.2. In order to have a more parsimonious specification the number of parameters for the credit spread curve can be reduced compared to the reference curve. The joint estimation for one issuer uses the following framework for the discount functions:

$$D_{c-ref}(t) = \sum_{k=1}^{k_{ref}} f_{ref,k}(t) \cdot a_{c-ref,k}$$ discount function of the reference curve for currency c

$$D_c(t) = D_{c-ref}(t) + \sum_{k=1}^{k_{spread}} f_{spread,k}(t) \cdot a_{c,k}$$ discount function for the issuer for currency c

k_{ref}	number of parameters for the reference zero-coupon yield curves
k_{spread}	number of parameters for the credit spread curves
$f_{ref,k}(t)$	component functions which use the chosen knots of the reference zero-coupon yield curves
$f_{spread,k}(t)$	component functions which use the chosen knots of the spread curves
$a_{c\text{-}ref,k}$	parameters of the reference zero-coupon yield curve for currency *c*
$a_{c,k}$	parameters of the credit spread curve of the issuer for currency *c*

This multi-curve model uses two features to improve the single-curve model. First it directly estimates the spread curves with a parsimonious splines model and second the reference zero-coupon yield curves and all spread curves are estimated jointly.

Employing this framework we obtain the following linear regression model for a issuer which has bonds outstanding in *C* currencies:

$$Y = X \cdot a + \varepsilon \Leftrightarrow \begin{pmatrix} \mathbf{Y}_{1-ref} \\ \mathbf{Y}_{2-ref} \\ \vdots \\ \mathbf{Y}_{C-ref} \\ \mathbf{Y}_1 \\ \mathbf{Y}_2 \\ \vdots \\ \mathbf{Y}_C \end{pmatrix} = \begin{pmatrix} \mathbf{X}_{1-ref} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_{2-ref} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{X}_{C-ref} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{X}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{S}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{S}_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{X}_C & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{S}_C \end{pmatrix} \cdot \begin{pmatrix} \mathbf{a}_{1-ref} \\ \mathbf{a}_{2-ref} \\ \vdots \\ \mathbf{a}_{C-ref} \\ \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_C \end{pmatrix} + \begin{pmatrix} \boldsymbol{\varepsilon}_{1-ref} \\ \boldsymbol{\varepsilon}_{2-ref} \\ \vdots \\ \boldsymbol{\varepsilon}_{C-ref} \\ \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_2 \\ \vdots \\ \boldsymbol{\varepsilon}_C \end{pmatrix}$$

$Y_{c\text{-}ref}$ and Y_c are vectors consisting of elements (suppressing index *c*)

$$\mathrm{y_i} = \mathrm{P_i} - \sum_{\mathrm{j}} \mathrm{Z_{ij}},$$

$X_{c\text{-}ref}$ and X_c are matrices with elements (suppressing index *c*)

$$\mathrm{x_{ik}} = \sum_{\mathrm{j}} \mathrm{Z_{ij}} \cdot \mathrm{f_{ref,k}}(\mathrm{T_{ij}}),$$

S_c is a matrix with elements (suppressing index c)

$$s_{ik} = \sum_j Z_{ij} \cdot f_{spread,k}(T_{ij}),$$

$a_{c\text{-}ref}$ and a_c are vectors of parameters, and

$\varepsilon_{c\text{-}ref}$ and ε_c are vectors of residuals assumed to be iid with $(0,\sigma_{c\text{-}ref}^2)$ and $(0,\sigma_c^2)$.

The residual term is allowed to have different variances because some bond prices a generally more noisy than others. To estimate the parameters a Restricted Feasible GLS procedure (see, e.g. Greene (2000)) is applied. Numerical examples for applications in the given framework show, however, that the relaxation of the variance structure is of minor importance for the results.

3.4 Multi-curve splines models with parallel shift

In the *multi-curve splines model with parallel shift* the reference zero-coupon yield curves for all currencies and one basic credit spread curve for a specific issuer are estimated jointly. Additionally there is a parameter for every currency which can shift the basic credit spread curve up or down in a parallel fashion. This shift is linear in the discount function and is equivalent to a first order Taylor series approximation of a parallel shift in the zero-coupon yield curve. This model has the advantage that one data point per currency is sufficient to estimate an entire credit spread curve. Employing a shift for the credit spread curve is similar to the approach used by Elton et al. (2000) where the shift is dependent on exogenous variables. In order to have a more parsimonious specification the number of parameters for the credit spread curve can be reduced compared to the reference curve. The joint estimation for one corporation uses the following framework for the discount functions:

$$D_{c-ref}(t) = \sum_{k=1}^{k_{\text{ref}}} f_{ref,k}(t) \cdot a_{c-ref,k}$$ discount function of the reference curve for currency c

$$D_c(t) = D_{c-ref}(t) + \sum_{k=1}^{k_{spread}} f_{spread,k}(t) \cdot a_{spread,k} + b_c \cdot t$$ discount function for the corporation for currency c

k_{ref}	number of parameters for the reference zero-coupon yield curves
k_{spread}	number of parameters for the credit spread curves
$f_{ref,k}(t)$	component functions which use the chosen knots of the reference zero-coupon yield curves
$f_{spread,k}(t)$	component functions which use the chosen knots of the spread curves
$a_{c\text{-}ref,k}$	parameters of the reference zero-coupon yield curve for currency c
$a_{spread,k}$	parameters of the credit spread curve of the corporation
b_c	parameters for currency c to shift the credit spread curve of the corporation up or down

The new parameter b_c is set to zero for the first currency and for all other currencies the parameter is estimated in the regression model.

Using this framework we obtain the following linear regression model for a corporation which has bonds in C currencies:

$$Y = X \cdot a + \varepsilon \Leftrightarrow \begin{pmatrix} \mathbf{Y}_{1-ref} \\ \mathbf{Y}_{2-ref} \\ \vdots \\ \mathbf{Y}_{C-ref} \\ \mathbf{Y}_1 \\ \mathbf{Y}_2 \\ \vdots \\ \mathbf{Y}_C \end{pmatrix} = \begin{pmatrix} \mathbf{X}_{1-ref} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_{2-ref} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{X}_{C-ref} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{X}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{S}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_2 & \mathbf{0} & \mathbf{0} & \mathbf{S}_2 & \mathbf{W}_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} & \vdots & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{X}_C & \mathbf{S}_C & \mathbf{0} & \mathbf{0} & \mathbf{W}_C \end{pmatrix} \cdot \begin{pmatrix} \mathbf{a}_{1-ref} \\ \mathbf{a}_{2-ref} \\ \vdots \\ \mathbf{a}_{C-ref} \\ \mathbf{a}_{spread} \\ \mathrm{b}_2 \\ \vdots \\ \mathrm{b}_C \end{pmatrix} + \begin{pmatrix} \boldsymbol{\varepsilon}_{1-ref} \\ \boldsymbol{\varepsilon}_{2-ref} \\ \vdots \\ \boldsymbol{\varepsilon}_{C-ref} \\ \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_2 \\ \vdots \\ \boldsymbol{\varepsilon}_C \end{pmatrix}$$

W_c is a matrix with elements (suppressing index c)

$$w_i = \sum_j Z_{ij} \cdot T_{ij},$$

b_c are parameters for the parallel shift.

The residual term as in the *multi-curve spline model* again is allowed to have different variances.

We develop this new model to be able to produce estimates for credit spread curves for each currency on a single issuer level given the scarcity of data in the corporate bond market. The advantage of the new model is that all bonds of one issuer are jointly used to estimate one basic credit spread curve and additional parameters are estimated simultaneously to take potential currency effects into account. Using the new model reduces the minimal number of necessary bonds significantly. This progress is only possible by using the multi-curve estimation approach which is a general improvement in the field of credit spread estimation (see Houweling et al. (2001) and Jankowitsch and Pichler (2002)).

4 Empirical Analysis

4.1 Data

The data set used in this study is based on a sample of 304 international corporate bonds of 30 issuers denominated in ten currencies. These data are aimed to represent a typical trading portfolio of a European bank. Since no comprehensive data base for international corporate bonds is available and most of these bonds are rather illiquid we have to restrict our analysis to a representative sample of liquid issues. However, the main results of this paper are not influenced by the choice of the sample (for a complete list of all bonds contained in the sample see appendix, table 6).

The data include daily prices of corporate bonds, money market rates and swap market rates from the period 01/2002 to 07/2002. The daily bond prices are quotations taken from Reuters. These quotations are collected between two and six p.m. CET. Outliers are eliminated and the average of the remaining quotations of a bond is used as price. The money market rates and swap market rates are also quotations taken from Reuters at four p.m. CET.

We restrict our analysis to coupon bonds without any option features and a time to maturity shorter than ten years. The data set includes the basic features of the bonds, e.g. the maturity, the currency and the exact cash flow schedule. Table 1 and 2 give an overview of the data set:

Business	Auto-mobiles	Banking	Beverage & Food	Energy	Govern. Agency	Insurance	Trans-portation	Telecom	Utilities – Electrical
Number of Corp.	4	10	3	1	2	1	1	4	4
Number of Bonds	52	125	21	7	21	6	9	17	46

Table 1: Number of corporate bonds for industries

Currency	CZK	DKK	EUR	GBP	HUF	NOK	PLN	SEK	USD	ZAR
Number of Bonds	12	11	150	33	10	12	14	8	30	24

Table 2: Number of corporate bonds for currencies

For all ten currencies money market and swap market rates are collected. We use money market rates for one, three, six and nine months and swap rates from one up to ten years.

It is important to realise one possible weakness of this data set. The data set only contains quoted prices and not the actual traded prices. The quoted prices may only hold for a relative small quantity, whereas traders demanding higher quantities cannot in advance determine the actual price for the entire quantity they wish to trade. However, practitioners use these quotations in their daily work and the quotes are regarded by many market participants to be good for a certain size, typically EUR 10 million, and most of the trades are taking place within the quoted bid-ask prices.

4.2 Methodology

We want to analyse the effect of the currency on the credit spread curves. We have shown in section 2 that in a general affine pricing framework the assumption of a zero covariance between the default variable and the evolution of the exchange rates deflated by the relevant money market accounts implies equal credit spreads across currencies. Now we want to test for differences between observed credit spreads in different currencies with our data set. With the results we want to critically analyse the pooling of bonds denominated in different currencies and the use of dollar denominated credit spread curves to price defaultable claims in other currencies. Additionally we have an indirect test for the hypothesis of zero covariance.

The starting point of our empirical analysis is the estimation of credit spread curves for each issuer in each currency using the splines model (see section 3.2) and the multi-curve splines model (see section 3.3). The money market and swap market rates are the input data for the reference curves. We use the splines model with four parameters and the multi-curve splines model with four parameters for the reference curve and three parameters for the spread curves. For both models we use equidistant knots. Analysing our data set we find 27 combination of issuers and currencies but only for four of these combinations we have sufficient corporate bonds for the two models. Having such a small sub-sample we cannot base statistical test on the resulting credit spread curves but we can get an intuition of the currency effect by conducting a visual inspection of these credit spread curves.

To analyse all 27 combinations we use the extended multi-curve splines model with parallel shift (see section 3.4). Again we use equidistant knots with four parameters for the reference curves and three parameters for the credit spread curves. This model has the advantage that only one credit spread curve is estimated for a issuer using all bonds in different currencies jointly and additional parameters are estimated simultaneously which control for potential currency effects. Having two currencies for a corporation we need one parameter to have two individual credit spread curves. For every additional currency one additional parameter is used to have a credit spread curve for this currency. We always set up the model in the way that these parameters shift the credit spread curve of a certain currency (e.g. USD) compared to the credit spread curve for euro denominated bonds. Using this model reduces the minimal number of necessary bonds significantly. Again we conduct first a visual inspection to observe whether there is significant difference between the credit spread curves in different currencies.

In order to test the hypothesis of a zero covariance between the default variable and the evolution of the exchange rates deflated by the relevant money market accounts we use several statistics on the parameter which shifts the spread curve. If there is a significant difference between the credit spread of one issuer in different currencies then the parameter should be significantly different from zero. First we use a sign test on the parameter determining whether the parameter is significantly positive or negative. Since we have at least 50 observations of the parameter per issuer and currency we can use a normal approximation for the test statistic. As a second test we use a t-test on the parameter in every single regression to test if this parameter is significant different form zero.

The third statistical test takes together the results of the t-test. To decide if the parameter is significantly different from zero for all regression models per issuer and currency a z-test is used. We test the hypothesis that the fraction of significant parameters using a 1-α confidence level t-test is less than the α. If the null hypothesis is rejected, then the parameter is significant meaning we have a difference in the credit spread between bonds in different currencies.

$$\frac{N^{\text{significant}} - \alpha}{\sqrt{\frac{\alpha \cdot (1-\alpha)}{n}}} \sim \mathrm{N}(0,1)$$

$N^{\text{significant}}$ number of observations where the parameter is significant

α	confidence level in the t-test
n	number of observations

4.3 Results

We start our analysis on the dependence of the credit spread of a corporation on the currency by estimating credit spread curves for each currency using the splines model and the multi-curve splines model as explained in section 4.2. To estimate reliable credit spread curves for an issuer in a currency we need at least three bonds in this currency distributed over the whole maturity interval. Analysing our corporate bond portfolio we can find only four issuers that have sufficient bonds in at least two currencies. So with these two models we can only analyse 4 out of 27 combinations of issuers and currencies as explained in section 4.2. After estimating the credit spread curves for this data set we conduct a visual inspection of these curves. We find that the curves are stable and economical reasonable (as an example see figure 1).

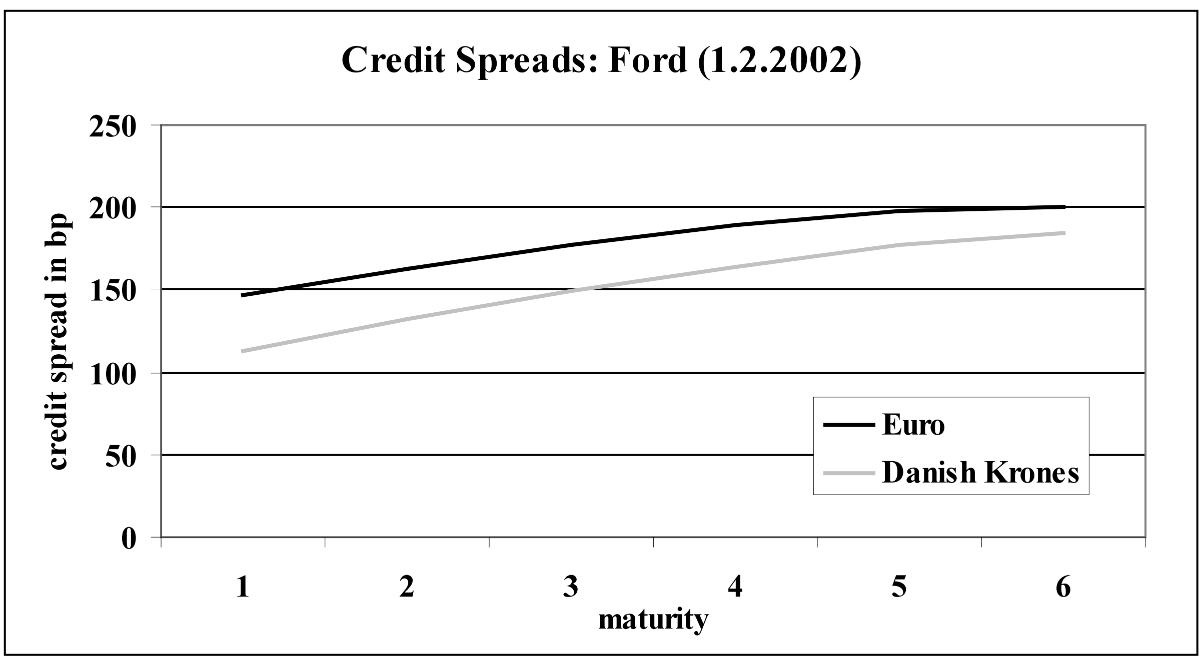

Figure 1: Credit spread curves for Ford on February 1, 2002 for Euro and Danish Krones bonds

From the visual inspection we see that the credit spread curves seem not to be equal and they seem to have different volatilities. Having only credit spread curves for four corporations in three currencies does not allow us to give a clear answer of the dependence of the credit spread on the currency.

This is the reason why most authors (e.g. Batten and Hogan (2003) or Kercheval et al. (2003)) estimate credit spread curves not on a single issuer basis but for rating classes. So we could have rating classes for each currency and repeat the procedure of curve estimation. The disadvantage of this strategy is that it does not reveal the true credit spread risk for every corporation but only an average spread risk for every bucket. To solve this problem we use the multi-curve splines model with parallel shift (see section 3.4). This model has the advantage that only one credit spread curve is estimated for each corporation using all bonds in different currencies together and additional parameters are estimated simultaneously which controls for currency effects. This model reduces the minimal data requirement for the calculations significantly. Now the minimal number of bonds in all currencies together is three. Analysing our corporate bond portfolio we have now 27 combinations consisting of fourteen corporations and nine different currencies. For these corporations we estimate credit spread curves for each currency using the multi-curve splines model with parallel shift and have a visual inspection of these curves. Again we find that the credit spread curves in different currencies seem not to be equal and to have different volatilities (as an example see table 3). To determine if there is a significant difference between the credit spread curves in different currencies we use the statistics described in section 4.2.

Currency / Maturity (years)	EUR	USD
1	3.024	6.832
2	2.837	6.891
3	3.344	6.844
4	4.382	7.652
5	4.617	8.658
6	4.061	9.915
7	4.552	10.236
8	4.331	10.894

Table 3: Volatilities of credit spreads for Electricite de France

To test if the number of positive or negative parameters is significant we use the sign test described in section 4.2. We use this test first on the estimated parameters for each issuer in each currency. We find a significant number of positive or negative parameters for 26 out of 27 cases (for details see appendix, table 7). Then we use the sign test taking the parameters of all issuers together for each currency. Table 4 contains the result for this test.

Currency	#negative parameters	#all parameters	test statistic	average of parameters
CZK	147	271	1.397	0.000862
DKK	255	464	2.135^+	0.000942
GBP	140	197	5.914^+	-0.001828
HUF	54	76	3.671^+	-0.000966
NOK	3	305	17.121^+	0.002371
SEK	0	100	10.000^+	0.001531
USD	314	358	14.269^+	0.007944
ZAR	79	260	6.326^+	0.001582
PLN	198	284	6.645^+	-0.000864

Table 4: Results of the sign test for each currency; $^+$significant at the 5% level

We find a significant number of positive or negative parameters for eight of the nine currencies (only for CZK we can find no significant result). Important to note is that the sign test does not take the magnitude of the effect into account. Therefore we want to judge if the results are also economically significant. So we take a closer look at the average parameter value for these currencies. Having a minimum bid-ask spread of around 10 basis points in the corporate bond market we argue that a difference between credit spread curves is economically significant if it is at least equal to this minimal bid-ask spread. A parameter below -0.0008 or above 0.0008 causes a parallel shift of around 10 basis points. Considering the average of the parameters we see that for all currencies the difference is economically significant.

As a second part of the analysis we use a t-test on the parameters in every single regression to test if these parameters are significant different from zero as explained in section 4.2. For the results on a single issuer level see table 8 in the appendix. Table 5 contains the number of significant parameters for each currency for all issuers together.

We can see from table 5 that most parameters are significant. Having this result we now test the hypothesis that the fraction of significant parameters using a 95% confidence level t-test is less than the 5% using a z-test as explained in section 4.2. If the null hypothesis is rejected, then the parameters are significant meaning we have differences in the credit spread between different currencies. We use this test first on the estimated parameters for each issuer in each currency. We find that the null hypothesis is rejected in 26 out of 27 cases (for details see appendix, table 8). Then we use this test taking the parameters of all issuers together for each currency. Table 5 contains the results for this test for each currency.

currency	# significant parameters	# all parameters	% of significant parameters	test statistic
All currencies	1461	2316	63.1%	128.25^{+}
CZK	95	271	35.1%	22.71^{+}
DKK	338	464	72.8%	67.05^{+}
GBP	99	197	50.3%	29.14^{+}
HUF	40	76	52.6%	19.05^{+}
NOK	295	305	96.7%	73.51^{+}
SEK	100	100	100.0%	43.59^{+}
USD	122	358	34.1%	25.24^{+}
ZAR	153	260	58.9%	39.84^{+}
PLN	218	284	76.8%	55.49^{+}

Table 5: Results of the z-test for each currency; $^{+}$significant at the 5% level

We see from table 5 that all relations for each currency and all currencies together are significant.

Taking all results together we can find a clear difference between the credit spread curves of different currencies. So for the purpose of pricing and risk management bonds of one issuer in different currencies cannot be treated together without explicitly taking into account potential currency effects, e.g. by using the extension of the multi-curve splines model proposed in this paper.

5 Summary

We provide empirical evidence on the currency dependence of corporate credit spreads. This dependence is important for pricing and risk management models that use these credit spread curves as an input. Assuming zero correlation between the default variables and the exchange rates deflated by the relevant money market accounts we show that credit spreads are expected to be equal across different currencies in a rather general pricing framework. We conduct formal tests of the hypothesis of zero correlation between credit risk and exchange rate risk and provide strong empirical evidence that there are significant differences of credit spreads across different currencies and thus reject this hypothesis. The magnitude of the effect is above the minimum bid-ask spread of 10 bp for all currencies making it also economically significant.

For the purpose of estimating credit spread curves we conclude that bonds denominated in different currencies cannot be pooled without explicitly taking into account potential currency effects, e.g. by using the extension of the multi-curve splines model proposed in this paper. Finally, our results imply that dollar related credit spread curves cannot be used without special care for pricing defaultable claims denominated in other currencies.

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Appendix

	TOTAL	CZK	DKK	EUR	GBP	HUF	NOK	PLN	SEK	USD	ZAR
ALL	304	12	11	150	33	10	12	14	8	30	24
AXA	6	-	-	5	1	-	-	-	-	-	-
Bank Austria	13	2	-	6	-	2	-	1	-	1	1
Caisse d'Amortissement de la dette sociale	13	-	-	7	3	-	-	-	-	3	-
Coca Cola	9	-	-	4	3	-	-	-	-	2	-
Commerzbank	20	2	-	4	-	-	-	1	-	-	13
Daimler	18	-	-	11	-	1	-	3	-	2	1
Deutsche Bahn	9	-	2	3	-	-	2	-	2	-	-
Deutsche Telekom	4	-	-	4	-	-	-	-	-	-	-
Dexia	6	-	2	1	-	-	-	-	3	-	-
Dsl Bank	11	-	1	7	-	-	-	-	3	-	-
Electricidade de Portugal	4	-	-	4	-	-	-	-	-	-	-
Electricite de France	8	-	-	4	3	-	-	-	-	1	-
Enel	4	-	-	4	-	-	-	-	-	-	-
Fiat	10	-	-	6	4	-	-	-	-	-	-
Ford	19	3	3	7	-	-	4	-	-	2	-
General Electric	30	2	1	8	6	-	3	3	-	5	2
General Motors	5	-	-	5	-	-	-	-	-	-	-
HVB	26	-	1	9	3	4	-	2	-	-	7
ING Bank	5	-	1	4	-	-	-	-	-	-	-
KPN	4	-	-	4	-	-	-	-	-	-	-
L. Rentenbank	11	-	-	5	4	-	-	-	-	2	-
LB Rheinland-Pfalz	12	1	-	4	-	-	3	4	-	-	-
Nestle	4	-	-	1	-	-	-	-	-	3	-
Parmalat	8	-	-	8	-	-	-	-	-	-	-
Quebec	8	-	-	6	2	-	-	-	-	-	-
Rabbobank	13	-	-	4	2	-	-	-	-	7	-
SNS Bank	8	2	-	3	-	3	-	-	-	-	-
Telefonica	4	-	-	4	-	-	-	-	-	-	-
Total	7	-	-	5	-	-	-	-	-	2	-
Vodafone	5	-	-	3	2	-	-	-	-	-	-

Table 6: List of all corporate bonds in the sample (total and per currency)

corporation	#negative parameters	#all parameters	test statistic	average of parameters
Deutsche Bahn – DKK	0	104	10.20^{+}	0.001731
Deutsche Bahn – NOK	0	99	9.95^{+}	0.001746
Deutsche Bahn – SEK	0	100	10.00^{+}	0.001531
Electricite de France - USD	75	75	8.65^{+}	-0.001249
Fiat - GBP	42	42	6.48^{+}	-0.007184
ING Bank - DKK	104	104	10.20^{+}	-0.002394
L. Rentenbank – GBP	33	76	1.15	0.000894
L. Rentenbank – USD	66	81	5.67^{+}	-0.000164
SNS Bank – CZK	86	88	8.95^{+}	-0.000376
SNS Bank – HUF	54	76	3.67^{+}	-0.000898
SNS Bank – PLN	88	90	9.07^{+}	-0.001388
Bank Austria - USD	38	38	6.16^{+}	-0.001046
Coca Cola - GBP	65	76	5.74^{+}	-0.000825
Commerzbank – CZK	3	93	9.02^{+}	0.002407
Commerzbank – PLN	4	87	8.47^{+}	0.001173
Commerzbank – ZAR	2	104	9.81^{+}	0.003034
Ford – CZK	58	90	2.74^{+}	0.000476
Ford – DKK	37	95	2.15^{+}	0.001415
Ford – NOK	0	104	10.20^{+}	0.003087
General Electric – DKK	28	78	2.49^{+}	0.006636
General Electric – USD	57	79	3.94^{+}	0.000275
General Electric – ZAR	63	77	5.58^{+}	0.000532
HVB – DKK	84	84	9.17^{+}	-0.001718
HVB – ZAR	59	79	5.74^{+}	0.000688
LB Rheinland-Pfalz - NOK	3	102	9.51^{+}	0.002246
LB Rheinland-Pfalz – PLN	106	107	10.15^{+}	-0.002079
Rabobank – USD	78	85	7.70^{+}	0.003493

Table 7: Results of the sign test for each company/currency; $^{+}$significant at the 5% level

corporation	# significant parameters	#all parameters	% of significant parameters	test statistic
All	1461	2316	63.1%	128.25^{+}
Deutsche Bahn – DKK	104	104	100.0%	44.45^{+}
Deutsche Bahn – NOK	99	99	100.0%	43.37^{+}
Deutsche Bahn – SEK	100	100	100.0%	43.59^{+}
Electricite de France - USD	19	75	25.3%	8.08^{+}
Fiat - GBP	42	42	100.0%	28.25^{+}
ING Bank - DKK	104	104	100.0%	44.45^{+}
L. Rentenbank – GBP	20	76	26.3%	8.53^{+}
L. Rentenbank – USD	21	81	25.9%	8.64^{+}
SNS Bank – CZK	0	88	0.0%	-2.15
SNS Bank – HUF	40	76	52.6%	19.05^{+}
SNS Bank – PLN	52	90	57.8%	22.97^{+}
Bank Austria - USD	17	38	44.7%	11.24^{+}
Coca Cola - GBP	37	76	48.7%	17.06^{+}
Commerzbank – CZK	49	93	52.7%	21.10^{+}
Commerzbank – PLN	61	87	70.1%	27.87^{+}
Commerzbank – ZAR	97	104	93.3%	41.30^{+}
Ford – CZK	46	90	51.1%	20.07^{+}
Ford – DKK	60	95	63.2%	26.01^{+}
Ford – NOK	97	104	93.3%	41.30^{+}
General Electric – DKK	27	78	34.6%	12.00^{+}
General Electric – USD	31	79	39.2%	13.96^{+}
General Electric – ZAR	36	77	46.8%	16.81^{+}
HVB – DKK	44	84	52.4%	19.92^{+}
HVB – ZAR	20	79	25.3%	8.29^{+}
LB Rheinland-Pfalz - NOK	99	102	97.1%	42.66^{+}
LB Rheinland-Pfalz – PLN	105	107	98.1%	44.20^{+}
Rabobank – USD	34	85	40.0%	14.81^{+}

Table 8: Results of the z-test for each company/currency; $^{+}$significant at the 5% level