The European corporate bond market and debt portfolio losses in a reduced-form factor model*

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Abstract

An accurate model for losses incurred by defaults of credit-risky securities is valuable information for the risk analysis of defaultable debt portfolios and for the pricing of derivative instruments that make portfolio default risk tradable, such as basket products or credit risk securitizations. Information about the market's assessment of the occurrence of individual and multiple default events is to a certain amount processed in the prices of defaultable securities.

The paper establishes the link between a market where default risk is traded and losses in defaultable debt portfolios. A factor model of the affine-yield class is specified, taking into account the effects of an economy-wide risk factor both on observed default risk premia and losses in a portfolio context. Model estimation is performed on the basis of the implied zero bond spreads of European corporate bonds.

Probability distributions of losses in defaultable debt portfolios are examined. The specified default mechanisms are in line with the estimated factor model, incorporating additional diversification among sectors. The risk analysis provides the basis for the pricing of derivatives referring to losses in debt portfolios, for which valuation results are obtained.

We find that the model specification assuming an economy-wide risk factor yields a good explanation of the joint evolution of default risk premia observed in the bond market. Translated to the portfolio context, the impact of the common factor on overall loss variation is high, rendering sector-related diversification benefits rather small.

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1 Introduction

An accurate model for losses from defaults in a portfolio of credit-risky assets is a main concern in several fields:

Firstly, all tasks related to the management of risk in debt portfolios - including the allocation of supervisory capital for default risk from outstanding claims in bank portfolios - require a probabilistic assessment of the losses that may be incurred from defaults.

Secondly, the valuation of contingent claims referencing to a portfolio of defaultable claims, subsequently denoted as a 'reference portfolio', requires a specification of the stochastic behavior of the underlying variable. The contingent claims on default-risky portfolios fall in two categories: basket credit derivatives generate payoffs when a specified number of defaults occurs in the reference portfolio or a certain loss amount is reached. Securitization structures that transfer the risk of defaultable debt portfolios to the market also exhibit credit-derivative features. From a financial engineering perspective, the claims issued in these transactions (so-called collateralized debt obligations, CDOs, within the class of asset-backed securities) can be seen as credit-linked notes (CLNs) that combine characteristics of basket credit default swaps (CDSs) with default-free claims, e.g. coupon bonds.[1] The values of both types of derivative instruments are determined by losses from defaults in the underlying reference portfolio. The valuation of these claims requires both a probabilistic assessments of the loss severity and the timing of default events. Debt portfolio losses are largely determined by the impact of joint credit events that lead to considerable deviations of total losses in a portfolio from the expected loss amount.

Despite the difficulties of measuring credit risk using historical default data, default risk as it is assessed by the markets is observable: reduced-form models in the tradition of Lando (1998) and Duffie and Singleton (1999a) set the fair credit risk yield premium in relation to the expected losses from defaults. The fair credit risk premia are expected to incorporate systematic credit risk factors that govern the dependencies among defaults of different claims in a debt portfolio leading to joint default events.

The paper establishes the link between a market where default risk is traded and losses in defaultable debt portfolios. A factor model of the affine-yield class is specified, taking into account the effects of an economy-wide risk factor both on observed risk premia and default losses in a portfolio context. The model provides enough flexibility to incorporate the effects of diversification among risk levels and sectors for a given defaultable debt portfolio. Model estimation is performed on the basis of the implied zero bond spot spreads of Euro-denominated bonds issued by European corporates rated investment grade in July 2000, selected on the basis of the issuers' appearances in the reference portfolios in

[1] Duffie and Gârleanu (2001) give a concise description of the constituents of CDOs relevant for valuation and present a simulative risk analysis of CDOs within the intensity framework.

securitization structures. Probability distributions of losses in defaultable debt portfolios are derived that are in line with the estimated factor model. Furthermore, we study a basic securitization example in order to assess the model implications for contingent claims valuation.

The structure of our study is as follows: Building on a short review of the reduced-form framework in section 2, the factor model is outlined in section 3. Besides the specification of dependencies captured by the factors, implications for correlations among observed variables are derived. The model estimation based on observed default risk premia as spreads over maturity-equivalent risk-free rates of European corporate bonds ist outlined in section 4. We describe the method used to infer premia on the basis of zero bond prices from coupon bond prices, estimate the model using a Kalman filter quasi maximum likelihood (KF-QML) method and assess the model's explanatory power, taking into account the explanation of observed correlations among corporate spreads. The translation of the estimated factor model into the defaultable debt portfolio context is performed in section 5. We put particular emphasis on the model extension to incorporate portfolio diversification to several sectors and derive the probability distributions of discounted losses, in the literature frequently referred to as 'loss distributions'. Finally, we derive pricing relationships for contingent claims from default risk securitzations that refer to the examined portfolios. Section 6 concludes.

2 Review of the reduced-form framework

Prices $P_r(t,\tau)$ and corresponding yields $R_r(t,\tau)$ of risk-free zero bonds paying one unit of currency at maturity $T = t + \tau$ with certainty are determined by the expected evolution of the instantaneous ("short") interest rate $f_r(t)$ under a risk-neutral measure

$$P_r(t,\tau) = \exp\left(-E_t\left[\int_t^T f_r(s)ds\right]\right).$$

$$R_r(t,\tau) = -\frac{\ln P_r(t,\tau)}{\tau}.$$

We decompose the yield of a zero bond $R_{corp}(t,\tau)$, that promises the payment of one unit of currency at time T, but may be subject to prior default, into the risk-free yield and the credit spread, i.e. the yield differential that compensates the bondholder for the default risk the corporate discount bond bears:

$$R_{corp}(t,\tau) = R_r(t,\tau) + S_{corp}(t,\tau). \tag{1}$$

Risk premia $S_{corp}(t,\tau)$ for positive times to maturity τ are denoted as spot spreads. The spot spread is determined by the relative price discount of a defaultable bond with price

$P_{corp}(t,\tau)$

$$S_{corp}(t,\tau) = -\frac{\ln\left(\frac{P_{corp}(t,\tau)}{P_r(t,\tau)}\right)}{\tau}.$$

Making the usual assumption of a doubly-stochastic exponential default mechanism, where default events are modelled as first jumps of Poisson-processes with stochastic intensities, we have stated the setup necessary to analyze the dynamics of the term structure of credit spreads $S_{corp}(t,\tau)$ analogously to the term structure of risk-free rates $R_r(t,\tau)$. We define the instantaneous (”short”) spread as a random variable, determined by the instantaneous default probability $h_{corp}(t)$ and the loss rate given default LGD_{corp}

$$s_{corp}(t) = h_{corp}(t)LGD_{corp}. \quad (2)$$

In order to make sure that in case of risky zero bonds, recovery payments do not exceed the pre-default market value of the defaulted claim, we assume that a defaulted zero bond recovers a fraction of its market value at an instant before default has occurred and work in the framework of Duffie and Singleton (1999a). Moody's survey of default behavior of European corporate bonds (Hamilton (2002)) gives an intuition about the scale of the constituents of expected losses from credit defaults: For the period between 1985 and 2001, 0.2% of European corporates having issues outstanding rated investment grade by Moody's defaulted on their obligations. For European senior unsecured bonds, Moody's reports an average recovery rate of 20.8% of the bonds' face values, which is by far lower than in the US with 50%.

Although we have a proxy for the historical LGD of European investment grade corporates, we do not correct for the LGD to obtain default intensities $h_{corp}(t)$. One reason is that we are unsure about the partition of the risk premium for variation in expected loss rates between the two sources of uncertainty, i.e. the stochastic intensity evolution during the time to maturity and the recovery risk due to the uncertain magnitude of the LGD. The other reason is that for a sufficiently large number of default events, expected losses from defaults are not affected by the ratio of intensity and LGD. In the Duffie/Singleton-framework intensity and LGD can be seen as substitutes in pricing defaultable bonds. Translated to a well-diversified debt portfolio, we may treat each defaultable position with positive recovery equivalently to a position with zero recovery and a lower default intensity equal to the short spread without changing the distribution of losses in a portfolio. The short spread may therefore be considered as an instantaneous expected loss rate from default of a zero bond and $-\ln\left(\frac{P_{corp}(t,\tau)}{P_r(t,\tau)}\right)$ as the expected loss as a fraction of the bond's market value for the entire time to maturity.

3 The reduced-form factor model

3.1 Factor specification for defaultable bond pricing and loss forecasting

The reduced-form framework allows for the expected loss rate of a corporate entity $h_{corp}(\mathbf{f}_s(t))$ being driven by several factors, collected in a latent state vector $\mathbf{f}_s(t)$. The specified factors both explain the spot spreads of defaultable corporate claims observed in the market and determine the level and riskiness of the corporate claims in a portfolio.

Both in the market and in the defaultable claims portfolio context, the default risk evolution among several borrowers is expected to be interrelated due to systematic risk factors that affect all corporate borrowers or a subset of them simultaneously. We parametrize a dependence structure as follows, taking into account an economic meaning of the latent factors while keeping the model still tractable for estimation.

We divide the corporate bonds into classes $corp := class = (risk, sec)$. Firstly, a corporate claim is characterized by the level of default riskiness of the borrower. In the bond market, we assume the risk level to be represented by the entity's letter rating at the start of the estimation period. Secondly, we aim to capture the effects caused by industry concentrations in a reference portfolio. One would expect the spot spreads as default risk premia of individual issuers within an industry to display a higher degree of dependence than between issuers of different sectors. The model setup therefore allows for the sector-dependency of the class-specific factor. Due to paucity of data on corporate bonds and fast-increasing model size, however, we cannot identify class-specific factors disaggregated for industries and form classes for model estimation according to the risk level only.[2] For loss forecasting for reference portfolios, however, a breakdown for sector affiliation is performed within diversification scenarios that are in line with the factor model estimated on the basis of a disaggregation according to risk levels only.

Besides the effects related to risk level and sector affiliation, it appears reasonable to allow for a factor affecting all reference claims jointly regardless of their class affiliation such that systematic risk on the macroeconomic level is accounted for.

We impose a simple additive structure on the expected loss rates or short spreads

$$h_{corp}(t) := h_{class}(t) = f_c(t) + f_{class}(t),$$

which implies a uniform expected loss rate evolution of all corporate entities within one risk/sector-class. The factors driving the stochastic expected loss rate evolution may all be considered as 'systematic', as they determine the risk level of several obligors. However, due to the doubly-stochastic property of the default mechanism, the actual occurrence of

[2] Modelling of systematic components requires simultaneous estimation of the respective factor processes. The fast increasing model size thus imposes restrictions on the richness of the model construction.

a default event and the realization of the default time is idiosyncratic for every obligor. Despite the uniformity of intensity evolution within one class, the probability of two claims defaulting simultaneously in the continuous-time limit is zero. With regards to the notion of conditional independence encountered in the literature, as for instance characterized by Frey and McNeil (2001), our model setup implies independence of default events *within* the same class conditional on the realizations of the factors $f_c(t)$ and $f_{class}(t)$ and independence of default events *between* classes conditional on the realization of factor $f_c(t)$.

The latent factors $\mathbf{f}_s(t) = (f_c(t), (f_{class}(t))_{class=(risk,sec)})'$ and $f_r(t)$ evolve stochastically over time. For the purpose of derivative pricing, a factor process specification under a risk-neutral measure is required.

The transitional behavior of every factor $f_k(t)$ is assumed to be described under a risk-neutral measure by the stochastic differential equation

$$df_k(t) = \kappa_k \left(\theta_k - f_k(t)\right) dt + \sigma_k \sqrt{f_k(t)} dW_k(t), \tag{3}$$

which has originally been introduced by Cox, Ingersoll, and Ross (1985) (CIR) for describing the dynamic evolution of the short rate.

The process specification implies both reversion to the long-run risk-neutral factor mean θ_k at an annualized rate κ_k and level-dependent process volatility in combination with a reflecting barrier at zero. This property is desirable for default modelling, as negative intensity realizations as instantaneous default probabilities are precluded. The Wiener processes $W_k(t)$ are assumed to be pairwisely uncorrelated.

To ensure absence of arbitrage, the existence of a probability measure is assumed such that bond prices discounted by a risk-less money market account are martingales.

Model estimation is to be performed on the basis of realizations of spot rates and spreads as they are explained conditional on the latent factor realizations. Spot rates and spreads of positive maturities carry term premia, compensating risk-averse investors for the risk of factor changes. Thus, the factor processes under the physical measure are of interest for estimation. The factor process under the risk-neutral measure (3) is derived from the factor process under the physical measure

$$df_k(t) = \kappa_k^* \left(\theta_k^* - f_k(t)\right) dt + \sigma_k \sqrt{f_k(t)} dW_k^*(t). \tag{4}$$

Specifying the market price of factor risk as proportional to the current factor realization as

$$\Lambda_k = \lambda_k \frac{\sqrt{f_k(t)}}{\sigma_k},$$

the risk-neutral reversion speed κ_k and the risk-neutral unconditional factor mean θ_k are obtained from the real-word parameters as

$$\kappa_k = \kappa_k^* + \lambda_k \tag{5}$$

$$\theta_k = \frac{\theta_k^* \kappa_k^*}{\kappa_k}. \tag{6}$$

Given the factor model specification, defaultable zero bond yields $R_{corp}(t, \tau) := R_{class}(t, \tau)$, as defined by (1) are class-specific and represent default-risky term structures of spot rates for the respective classes.

As the CIR-model belongs to the class of affine yield factor models (Duffie and Kan (1996)), risky zero bond prices are exponential-affine functions in the state variables:

$$P_{class}(t, \tau) = \exp\left(\sum_{k=r,c,class} \left(a_k^{CIR}(\tau, \phi_k) - b_k^{CIR}(\tau, \phi_k) f_k(t)\right)\right). \tag{7}$$

The time-invariant vector ϕ with elements $\phi_k = (\kappa_k^*, \theta_k^*, \sigma_k, \lambda_k)$ collects the parameters driving the unobserved factor dynamics and the market prices of factor risk parameters. The closed form solutions for functions $a_k^{CIR}(0, \phi_k), b_k^{CIR}(0, \phi_k)$ are given in Appendix A.

The exponential-affine characterization of the defaultable bond prices has the desirable implication that risky zero bond spot rates are affine in the underlying factors:

$$R_{class}(t, \tau) = \sum_{k=r,c,class} \left(A_k(\tau, \phi_k) + B_k(\tau, \phi_k) f_k(t)\right), \tag{8}$$

with functions $A_k(\tau, \phi_k) = -\frac{1}{\tau} a_k^{CIR}(\tau, \phi_k)$ and $B_k(\tau, \phi_k) = \frac{1}{\tau} b_k^{CIR}(\tau, \phi_k)$.

Setting

$$R_r(t, \tau) = A_r(\tau, \phi_r) + B_r(\tau, \phi_r) f_r(t),$$

$$C(t, \tau) = A_c(\tau, \phi_c) + B_c(\tau, \phi_c) f_c(t),$$

$$Class(t, \tau) = A_{class}(\tau, \phi_{class}) + B_{class}(\tau, \phi_{class}) f_{class}(t), class = (risk, sec),$$

we make use of the fact that the term structure of risk-free spot rates $R_r(t, \tau)$ is observed separately.

The yield components $C(t,\tau)$, $Class(t,\tau)$ are observed jointly in form of the term structures of spot spreads of the respective corporate classes

$$S_{class}(t,\tau) = C\left(t,\tau\right) + Class\left(t,\tau\right). \tag{9}$$

The subsequent analysis is facilitated by the observability of risk-free zero bond yields $R_r(t,\tau)$. We therefore estimate a term-structure model first using risk-free bond data, and can then estimate the spread model. It is conceivable that part of the observed spreads is due to the presumed lower liquidity of the corporate bond market. The expected loss rate implied in the prices we use for estimation will therefore reflect the investors' utility loss from constrained trading opportunities. Therefore we do not claim that the factors driving the spread dynamics exactly represent expected loss rates from default risk. With respect to the valuation of contingent claims referencing to debt portfolios, however, the bias may be less troublesome: In a no-arbitrage context, the fair values of these products are determined as derivatives, whose payoff distributions are driven by the same factors as risky bond prices. Assuming that with respect to liquidity, the market for the relevant contingent claims is comparable to the corporate bond market, we price these derivatives accordingly to the market from which the factor dynamics have been inferred.

3.2 Dependence in the corporate bond market: term structures of spread correlation

The model is to be calibrated to the first two conditional moments of observed quantities, i.e. to observed zero bond spot rates and spreads. The estimated parameter vector is therefore expected to maximize explanatory power with respect to the time series of observed quantities and their variance. In view of the model application to reference portfolios of defaultable claims, a vital model part is the correlation structure between losses from different reference obligors. However, it is not explicitly calibrated to cross-moments of observable quantities. A reasonable specification check is therefore to enquire wether the model can explain empirical correlations between observed variables, i.e. the spot spreads of obligors of different classes. A correlation structure between spot spreads $S_u\left(t,\tau\right)$ and $S_v\left(t,\tau\right)$ of classes $class = u, v$ is defined by the parameter vectors ϕ_c, ϕ_u and ϕ_v :

The pairwise instantaneous correlations between the short spreads of two classes are determined as

$$corr(s_u(t), s_v(t)) = \frac{\frac{\theta_c^*\sigma_c^2}{2\kappa_c^*}}{\left[\left(\frac{\theta_c^*\sigma_c^2}{2\kappa_c^*} + \frac{\theta_u^*\sigma_u^2}{2\kappa_u^*}\right)\left(\frac{\theta_c^*\sigma_c^2}{2\kappa_c^*} + \frac{\theta_v^*\sigma_v^2}{2\kappa_v^*}\right)\right]^{1/2}} \tag{10}$$

as unconditional factor and short spread moments are

$$
\begin{aligned}
var(f_k(t)) &= \frac{\theta_k^* \sigma_k^2}{2\kappa_k^*} \\
var(s_{class}(t)) &= \frac{\theta_c^* \sigma_c^2}{2\kappa_c^*} + \frac{\theta_{class}^* \sigma_{class}^2}{2\kappa_{class}^*} \\
cov(s_u(t), s_v(t)) &= \frac{\theta_c^* \sigma_c^2}{2\kappa_c^*}.
\end{aligned}
\tag{11}
$$

For spot rates or spreads, respectively, volatilities and correlations are characterized by term structures that are determined by the affine functional relationship between factors and defaultable spot rates.

Note that when working under the risk-neutral measure, i.e. within the context of contingent claims valuation, the unconditional moments given the risk-neutral parameters θ_k and κ_k are of interest. Defined analogously, these differ from the second moments under the physical measure in case of the CIR square-root processes.

The unconditional factor variances translate to the variances of spot rates and spreads as

$$
\begin{aligned}
var(R_r(t,\tau)) &= B_r^2(\tau, \phi_r) \frac{\theta_r^* \sigma_r^2}{2\kappa_r^*} \\
var(S_{class}(t,\tau)) &= B_c^2(\tau, \phi_c) \frac{\theta_c^* \sigma_c^2}{2\kappa_c^*} + B_{class}^2(\tau, \phi_{class}) \frac{\theta_{class}^* \sigma_{class}^2}{2\kappa_{class}^*}.
\end{aligned}
\tag{12}
$$

Due to the maturity-dependence of the unconditional variance of the observable variable, the square-root of this relationship will be referred to as a 'term structure of volatility'.

The instantaneous short spread correlations translate to the correlations between spot spreads of different risk classes as

$$
\begin{aligned}
&corr(S_u(t,\tau), S_v(t,\tau)) \\
&= \frac{B_c^2(\tau, \phi_c) \frac{\theta_c^* \sigma_c^2}{2\kappa_c^*}}{\left[\left(B_c^2(\tau, \phi_c) \frac{\theta_c^* \sigma_c^2}{2\kappa_c^*} + B_u^2(\tau, \phi_u) \frac{\theta_u^* \sigma_u^2}{2\kappa_u^*} \right) \left(B_c^2(\tau, \phi_c) \frac{\theta_c^* \sigma_c^2}{2\kappa_c^*} + B_v^2(\tau, \phi_v) \frac{\theta_v^* \sigma_v^2}{2\kappa_v^*} \right) \right]^{1/2}}.
\end{aligned}
\tag{13}
$$

This relationship will subsequently be denoted as a 'term structure of correlation'.

4 The European corporate bond market in the factor model

4.1 Data description

The default risk to be analyzed shall represent a relevant part of debt portfolios with exposure to European corporates. The bonds constituting the default-risky sample are chosen to mimic a typical reference portfolio as it is traded in securitized form. The selection of bonds on the basis of securitized debt is based on two considerations: Firstly, the mere existence of these products indicates that parties indeed do hold similar portfolios they attempt to hedge by selling portions of the respective default risk in the market. Secondly, our study provides a basis for valuing baskets and CDOs referencing to European corporate credit default risk, which is implemented in the final section.

All bonds are specified as straight bonds paying yearly coupons evenly. Odd first or last coupons are allowed and accounted for in the respective cash-flow series, but sinking fund bonds and bonds with any derivative features such as call or put provisions or rating-sensitive coupons are excluded. With respect to prioritization in case of bankruptcy, the choice is limited to senior unsecured obligations.

Concerning the risk-free interest rate model, we regard the term structure of interest rates of German government debt (Bundesanleihen) to be the relevant term structure $R_r(t, \tau)$ for the Eurozone.

The corporate bond sample consists of 334 bonds issued by 105 firms rated investment grade by Moodys or, subsidiarily, by Standard & Poors at the beginning of the 18 months sample period or obtained an investment grade rating in the first half year of the sample period.

Due to the need for the large number of bonds necessary for the derivation of an implied zero bond spot rate curve, for estimation, the partition of bonds is being restricted to classes according to the risk level only. For implementation, the class indices $class =: risk = 1, 2, 3$ refer to the letter ratings Aaa/Aa, A and Baa, respectively. The composition of classes remains fixed during the estimation period because letter ratings only serve as a classification device according to the issuers' initial risk level. Every class shall reflect the entire variability of the level of riskiness, i.e. the implied spot rates and spreads, respectively, through time. An overview of the bonds' characteristics is given in table 1.

The corporate bond sample, initially fully rated investment grade, encountered significant downgrading, as can be seen from the bonds' migration behavior shown in table 2, but none of the issuers defaulted during the estimation period. In accordance, median yield spreads with respect to Bund coupon bond yields widened about 36 to 49 basis points for different risk classes.

As discussed above, separate estimation for different sectors is not feasible. However, a disaggregation of the migration events in the sample among sectors shows that sector heterogeneity is present with respect to the evolution of riskiness. We use the major level of the FTSE (2002) industry classification scheme: basic industries, consumer goods (cyclical & noncyclical), services (cyclical & noncyclical), financials and utilities.

Quality deterioration has been worst for noncyclical services: the 44 bonds of this sector encountered 50 downgrade events, but no upgrades. This has been due to the distress in the telecommunications sector, which accounts for 38 out of 44 bonds issued by noncyclical services. This is followed by 24 downgrades on bonds issued by mainly banks and few insurance companies (subsumed as the financial sector), that lost their Aa-rating. In the remaining sectors, downgrading activity was less severe.

Pricing information of the corporate sample is obtained from Datastream.[3] All bond specifications are obtained from Bloomberg.

The evolution of the risk-free term structure is estimated from of 56 German Bundesanleihen paying yearly coupons whose prices and specifications were obtained from the German Bond Database at Mannheim University.

In both risk-free and corporate segment, weekly clean prices on Wednesdays from 5/7/00 to 19/12/01 are used, accrued interest is added on an actual/actual basis. Choosing a narrower sampling interval would not be useful due to infrequent trading in corporate bonds.

4.2 Default risk premia in the European corporate bond market

4.2.1 Estimation of implied term structures of risk-free spot rates and corporate spot spreads

Estimation of the factor model requires risk-free spot rates and spot spreads of given maturities. Pricing information, however, is only available through coupon bond data. We therefore make use of the zero bond decomposition that relates the observed corporate and risk-free coupon bond prices to defaultable and risk-free zero bond spot rates. Implied yield curves are derived for each subsample, representing a corporate risk level or the risk-free Bundesanleihen $sub = r, class$ at every observation date. Implied term structures of risk-free and defaultable spot rates of the three specified risk levels, $R_r(t, \tau)$ and $R_{class}(t, \tau)$ and the associated spreads $S_{class}(t, \tau)$ as affine functions of the latent factors are obtained by interpolation.

We use a smoothing procedure proposed by Düllmann and Windfuhr (1999) on coupon bond prices $V_i(t, sub, \tau)$, where i denotes a price observation which may be part of any subsample. The implied term structures represent observations of risk-free or risky rates $R_r(t, \tau)$, or $R_{class}(t, \tau)$, respectively.

The zero bond decomposition assumes that each coupon- or principal payment $CF(\tau)$ is discounted with the appropriate risk-free or risky spot rate $R_{sub}(t, \tau)$ of a zero bond with

[3]The pricing information provided consists of price quotes from traders or exchange prices and not of matrix prices. Redundant price quotes that did not change for one week were dropped. Outliers were identified on the basis of yields to maturity and some prices with abnormal deviations that were apparently not due to rating changes were dropped.

maturity τ applying to the respective risk-free or corporate subsample

$$V_i(t, sub, \tau) = \sum_{\tau=\tau_1}^{\tau_N} e^{-R_{sub}(t,\tau)\tau} \cdot CF(\tau) + \varepsilon_i(t, \tau). \tag{14}$$

$\varepsilon_i(t, \tau)$ denotes the observation error of an individual coupon bond price when the bond is priced at the appropriate term structure of discount rates $R_{sub}(t, \tau)$. Prices $V_i(t, sub, \tau)$ are therefore explained by a nonlinear regression, regressors being represented by the coupon- or principal payment dates, collected in the vector $\tau = (\tau_{1,...,}\tau_N)$. In order to derive a smooth term structure, a parametrization $R_{sub}(t, \tau) = R_{sub}(\beta_{sub}(t), \tau)$ of the yield/maturity relationship is required, where the entries of the parameter vector $\beta_{sub}(t)$ are specific for every week t and every subsample $sub = r, class$.

Due to the nonlinearity of the estimation problem caused by the large number of payment dates of long-maturity bonds, we use the simple functional form for zero bond yields proposed by Nelson and Siegel (1987)

$$R_{sub}(\beta_{sub}(t), \tau) = \beta^0_{sub}(t) + \left(\beta^1_{sub}(t) + \beta^2_{sub}(t)\right) \cdot \frac{1 - \exp(-\frac{\tau}{\beta^3_{sub}(t)})}{\frac{\tau}{\beta^3_{sub}(t)}} - \beta^2_{sub}(t) \cdot \exp(-\frac{\tau}{\beta^3_{sub}(t)}). \tag{15}$$

As only a small number of very short term bonds are available, the estimation tends to be unstable for short maturities due to the low impact of coupon payments on the objective function (see below). We therefore fix the intercepts $\beta^0_{sub}(t) + \beta^1_{sub}(t)$, i.e. the short rates, as the median yields of bonds with remaining maturities of less than one year.

For implementation, the cash-flow weights $CF(\tau)$ are individually determined by the contractual coupon size of every risk-free or corporate bond, including odd first and last coupons in the corporate sector. The shortening of maturities of all coupon and principal payments over time is accounted for.

Imposing the short rate restriction to hold, the free parameters $\beta^0_{sub}(t), \beta^2_{sub}(t), \beta^3_{sub}(t)$ are estimated by solving nonlinear least squares problems of the form

$$\hat{\beta}_{sub}(t) = \arg\min \sum_{i \in sub} \left[V_i(t, sub, \tau) - \widehat{V}_i(t, sub, \tau)\right]^2$$

for every subsample $sub = r, class$ at every observation date and $\widehat{V}_i(t, sub, \tau)$ priced at the relevant Nelson/Siegel term structure.

The obtained estimates $\hat{\beta}_{sub}(t)$ characterize the joint evolution of the implied term structures during the estimation period such that risk-free and corporate zero bond spot rates are generated by the parametric form (15).

4.2.2 Statistical analysis of risk-free spot rates and corporate spot spreads

A short statistical analysis of cross-sectional and time-series properties of implied risk-free spot rates and corporate spot spreads of the respective risk levels indicates wether the previously specified multi-factor reduced-form model with the additive factor restructure and the CIR-transition behavior complies with the observed spread behavior.

Figure 1 shows the evolution of the implied spot rates $R_r(t,\tau)$ and $R_{class}(t,\tau)$ as represented by the respective term structures for the $\tau = 2, 4, 6, 8, 10$ maturity classes. The corresponding spot spread evolution $S_{class}(t,\tau)$ for the corporate classes has been calculated preliminarily using the fitted yield curves. As expected, the spreads of all classes are almost always positive and increasing with the risk level of the class for most observation dates. The evolution reflects the deterioration of credit quality and the increase in yield spreads of coupon bonds during the estimation period.

A summary of the time series properties of the implied risk-free spot rates and spot spreads is given in table 3. Mean spot spreads of all maturities are increasing with the risk level of the class. The entire term structure of the risk class two spot spreads $S_2(t,\tau)$ however, is more volatile than the term structure of class three $S_3(t,\tau)$.

Unlike risk-free rates, spot spreads are significantly skewed to the right and the skewness decreases with maturity. This is incurred by the moderate variation at low levels during the first year of the sample period 2001.

Term spreads for every term structure $R_r(t,\tau)$ and $S_{class}(t,\tau)$ are calculated as the difference between the two year spot rate or spread and its ten year counterpart. The risk-free term structure and the term structures of spot spreads of classes one and two are upward sloping in about 95% of the observation dates and in 90% for class three. Setting the slopes in relation to the levels of the repsective term structures reveals that relative term spreads of the corporate credit spread curves exceed those of the risk-free term structure by far. Corporate spreads of long maturities therefore carry large term premia. Furthermore, long term rates and spreads are less volatile than short term rates and spreads, which is in line with a time series behavior characterized by mean reversion. Diminishing autocorrelation functions of all series indicate the absence of unit roots. Together with the autocorrelational pattern of first differences, they give evidence of moderate mean reversion of spot rates and spreads of all classes.

The implied term structures of spot spreads display properties that are in line with the implications of a CIR-term structure model for spot spreads as affine functions of the latent factors: in the CIR model, both factors and spot rates and spreads are noncentrally-χ^2-distributed.[4] The implied nonnegativity of spot spreads and their skewness to the right

[4] A linear combination of independent noncentrally χ^2-distributed random variables is noncentrally χ^2-distributed, see Jamshidian (1996).

incurred by the possible level-dependence of volatility are fully reflected by the implied term structures of spot spreads, which makes their transitional behavior indeed a promising candidate to be explained by the proposed CIR-model.

The presumed presence of a common factor affecting all spreads, which has been specified in the factor model governing expected loss rates in a portfolio of defaultable claims can be identified in the comovements of spot spreads between the corporate classes.

A comparison of the spot spreads' term structures in table 4 reveals the considerable positive correlation between spot spreads $S_{class}(t, \tau)$ of different risk levels. The strength of the linear relationship is measured by OLS. For regressions between spot spreads of the same maturity class, we choose the time series of the higher risk level to be the dependent variable. For the comparison of risk-free rates with spreads, we choose the spot spread of the corresponding maturity to be the dependent variable. Correlation between spot spreads is strongest for short maturities and is also present in first differences of the fitted series, as shown below.

The sharp increase of all spreads in autumn 2001 affects mainly observed correlation between spreads of short maturities. Correlations between the two year spreads decrease considerably when only a shorter sample period ending on 5/9/2001 is considered.

Furthermore, all spreads exhibit negative correlation with risk-free rates, which is both in line with theory, as explained by Longstaff and Schwartz (1995) in the context of a structural model in the Merton (1974) framework and with existing empirical evidence in the US corporate bond market, as for instance, has been shown by Longstaff and Schwartz (1995), Duffee (1999) and Duffee (1998). The relationship turns out to be strong, as spreads, the dependent variables, are considerably lower than risk-free rates.

4.2.3 Error analysis: Residual effects of liquidity proxies and sector affiliation

Table 5 gives summary statistics on the pricing errors $\varepsilon_i(t, \tau) = V_i(t, sub, \tau) - \hat{V}_i(t, sub, \tau)$ of individual coupon bonds priced at the implied term structures of risk-free and default-risky sport rates, broken down for the subsamples $sub = r, class$. To get an idea about the scale to which estimated pricing errors $\widehat{\varepsilon_i}(t, \tau)$ translate to differences on the return level, we calculate, as explained in the annotations of table 5, for each subsample a "median" yield error based on the median characteristics with respect to coupon, maturity and yield to maturity of the bonds contained in the subsample, as displayed in the last column of table 1. Principals are normalized to 100.

The location parameters indicate that the Nelson-/Siegel-interpolation underprices both German Bundesanleihen and European corporate bonds. However, the level of the pricing errors is small in all four subsamples: Converting median pricing errors to median yield errors reveals that the median yield error is smaller than ten basis points for every subsample. For all subsamples, it is in the same order of magnitude. Therefore, the small biases should not affect an analysis of spot spreads as the differences between risky and risk-free rates.

The pricing errors of corporate bonds are widely dispersed. This is not necessarily entirely due to individual differences in credit risk but to market imperfections, such as individual liquidity differences due to infrequent trading in corporate bonds and the sample heterogeneity.

The pricing errors are analyzed with regard to two objectives: Firstly, possible liquidity differences within the corporate bond market shall be identified. Secondly, the factor model for credit portfolio risk is flexible enough to incorporate sector-specific differences in default risk. Although the data basis permits no estimation for several sectors - the derivation of implied spot spreads requires a large number of bond prices - sector-specific effects are expected to become evident in the residuals.

As liquidity-related information such as trading volumes or bid/ask-spreads is not available for the mainly OTC-traded corporate bonds, we introduce three variables that may convey information about the liquidity of a specific price observation.[5] Assuming that large issues are liquid and the liquidity of an issue decreases with time, a positive relationship is expected between the pricing error and the issue size and a negative one between the residuals and the age of an issue. Furthermore, it is conceivable that the amount of new pricing information obtained in the prices' time-series is related to the trading intervals, a liquidity-related feature. We therefore include the number of price changes observed, normalized by the number of time-series observations. The sample means of the liquidity-related independent variables are EUR 546 Mio. (issue size), 2.73 years (age) and 0.893 (price changes per week). The time to maturity is included to quantify a possible pricing bias incurred by the simple functional for of the Nelson/Siegel interpolation.

With respect to sector-specific differences conveyed by the residuals, we hypothesize that industries considered as cyclical generally bear larger default risk than noncyclical ones. This can be motivated in a firm value setup in the tradition of Merton (1974) by the put option character of debt, whose value decreases with the presumed higher volatility of the firm value of cyclical businesses. Empirically, this hypothesis has been examined by Longstaff and Schwartz (1995) for an US corporate sample, who find a positive relationship between the spread's sensitivity on industry-specific stock index changes and the spread level. Furthermore, according to the severe observed downgrade activity in the noncyclical services (mainly represented by telecommunication companies) and financial sectors during the sample period, we expect a considerable price discount on these bonds,

[5] Similar proxies have been studied in the literature, e.g. Sarig and Warga (1989) examine the explanation of liquidity discounts by age of the issue, amount outstanding and the magnitude of bid/ask-spreads in the risk-free sector.

as the subsequent downgrades are obviously not reflected in the initial letter rating. The basic-industries-dummy is included in the constant.

The residual regression is given in table 6, broken down for the various risk levels. Neither of the presumed liquidity-related variables has the expected effects throughout all risk levels - the small effects are not even significant despite the large sample size and the average weekly price changes do not display the expected sign. For the European corporate bond sample at hand, we therefore reject any explanatory power of the liquidity proxies used in the literature.

The remaining effect of unexplained time to maturity on errors is insignificant at the one-percent level and negligible when converted into yields, which indicates that the term-structure relevant information contained in coupon bond prices is adequately captured by the Nelson/Siegel curves.

The regression of residuals on sector dummies gives clear evidence of sector-specific price differences.

As expected from the observed rating deterioration, the bonds issued by noncyclical services are significantly overpriced by the implied risky yield curves that reflect the mean spot rate evolution for the respective risk classes. This contradicts the hypothesis that industries considered as cyclical are generally riskier and therefore should trade at a price discount, but the result is in the present sample due to the telecommunications quality deterioration. In the consumption goods sector, we observe that cyclical businesses of class one do indeed trade at a discount, which does not occur in conjunction with downgrade activity from Aa to A. However, based on our sample data, we must reject the hypothesis of discounts on cyclical businesses. However, a sector-specific pricing difference is evident between consumption good producers and service providers.

It is surprising that in spite of the considerable downgrades in the financial industry, on average, bonds still trade at a price premium on all other bonds except utilities and that the discount in class one, in which all downgrades occur, is small compared to the corresponding discounts in the services and consumer goods industries.

A conversion of the regression intercepts explained by sector dummies reveals that the sector-specific differences are substantial: Considering a bond paying a coupon of six percent and five years to maturity priced at par, the largest average pricing difference taking the entire sample into account of 2.60% between services and utilities translates to a difference in yield to maturity of 60 bps. The analysis shows that the introduction of sector-specific factors into the valuation model is justified by the pricing differences observed in the corporate bond market.

4.3 Factor process estimation

4.3.1 Outline of estimation procedure

Estimation is facilitated by the fact that risk-free spot rates and corporate spot spreads are observed separately, which permits the separate estimation of a standard risk-free term-structure model and the estimation of the model part explaining corporate spot spreads.

The factors explaining implied risk-free rates and corporate spreads are unobserved. Factors related to default risk cannot be replaced by a proxy variable, as opposed to the risk-free short rate, that can be approximated by a short-term money market rate as used by Chan, Karolyi, Longstaff, and Saunders (1992) and examined by Chapman, Long, and Pearson (1999). Furthermore, the observed spot spreads are sums of two affine functions of the latent factors $f_c(t)$ and $f_{class}(t)$. The information set therefore requires the use of an estimation method that implies the unobserved factor evolution from observed spot spreads.

For the problem at hand, an inversion of observed term structures following Pearson and Sun (1994) in combination with exact maximum likelihood estimation of the parameter vector ϕ based on the true transition densities of the CIR-processes would be preferable, as all distributional properties would be respected during estimation. The large model size and the dependence of every spread observation on two factors, however, appear to render an exact inversion of the term structures infeasible.[6]

Instead, we use a Kalman filter quasi maximum likelihood (KF-QML) method based on the prediction errors in the cross section of observed risk-free spot rates and corporate spot spreads of the three risk levels. The KF makes a linear projection of observed spot spreads on factors, which is computationally less intensive. The implemented KF-QML-method has been used by Duan and Simonato (1995) on non-Gaussian interest rate models and Duffee (1999) for yield spreads of corporate bonds.

For an outline of the KF-algorithm we give the standard reference (Harvey (1989)). The KF-algorithm is based on a state-space formulation of the latent factor model. This takes into account both the time-series dimension in form of a 'transition equation', which is determined by the factor processes under the physical measure, and the cross-sectional dimension in form of a 'measurement equation', which is given by the terms structures of spot rates and spreads, represented by the 2,4,6,8 and 10 year constant maturity classes. In order to provide consistency with the literature on filtering, these implied yields and

[6] The large computational burden is due to the fact that the numerical maximization algorithm would require one inversion step at every observation period for every trial parameter vector. The same choice has for instance been made by Liu, Longstaff, and Mandell (2000) in the case of a factor model assuming Gaussian factor transitions (Vasicek (1977)).

spreads are referred to as 'observables'. Observables are treated as 'noisy measurements' of the model-implied affine functions of the factors. Noise covariances $\mathbf{M}_r$ and $\mathbf{M}_s$ for the two estimation problems are imposed to be diagonal, where the variances of measurement noise are expected to be small due to the smoothness of the interpolation function.

Appendix B documents the state-space formulation on whose basis parameter estimation is performed.

The KF-recursion requires the information provided by first two conditional moments of the factor transitional distribution. In case of a Gaussian model, this captures the full distributional information on factor transitions. In the non-Gaussian case at hand, the linearized projection of observations on factors ignores the higher moments of the noncentral χ^2 distributions, whereas the level dependence of the factor variances is taken into account. The joint density of observation errors is the approximated by a quasi likelihood, where the inherent deviation from the true likelihood is incurred by the inference about the factor realization.

4.3.2 Results on factor processes

Parameter estimates for the multi-factor CIR-model and the standard deviations of unexplained i.i.d. measurement noise of observable spot rates and spreads $\sqrt{m_r(\tau)}$ and $\sqrt{m_{class}(\tau)}$ are displayed in table 7.

Heteroscedasticity-consistent estimators of asymptotic standard errors (White (1980)), whose computation is outlined in appendix B, are given in parentheses.

The parameter estimates determining the short spread evolution for the three corporate classes considered vary qualitatively from the parameters characterizing the risk-free short-rate process, which is due to the relatively low level of observed spot spreads in combination with their volatile time series.

The factor processes explaining the observed spread evolution of the three risk levels under the physical measure imply short spreads that are both increasing in unconditional expected value θ_k^* and volatility with the risk level of the class. Mean reversion is present both in the short rate process and in the factor processes explaining the corporate spreads. The reversion speed κ_k^* under the physical measure is considerably higher for all corporate spread factors. The strength of mean reversion, however, still involves high unconditional process volatilities for all factors, as the volatility coefficients σ_k are high for all spread factors. As a result, the unconditional process standard deviation is one third of the unconditional mean for the short rate process, but twice the mean level for all corporate spread factors.

The corporate spreads carry high term premia, compensating risk-averse individuals who invest in corporate securities with long maturities for the risk of factor change. The estimates for the market price of risk parameters λ_k are strongly negative in the corporate spread segment. This result has been expected due to the steepness of the implied term structures. It implies that the risk-neutral factor processes specified under the equivalent martingale measure relevant for the contingent claims valuation differ considerably from their counterparts under the physical measure. Unconditional means of all factors driving the short spreads of the classes increase pronouncedly with the measure transformation. Furthermore, the unconditional process variances, which determine the degree of concentration risk in a reference portfolio, change significantly in relative magnitude. The variance of the common spread factor, i.e. the covariance between the risk-neutral short spreads of the classes in a reference portfolio, experiences a larger increase than the class-specific factor variances, which implies in increase of dependence in default riskiness of the claims.

The risk-neutral factor mean of class two θ_2 as implied by the parameter estimates exceeds its class-three counterpart by three basispoints, which contradicts the assumed higher risk level of class three claims. It will be outlined in the following section that this is due to the good explanation of volatilities of observed spot spreads, as summarized in table 3. As the entire term structure of spot spreads of class two is more volatile than the term structure of class three, a high unconditional volatility of the risk-neutral process of the class-specific factor two is obtained, as expected. Equation (11) applied to risk-neutral parameters shows that this requires either a higher volatility parameter σ_2, which is not the case, or a higher risk-neutral mean θ_2, which has been obtained.

For the application of the factor model in a valuation example, the risk-neutral mean of factor $f_2(t)$ is reset equal to $(\theta_1 + \theta_3)/2$, leaving all other parameters unchanged in order to obtain expected default losses meaningfully related to the level of riskiness of the claims.

4.3.3 Explanatory power of the model: individual term structures and term structures of spread correlation

The residual statistics summarized in table 8 permit an assessment of the explanatory power of the factor model with respect to the cross-sectional dimension of observables, as it is represented by the individual term structures of risk-free spot rates and corporate spreads and of their observed evolution through time.

For all observables, fitting errors and, correspondingly, the estimated standard deviations of observations noise as depicted in the lower half of table 7 are particularly small for medium maturities: risk-free dynamics are almost exactly matched for the six year rate and mispricings at both ends of the maturity range are not severe. With repect to spot rates, mispricings occur for both short and long maturities: The CIR functional form allows for less curvature than the Nelson/Siegel curves for spreads, overestimating spreads of short and long maturities, where mean errors at the long ends are still smaller than three basispoints, but considerably higher at the short ends of the spread curves.

With respect to time series behavior, the explanatory power is similar: judging from the autocorrelational pattern of residuals shown in table 8, the factor model explains a considerable part of serial correlation of those observed spot rates and spreads whose levels are well explained by the model. In case of risk-free rates, the slow decay of residual autocorrelation in combination with the maturity-dependence of fitting errors gives a hint on the presence of a second factor related to the shape of the term structure not captured by the single-factor CIR model.

The model has been estimated using the information provided by the first two conditional moments of the error structure generated by the KF-procedure, which itself relies on the first two moments of observed quantities, i.e. spot rates and spreads.

An adequate model can therefore be expected to explain the unconditional observed moments of the observable variables as well. The fitting ability of the unconditional mean is captured by the residual means, as discussed above. The second unconditional moments of the observables is represented by the term structures of volatilities, as defined in equation (12).

Figure 2 compares the term structure of observed volatilities, i.e. the standard deviations of spot rates and spreads as reported in table 3, and the term structure of volatility that would by forecasted by the factor model. Whereas the risk-free model underestimates observed systematic spot rate volatility, the three term structures of systematic spread volatilities are remarkably well explained by the two factors affecting every class. However, as discussed above, this comes at the cost of the inconsistency in risk-neutral factor means of classes two and three.

The residual and volatility analysis shows that both individual cross-sectional observations and their volatility and time series behavior of medium term spot spreads are almost completely explained by the factor model.

In the debt portfolio context, the model's ability to provide forecasts of joint expected losses that are in line with the observed joint evolution of the three classes of spot spreads is of relevance.

The model has not explicitly been calibrated to cross-moments of spot spreads of different classes. A comparison of the model-implied term structures of correlation, as have been defined in equation (14), with the empirically observed dependence gives insight about the explanation of the dependencies in the corporate spreads.

Figure 3 shows that for all term structures of spreads, the observed correlation is matched for the eight years maturity class. Pairwise intantaneous correlations among the three risk classes, as defined in (10), are between 0.3 and 0.6 and decreasing for classes bearing higher default risk, which is due to the higher unconditional process volatility of riskier class-specific factors.

The empirically observed decrease in dependence with maturity is not reflected by the model-implied correlation structure. Furthermore, the within-sample correlations between estimated class-specific factor series, as displayed in table 9, are not always near zero. The estimated factor series of the sample period displayed in figure 4 reveal that the strong positive correlation between the class-specific factors of classes two and three is due to the inability to explain the strong spread increase of all three risky term structures only by dependence on one additive common factor.

In order to assess the severity of this problem, we firstly determine the observed term structure of correlation for a shorter period ending on 5/9/01, which differs substantially from its full period counterpart. It is plotted in figure 3 and corresponds by far better to the theoretical dependence. The observed very high correlation between spot spreads of short maturities is entirely induced by a strong reaction to the events in September 2001. Secondly, the spread model is re-estimated for the shorter period displaying more homogeneous time series behavior. Obtained parameter estimates are rather robust against the omission of the 'extreme' period, with the relevant exception that the class-specific factor $f_3(t)$ is less volatile. This is plausible given the strong increase of all class-three-spreads in autumn 2001. These findings have the desirable implication that the estimation procedure and the assumed model type are robust to short term strong deviations from "mean" time series behavior, even for a sample period as short as one year. The drawback, however, is that the simple affine structure cannot capture both transitional and cross sectional properties during the "extreme" period whilst preserving factor uncorrelatedness by a suitable adjustment of parameter estimates.

A final analysis of residual correlation, as displayed in table 10, should reveal dependence between spot spreads that is neither explained by the factor model, nor captured in the comovements of factors. Low residual correlation between the six year spot spreads of classes one and two as well as classes one and three suggests that the observed correlation is well explained by the model for medium maturities of the respective term structures, as correlations among factor series are weak. Concerning the residual correlation among the riskier classes two and three, both strong correlation between factor series and the out-of-sample prediction still leave considerable correlation between medium maturity spreads unexplained.

5 Portfolio losses and the pricing of contingent claims in the factor model

5.1 The distribution of portfolio losses

Losses in portfolios of defaultable claims serve as the underlying variables for credit risk securitzations, subsumed as collateralized debt obligations (CDOs). The heart of standard securitization structures is the pooling of credit risk exposure of several entities and the issuance of credit-linked notes (CLNs). Credit-linked notes combine standard fixed-income cash-flows with derivative features related to the default risk in the reference portfolio. The arbitrage-free values of these claims depend not only on the total loss amount in the reference portfolio, but also on the timing of default events. In order to assess the implications of the factor model for losses in European investment-grade defaultable debt portfolios, we derive the probability distribution of discounted losses from defaults, often referred to as the 'loss distribution', for a given time horizon for reference portfolios that vary with respect to diversification among sectors. We then study a basic securitization example and determine the arbitrage-free spreads of the issued CLNs as derivatives on the same factors that affect corporate bond prices.

The partition of reference claims between risk levels is set equal to the dispersion among agency ratings in investment-grade securitizations that have provided the basis for the selection of bonds for model estimation. The letter ratings are rather homogeneously among all transactions considered, which yields the partition

#reference claims (risk=1): 10%

#reference claims (risk=2): 50%

#reference claims (risk=3): 40%.

In order to assess the impact of sector diversification on the loss distribution and the prices of contingent claims, we examine three benchmark cases that are in line with the estimated factor dynamics:

minimal Reference claims are diversified across risk levels only. As for the estimation setup, we set $class = risk, risk = 1, 2, 3$. The default risk in the portfolio is driven by four factors, determining three expected loss rates that are identical for all claims within one risk class.

sectors Reference claims are characterized be their risk level and sector affiliation, where the relevance of the latter has become evident within the residual analysis performed on coupon bonds. We set $class = (risk, sec), risk = 1, 2, 3, sec = 1, 2, 3$, assuming a homogeneous dispersion of sector within one risk level. The default risk in the portfolio is driven by ten factors, determining nine expected loss rates.

maximal The expected loss rates of individual reference claims are driven by a borrower-specific factor. This is implemented as the limiting case where every reference borrower represents his own sector, i.e. $class = (risk, sec), risk = 1, 2, 3, sec = 1, ...,$#reference claims. The default risk in the portfolio is driven by (#reference claims+1) factors, the number of expected loss rates is equal to the number of reference claims.

In the scenarios *sectors* and *maximal*, claims of different classes may have the same risk level, i.e. identical unconditional expected loss rates. We therefore assume that class-specific factors of the same risk level but different sectors are identically independently distributed processes.

Risk-neutral expected loss rates between classes are interrelated by the common factor $f_c(t)$. The pairwise correlations between expected loss rates caused by the common factor determine the entire dependence structure characterizing the dispersion of reference portfolio losses.[7]

The relevant correlations, as obtained by equation (10) applied to risk-neutral parameters κ_k , θ_k , are depicted in table 11.

Correlations between claims characterized by the same risk level, but of different sector affiliation are displayed on the main diagonal. The off-diagonal correlations apply to obligors of different risk levels. The lower half of the table depicts the pairwise correlation coefficients for the adjusted parameter θ_2 to be subsequently used for the valuation example.

All expected loss correlations between classes are high due to the high proportion of the risk-neutral common factor variance on the total variances. The overall decrease of correlation is an implication of the factor variance properties in the CIR-model: As factor variances depend on the mean factor level of the class considered, class-specific variance components are increasing in the risk-level of the class, reducing pairwise loss correlation for riskier claims.

The three reference portfolios consist of 100 claims with principals normalized to one. Interest rate risk is precluded, as CDO transactions normally attempt a perfect interest hedge. Risk-free interest rates are therefore assumed to be constant, the risk-free term structure is determined by the one-factor CIR-model evaluated at the risk-neutral mean θ_r.

Defaultable claims are straight coupon bonds that mature simultaneously at the end of the horizon, which is assumed to be five years. Claims may be subject to prior default, in which case a loss of the defaulted claim's current market value occurs. Market values

[7] This holds for every maturity, as term premia do not exist in the risk-neutral valuation setup.

are determined by the affine pricing relationship (7) holding for each coupon or principal payment outstanding.

Consistently with the usances in the credit derivatives market, coupons are paid quarterly such that the (discrete) impact of defaulted coupon payments on losses is kept small. Annualized coupon rates are set such that defaultable claims initially trade at par, i. e. $c_r = 5.476\%, c_1 = 6.683\%, c_2 = 6.693\%, c_3 = 7.190\%$.

Defaults occur at the first jump times of a vector of dependent Poisson processes with stochastic intensities $h_{class}(t)$. At the instant of default, the loss amount corresponds to the entire current market value, i.e. LGD=1, such that intensities equal expected loss rates. Individual default times are obtained by compensator simulation, as suggested by Lando (1998) and Duffie and Singleton (1999b)

$$\Upsilon_{corp} = \inf\left[t : \Lambda_{class}(t) \geq E_{corp}[1]\right],$$

where $\Lambda_{class}(t) = t \cdot \Delta t \sum_{s=1}^{t} h_{corp}(s)$ represents the default compensator and $E_{corp}[1]$ the claim-specific default barrier, an exponential variable with unit expectation.

The CIR-square-root processes governing the factor processes imply future factor levels at time $t + \Delta t$ conditional on the factor realizations at a given time t to be functions of noncentrally χ^2-distributed variables, with noncentrality parameters and degrees of freedom depending on the current factor realization, the interval length Δt and the respective parameter vector ϕ_k. The conditional factor probability distribution functions and the algorithm used for generation of random factor path realizations are given in appendix C.[8] The uniform variates determining the exponentially-distributed default barriers $E_{corp}[1]$ are simulated as antithetic variables, 10000 replications are simulated.

The empirical distribution functions of the discounted losses in the three benchmark portfolios are characterized by the statistics given in table 12, the corresponding number of default events are given in parentheses. Quantiles indicate the probability mass applicable to low and extreme losses, range statistics are given as robust measures of dispersion. As has been expected from the high correlations between risk-neutral expected loss rates implied by the parameter estimates, the impact of sector diversification on loss variation is comparatively small. The diversification effect is largely realized by the introduction of three sectors, the scenarios *sectors* and *maximal* differ with respect to the loss in market value given default only.

[8] We must assert that discretization of the processes using the normal approximation with level-dependent factor variance suggested by the Kalman-filter based estimation procedure outlined in Appendix B leads to very poor convergence of default rates due to the undesirable behavior at zero, even if negative factor realizations are allowed to contribute to the compensator $\Lambda(t)$.

5.2 A contingent claims valuation example

Given the default event times Υ_{corp} and the related loss amounts, the risk-neutral model can straightforwardly be used for the pricing of derivatives whose payoffs are determined by losses from defaults in the reference portfolio. We therefore study the price discovery for derivatives referring to specific loss ranges of the underlying reference portfolio.

Derivatives on portfolio losses exist in the form of basket derivatives and default risk securitizations (CDOs). Whereas baskets normally refer to a portfolio consisting of a small number of obligors, securitization portfolios are diversified among a large number of individual entities. In the case for baskets, discretization effects due to indivisibility of losses are expected to have an impact on prices of loss derivatives. In the case of securitizations, however, diversification is normally sufficient to render the impact of indivisibility acceptably small, which is of particular relevance when working with expected loss rates. The assumption of a portfolio of 100 single names is expected to realize most of the diversification potential.

Economically, the simplest type of a CDO can be considered as a basket default swap on a large reference portfolio bearing losses in specified ranges.[9] Institutionally, baskets are written in form of CDSs, whereas tranches from CDOs are written both as CDSs and CLNs, depending on the requirement for exclusion of counterparty risk. For valuation purposes, both derivative types are equivalent: A CLN is replicated by a CDS and a risk-free interest claim on the principal amount serving as basis for loss determination of the CDS.

Tranches are characterized by their loss thresholds $\underline{K}_{tr}$, i.e. the aggregate loss amount in the reference portfolio from which the tranche is affected, and their principals at initialization $P_{tr}(0)$, both of which determine the loss range to be covered.

According to CDO terms, we assume that premia are paid quarterly in arrears, where premia claims are calculated on the basis of the principal of the respective tranche outstanding at the beginning of the period $P_{tr}(t-1)$. In case of loss occurrence in the reference portfolio affecting the respective tranche, the loss amount $loss(t)$ is paid by the tranche protection seller and the principal outstanding is reduced accordingly:[10]

[9] The equivalency holds for synthetic CDOs, i.e. structures where the reference portfolio remains on the issuer's balance sheet and a sale of defaultable claims does not take place. Default protection is instead acquired through tranched derivatives (CDSs/CLNs) as described. The payment obligations under the derivatives are direct obligations of the issuer and therefore not limited to proceeds generated by the reference portfolio. When dealing with true-sale structures, however, the funds available for payments to CDO tranche investors are limited to the cash-flow generated by the reference portfolio. Thus, the 'clean break' from the originator generates cash-flow constraints that may become binding.

[10] $loss(t)$ is reduced by the loss already borne by the directly subordinate tranche in case of period losses affecting two tranches.

$$\begin{aligned} P_{tr}(t) &= \mathbf{1}_{\left\{\sum_{s=1}^{t} loss(s) < \underline{K}_{tr}\right\}} \cdot P_{tr}(0) \\ &\quad + \mathbf{1}_{\left\{\sum_{s=1}^{t} loss(s) > \underline{K}_{tr}\right\}} Max\left[P_{tr}(t-1) - loss(t), 0\right]. \end{aligned}$$

The valuation problem for tranched CDS consists in determining the annualized premium s_{tr} such that the CDS is initialized at zero value

$$\begin{aligned} s_{tr} : CDS_{tr}(0) &= 0 \\ E_0\left[\sum_{t=1}^{T} P_r(0,t)\left[\left(\frac{s_{tr}}{4} - 1\right) P_{tr}(t-1) + P_{tr}(t)\right]\right] &= 0. \end{aligned}$$

In case of the issuance of CLNs and absence of interest rate risk, this implies that a tranche bearing a quarterly coupon exceeding the the risk-free rate by a spread equal to the fair CDS-premium can be issued at its par value $P_{tr}(0)$.

Besides the determination of arbitrage-free tranche spreads, we are interested in the effect of diversification of the reference portfolio on the tranche valuation for the three benchmark cases *minimal*, *sectors* and *maximal*.

We determine the arbitrage-free premia for CDO-tranches for two subordination structures, differing with respect to the level of overcollateralization (OC) provided by more subordinate tranches, i.e. tranches that absorb losses below the threshold $\underline{K}_{tr}$ of the tranche considered. A senior and a mezzanine tranche are issued in form of CDSs with maturity of five years, bearing a regular premium claim. The low-OC scenario assumes loss thresholds and tranche principals of

$$\begin{aligned} P_{sen}(0) = 90 \quad \underline{K}_{sen} = 10 \\ P_{mez}(0) = 8 \quad \underline{K}_{mez} = 2 \end{aligned}$$

and the high-OC scenario of

$$\begin{aligned} P_{sen}(0) = 84 \quad \underline{K}_{sen} = 16 \\ P_{mez}(0) = 10 \quad \underline{K}_{mez} = 6 \end{aligned}$$

A junior tranche with principals $P_{jun}(0) = 2$ and $P_{jun}(0) = 6$, respectively, absorbs first loss amounts before the issued tranches are impaired. Junior tranches are generally retained by the note issuer and thus do not carry premium claims. By establishing the securitization structure, the issuer acquires a cap on portfolio losses at $\underline{K}_{mez}$. For the retention of the first loss amounts he is compensated by the residual claim on spread income, which is the difference between spread income generated by the reference portfolio - each claim generates a regular spread payment of $\frac{c_{risk} - c_r}{4}$ prior to possible default - and the premia payable to tranche protection sellers.

Arbitrage-free premia of senior and mezzanine claims together with expected losses borne by the junior claimholder are given in table 13.

With an increase of reference portfolio diversification, the senior premium is slightly reduced for both overcollateralization scenarios. A senior tranche has the character of a long call position on portfolio losses with a basis price of $\underline{K}_{sen}$, from which it participates in portfolio losses. A decrease in loss volatility due to an increase in portfolio diversification implies a lower probability for high losses and thus a spread reduction. However, as the underlying loss distributions do not differ substantially in the right tail, the effect is rather small for both subordination scenarios.

For the mezzanine tranche, the effect of diversification on the premium is not expected to be unique: A tranche bearing losses in a low range is expected to bear higher losses in case of an amelioration of portfolio diversification and thus trades at a reduced spread. This assertion becomes evident when the present value of the expected losses assumed by the junior tranche, effectively a short put with basis price $\underline{K}_{mez}$ are considered. Junior losses actually increase with portfolio diversification.

For the given parameter estimates, the mezzanine tranche reacts like the senior tranche on the variation of diversification, where the effect is clearly more pronounced in the scenario where the mezzanine if 'safer' due to higher overcollateralization and larger size.

Figure (5) identifies the regions where tranches exhibit the specific optional characteristics when the portfolio diversification changes from *minimal* to *maximal* and the estimated factor model holds. In the region below the zero point, the cumulative loss distribution function *minimal* exceeds the cumulative loss distribution function *maximal*. With respect to losses born by a tranche protection seller, the *maximal* portfolio first-order stochastically dominates the *minimal* portfolio over losses smaller than the mean. This implies a spread increase with diversification for mezzanine tranches whose loss participation range is lower than for the low-OC scenario implemented, at which the mezzanine spread is almost insensitive to a change in diversification.

A final remark on the absolute magnitude of the premia is in order. The premia on the senior tranche (22-26 bps and 6-10 bps) correspond closely to senior premia as are observed in market for (virtually risk-free) Aaa-rated senior tranches of the respective size. In the medium region, however, observed spreads are by far lower than the arbitrage-free estimates. The market participants therefore judge loss occurrence in the mezzanine range less probable or severe than it is implied by the factor model, whereas the market's assessment about loss occurrence in the extreme senior range is in line the the predictions obtained by the factor model.

6 Conclusion

The paper investigates to what extent a reduced-form factor model of the affine class can explain the observed joint evolution of European corporate bond spreads and examines the implications of the estimated model on loss forecasting in a defaultable debt portfolio context. The main findings are summarized as follows:

The inclusion of a market risk factor affecting all defaultable debt claims appears appropriate in the light of the considerable comovement of observed spreads of different risk levels. The model provides additional flexibility to incorporate factors related to risk levels and the industry affiliations of an obligor. Empirical support for the latter is provided by the detection of sector-specific price deviations.

With respect to model validation within the estimation context of the bond market, we find that both cross-sectional and time series behavior of medium-maturity spreads are considerably well explained. The estimated correlation structure explains the observed dependence among observed spread series acceptably well during the period before autumn 2001, where spread evolution has been homogeneous, but fails to incorporate the extreme spread behavior at the end of the sample period. This lets us conclude that the affine factor model is suited for modelling regular spread behavior and will provide reasonable out-of-sample correlation forecasts for homogeneous scenarios.

The debt portfolio analysis incorporates the effects of diversification among sectors. However, for the obtained parameter estimates, the proportion of expected loss variation of particular classes of defaultable claims caused by common factor variation is high. Therefore, the impact of in increase in diversification with respect to sectors on portfolio loss variation is considerably small for the estimated model.

A model application examines the impact of diversification on the premia on CDOs, applying basket-CDS pricing methodology. We find that occurrence of severe losses is priced by the market according to the estimated factor model, whereas in the segment of moderate losses considerable mispricings are prevalent.

A Pricing and yield relationships for defaultable zero bonds in the CIR-model

The coefficients $a_k^{CIR}(\tau,\phi_k)$ and $b_k^{CIR}(\tau,\phi_k)$ of the pricing relationship

$$P_{class}(t,\tau) = \exp\left(\sum_{k=r,c,class}\left(a_k^{CIR}(\tau,\phi_k) - b_k^{CIR}(\tau,\phi_k) f_k(t)\right)\right)$$

of a defaultable zero bond whose factor dynamics are characterized by mean-reverting square-root processes are given by Cox, Ingersoll, and Ross (1985) as

$$\begin{aligned}
a_k^{CIR}(\tau,\phi_k) &= \ln\left(\frac{2\gamma_1(\phi_k)\exp\left(\frac{1}{2}\gamma_2(\phi_k)\tau\right)}{\gamma_4(\tau,\phi_k)}\right)^{\gamma_3(\phi_k)} \\
b_k^{CIR}(\tau,\phi_k) &= \frac{2\left(\exp\left(\gamma_1(\phi_k)\tau\right)-1\right)}{\gamma_4(\tau,\phi_k)} \\
\gamma_1(\phi_k) &= \sqrt{\left(\kappa_k^* + \lambda_k\right)^2 + 2\sigma_k^2} \\
\gamma_2(\phi_k) &= \kappa_k^* + \lambda_k + \gamma_1(\phi_k) \\
\gamma_3(\phi_k) &= \frac{2\kappa_k^*\theta_k^*}{\sigma_k^2} \\
\gamma_4(\tau,\phi_k) &= \exp\left(2\gamma_1(\phi_k) + \gamma_2(\phi_k)\cdot\exp\left(\gamma_1(\phi_k)\tau\right) - 1\right).
\end{aligned}$$

B The factor model in state-space form

Transition equation

Factors, or state variables, are collected in a $(5x1)$ state vector $\mathbf{f}(t)$. The intertemporal behavior of the latent state vector is assumed to be determined by the first two conditional moments of the CIR-processes governing the factors' dynamics under the physical measure, collected in a vector ϕ^- with elements $\phi_k^- = (\kappa_k^*, \theta_k^*, \sigma_k)$ excluding the market price of risk parameters.

Following the exposition of Harvey (1989), the transition system equation governed by parameters ϕ^- and 5-dimensional white noise $\eta(t, \mathbf{\Sigma}(\phi^-))$ with covariance matrix $\mathbf{\Sigma}(\phi^-)$

$$\mathbf{f}(t + \Delta t) = \mathbf{T}(\phi^-)\mathbf{f}(t) + \eta(t, \mathbf{\Sigma}\left(\phi^-\right)) \tag{16}$$

takes a particularly simple form, implied by the conditional factor expectations and variances of the factors, as determined by the continuous time dynamics prescribed by the CIR-model. As pairwise uncorrelatedness among all factors is assumed, the deterministic transition matrix, $\mathbf{T}(\phi^-)$ is diagonal with the conditional expectations of the respective state variables for the next discrete observation interval with length $\Delta t = 1/52$ as diagonal elements $\mathbf{T}_{ii}(\phi^-)$:

$$\mathbf{T}_{ii}(\phi^-) = E\left[f_k(t + \Delta t)\,|f_k(t)\right] = \left(1 - e^{-\kappa_k^*\Delta t}\right)\theta_k^* + e^{-\kappa_k^*\Delta t} f_k(t).$$

Analogously, the covariance matrix $\mathbf{\Sigma}(\phi^-)$ is diagonal with the factors' conditional transition variances for our discrete sampling interval as elements $\mathbf{\Sigma}_{ii}(\phi^-)$:

$$\begin{aligned}\mathbf{\Sigma}_{ii}\left(\phi^-\right) &= Var\left[f_k(t + \Delta t)\,|f_k(t)\right] \\ &= \frac{\sigma_k^2}{2\kappa_k^*}\left(1 - e^{-\kappa_k^*\Delta t}\right)^2\theta_k^* + \frac{\sigma_k^2}{\kappa_k^*}\left(e^{-\kappa_k^*\Delta t} - e^{-2\kappa_k^*\Delta t}\right) f_k(t).\end{aligned}$$

Measurement equations

The measurement equations account for the fact that the risk-free term structure model is estimated separately. Observables are the risk-free spot rates and corporate spreads of maturities $\tau = (\tau_1, ..., \tau_5) = (2, 4, 6, 8, 10)$. For the state-space setup, observable variables are collected in two vectors $\mathbf{R}_r(t) = (R_r(t, \tau))$ and $\mathbf{S}(t) = (S_{class}(t, \tau))$, with maturities $\tau = \tau_1, ..., \tau_5$ and risk levels $class = 1, 2, 3$ for the spot spreads. With regards to separate estimation, the state vector $\mathbf{f}(t)$ is partitioned into $f_r(t)$ and $\mathbf{f}_s(t) = (f_c(t), f_1(t), f_2(t), f_3(t))'$ and the parameter vector ϕ into ϕ_r and $\phi_s = (\phi_c, \phi_1, \phi_2, \phi_3)$, including the market price of risk parameters. The covariance matrices of measurement noise $\mathbf{M}_r$ and $\mathbf{M}_s$ are diagonal with elements $m_r(\tau_1), ..., m_r(\tau_5)$ and $m_1(\tau_1), ..., m_1(\tau_5), m_2(\tau_1), ..., m_2(\tau_5), m_3(\tau_1), ..., m_3(\tau_5)$, for the respective subsamples. The measurement noise $\varepsilon_r(t)'$ and $\varepsilon_s(t)'$ is represented by independently $N(0, 1)-$distributed random vectors.

The measurement equations for the KF-setup reflect the functional dependence of the CIR-model implied spot rates and spot spreads on the state variables, as given by (8):

$$\mathbf{R}_r(t,\phi_r) = \mathbf{A}_r(\phi_r) + \mathbf{B}_r(\phi_r) f_r(t) + \mathbf{M}_r \varepsilon_r(t)'$$

$$\mathbf{S}(t,\phi_s) = \mathbf{A}_s(\phi_s) + \mathbf{B}_s(t,\phi_s)\mathbf{f}_s(t) + \mathbf{M}_s \varepsilon_s(t)'.$$

For the model specification at hand, the arrays are given as

$$\mathbf{A}_r(\phi_r) = (A_r(\tau_1,\phi_r), ..., A_r(\tau_5,\phi_r))'$$

$$\mathbf{B}_r(\phi_r) = (B_r(\tau_1,\phi_r), ..., B_r(\tau_5,\phi_r))'$$

such that the short rate is mapped to five risk-free spot rates and

$$\begin{aligned}\underset{(15x1)}{\mathbf{A}_s(\phi_s)} &= (A_c(\tau_1,\phi_c) + A_1(\tau_1,\phi_1), ..., A_c(\tau_5,\phi_c) + A_1(\tau_5,\phi_1),\\ &\quad A_c(\tau_1,\phi_c) + A_2(\tau_1,\phi_2), ..., A_c(\tau_5,\phi_c) + A_2(\tau_5,\phi_2),\\ &\quad A_c(\tau_1,\phi_c) + A_3(\tau_1,\phi_3), ..., A_c(\tau_5,\phi_c) + A_3(\tau_5,\phi_3))'\end{aligned}$$

$$\underset{(15x4)}{\mathbf{B}_s(\phi_s)} = \begin{pmatrix} B_c(\tau_1,\phi_c) & B_1(\tau_1,\ \phi_1) & 0 & 0 \\ ...\tau_2,\tau_3,\tau_4... & ...\tau_2,\tau_3,\tau_4... & ...0... & ...0... \\ B_c(\tau_5,\phi_c) & B_1(\tau_5,\phi_1) & 0 & 0 \\ B_c(\tau_1,\phi_c) & 0 & B_2(\tau_1,\phi_2) & 0 \\ ...\tau_2,\tau_3,\tau_4.. & ...0... & ...\tau_2,\tau_3,\tau_4... & ...0... \\ B_c(\tau_5,\phi_c) & 0 & B_2(\tau_5,\phi_2) & 0 \\ B_c(\tau_1,\phi_c) & 0 & 0 & B_3(\tau_1,\phi_3) \\ ...\tau_2,\tau_3,\tau_4.. & 0 & ...0... & ...\tau_2,\tau_3,\tau_4... \\ B_c(\tau_5,\phi_c) & 0 & 0 & B_3(\tau_5,\phi_3) \end{pmatrix}$$

such that four factors are mapped to 15 spot spreads at every observation period.

Likelihood construction

Let $\mathbf{v}_r(t)$ and $\mathbf{v}_s(t)$ denote the vectors of prediction errors of the filter algorithm and $\mathbf{V}_r(t)$ and $\mathbf{V}_s(t)$ the respective prediction errors' covariance matrices. The initial expectations and variances of the state variables are set equal to the unconditional moments of the respective CIR-processes, i.e. $E[f_k(t)] = \theta_k$ and $Var[f_k(t)] = \frac{\theta_k \sigma_k^2}{2\kappa_k}$.

The KF-algorithm generates factor estimates as linear projections of observed variables $\mathbf{R}_r(t,\phi_r)$ or $\mathbf{S}(t,\phi_s)$, respectively, into the state vector, solving a least-squares problem on prediction errors $\mathbf{v}_r(t)$ or $\mathbf{v}_s(t)$, respectively, for a linearized model. The error-minimizing regression coefficient is obtained as the proportion of the conditional factor variances $\mathbf{\Sigma}(\phi^-)$ and variances of observed variables, adjusting the prediction made on the basis of the transition equation (16) by a linear function of the prediction error incurred. The linearization has the undesirable implication that negative factor updates may occur for the discrete observation intervals. Negative state variables, however are precluded by the CIR-model and must be suppressed for estimation. For the estimation problems at hand, negative updates actually are obtained in the model part referring to corporate spreads due to the low mean spread level combined with high volatility. For the relevant observation periods, no update is made of the corresponding entry k of the state vector $\mathbf{f}_s(t)$, and the entries of its covariance matrix related to k, i.e. we set

$$E[f_k(t)\,|\mathbf{S}\,(t)] = E[f_k(t)\,|\mathbf{S}\,(t-1)],$$

$$Cov[f_k(t), f_l(t)\,|\mathbf{S}\,(t)] = Cov[f_k(t), f_l(t)\,|\mathbf{S}\,(t-1)] \text{ for } k, l = c, 1, 2, 3.$$

Taking into account the first two moments of the prediction error distribution, we maximize the following Quasi-log-likelihood functions (apart from a constant) with respect to parameter vectors ϕ_r and ϕ_s, state vectors $f_r(t)$ and $\mathbf{f}_s(t)$ and the variances of the spot rates' and spreads' measurement noise $\mathbf{M}_r$ and $\mathbf{M}_s$:

$$\mathcal{L}^r(f_r(t), \phi_r, \mathbf{M}_r) = -0.5 \sum_{t=1}^{T} \left(\ln |\mathbf{V}_r(t)| + \mathbf{v}_r'(t)\mathbf{V}_r^{-1}(t)\mathbf{v}_r(t) \right)$$

$$\mathcal{L}^s(\mathbf{f}_s(t), \phi_s, \mathbf{M}_s) = -0.5 \sum_{t=1}^{T} \left(\ln |\mathbf{V}_s(t)| + \mathbf{v}_s\prime(t)\mathbf{V}_s^{-1}(t)\mathbf{v}_s(t) \right).$$

Technical aspects of the implementation of QML estimation

Some technical remarks on the solution of the maximization problems and statistical inference are in order:

The treatment of negative state variables by the suppression of the respective factor updates has the desirable effect to penalize the likelihood for negative factor updates. The information provided by the negative update is discarded and thus, the assumption works like a restriction on parameter estimation imposed to draw the optimization algorithm away from parameter sets that imply a high probability of negative state variables. Experimentation with the alternative choice, which consists in setting negative updates equal to zero, shows that the 'uninformative' case of negative updates is avoided more frequently. However, this comes at the cost of points of discontinuity of the likelihood due to the change in the information set that occurs when state variables reach zero, which makes the solution of the maximization problem tedious.

Due to the discontinuity inherent to the optimization task, a grid search is performed by evaluating a large number of function values for random trial parameters in plausible ranges. We then apply a derivative-free optimization algorithm on starting values with high function values. For the corporate spread model, we start on maximization of a restricted version, imposing equality on the parameters κ_k^* and σ_k of all factors, effectively explaining different factor levels and volatilities entirely by factor-specific θ_k^*-estimates. The maximizer of the restricted version is shocked within 50% and 150% of the parameter values to obtain starting values for the maximization of the full model. Convergence behavior of $\mathcal{L}^s$ shows that the likelihood function has local maxima with parameter constellations leading to almost identical risk-neutral parameters. This is due to the small likelihood contribution of time-series information. Local optima are characterized by unstable factor behavior caused by the suppression of negative updates. In order to impose factor orthogonality as far as possible, correlations between the extracted common factor series and the series of the three class-specific factors have been restricted not to exceed 0.4.

A second inconvenience caused by the discontinuous nature of the problem is related to statistical inference and parameter identification. Heteroscedasticity-consistent estimators of the asymptotic covariance of parameter vectors ϕ_r or ϕ_s, respectively, are obtained as (White (1980))-estimators. The computation of standard errors of the parameter estimates requires derivatives of observables with respect to parameters evaluated at the parameter estimates to be well-defined. Due to the use of both time series and cross-sectional information, both marginal effects of parameter changes on factor path estimates and estimated spot rates and spreads as functions of parameter estimates and state variables have to be taken into account. When approximating the relevant derivatives numerically, the possible discontinuity of the factor path has to be taken care of. We therefore make sure that the truncation times do not change in order to keep derivatives computable. If continuity were not being assured, infinite derivatives would be obtained and standard errors would become unreasonably small.

C Conditional factor probability distributions in the CIR mean-reverting square-root-process

As given by Cox, Ingersoll, and Ross (1985), a linear transformation of the realization of factor $f_k(t+\Delta t)$ conditional on its previous value $f_k(t)$ is distributed as

$$\begin{aligned}
2c_k f_k(t+\Delta t) &\sim \chi^2\left[2q_k+2, 2u_k\right], \\
c_k &= \frac{2\kappa_k}{\sigma_k^2\left(1-\exp(-\kappa_k\Delta t)\right)} \\
u_k &= c_k f_k(t)\exp(-\kappa_k\Delta t) \\
q_k &= \frac{2\kappa_k\theta_k}{\sigma_k^2}-1,
\end{aligned}$$

with $df = 2q_k + 2$ and the level-dependent noncentrality parameter $2u_k$.

In order to obtain random variates from the respective noncentral χ^2-distributions, we adapt the algorithm suggested by Duan and Simonato (1995). The algorithm makes use of the fact stated by Johnson and Kotz (1970) that the noncentral χ^2-distribution can be represented as the mixture $\chi^2\left[2q_k+2+Poi(h^{Poi}(f_k(t)))\right]$, i.e. a central χ^2-distribution, whose degrees of freedom are determined by an affine transformation of the Poisson distribution. Factor realizations are simulated for weekly time intervals $\Delta t = 1/52$, factors are initialized at the risk-neutral means $\tilde{\theta}_k$. For each time step with a given previous factor realization $f_k(t)$ we generate

i) the degrees of freedom as $df = 2q_k + 2 + Poi(h^{Poi}(f_k(t)))$:

   The Poisson-distributed variable with mean (intensity parameter) $h^{Poi}(f_k(t)) = c_k f_k(t)\exp(-\kappa_k\Delta t)$ is straightforwardly obtained as the number of events that occur at exponentially-distributed event times during the unit interval. Exponentially-distributed variates are obtained by the transformation of $U[0,1]$-distributed random variates by the inverse exponential CDF.

ii) the χ^2-distributed current realization of $2c_k f_k(t+\Delta t)$ with the simulated df :

   This is straightforwardly obtained by the transformation of $U[0,1]$-distributed random variates by the inverse central χ^2 CDF.

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Table 1: Sample overview: median (5%-fractile/95%-fractile) of contract specifications of German Bundesanleihen and Euro-denominated default-risky bonds issued by European corporates. Corporate classes are formed according to the level of riskiness as given by issuer's letter rating at the start of the estimation period.

	Start date (Jul 00)	End date (Dec 01)	Total period
German Bundesanleihen			
Valid prices	53	40	3523
Maturity (years)	2.82 (0.39/9.00)	2.05 (0.36/7.57)	2.50 (0.28/8.53)
Coupon (%)	6.50 (3.15/8.80)	6.38 (4.12/8.00)	6.50 (3.75/8.50)
Yield to maturity (%)	5.02 (4.55/5.17)	3.84 (3.49/4.56)	4.54 (3.56/5.17)
Aaa/Aa corporate class (risk=1)			
Valid prices	88	91	6751
Maturity (years)	4.66 (1.05/10.30)	3.98 (0.95/9.15)	4.28 (0.56/9.77)
Coupon (%)	5.75 (3.50/9.00)	5.75 (3.63/8.34)	5.75 (3.50/9.00)
Yield to maturity (%)	5.62 (5.15/6.27)	4.84 (4.84/6.14)	5.23 (4.06/6.22)
A corporate class (risk=2)			
Valid prices	83	112	7294
Maturity (years)	5.31 (1.20/9.73)	4.61 (0.86/8.84)	4.83 (1.17/9.33)
Coupon (%)	5.63 (3.64/7.74)	5.75 (3.75/7.06)	5.63 (3.63/7.13)
Yield to maturity (%)	5.87 (5.27/6.42)	5.05 (3.98/6.43)	5.46 (4.41/6.46)
Baa corporate class (risk=3)			
Valid prices	57	74	4569
Maturity (years)	5.96 (1.93/9.02)	4.53 (0.86/7.49)	4.86 (1.00/8.55)
Coupon (%)	5.75 (4.50/7.50)	5.88 (4.58/7.09)	5.88 (4.50/7.40)
Yield to maturity (%)	6.06 (5.32/6.77)	5.37 (4.41/6.96)	5.74 (4.59/6.85)

Table 2: Rating migrations of European corporate bonds during sample period.

	n° bonds	Aa to A	Baa to A	A to Baa	Baa to spec.	net migration
Entire sample	334	53	4	45	2	-96
Breakdown for industries						
Basic	74	6	0	1	0	-7
Services, cyclical	12	0	0	0	0	0
Services, noncylical	44	19	0	29	2	-50
Consumer, cyclical	66	0	0	12	0	-12
Consumer, noncyclical	41	0	0	3	0	-3
Financials	63	24	0	0	0	-24
Utilities	34	4	4	0	0	0

Table 3: Time series analysis of term structures of risk-free interest rates and spot spreads for the corporate classes.

	Summary statistics on term structures			Term spreads			Autocorrelation			Autocorr. 1st diff.	
	Mean	Std deviation	Skewness	Mean	Min	Max	$Lag = 1$	$Lag = 2$	$Lag = 10$	$Lag = 1$	$Lag = 2$
$R_r(t, 2)$	4.424	0.604	-0.640				0.985	0.967	0.778	0.060	-0.033
$R_r(t, 4)$	4.556	0.460	-0.261				0.974	0.948	0.724	-0.064	-0.052
$R_r(t, 6)$	4.709	0.331	-0.158	0.499	-0.141	1.337	0.961	0.925	0.630	-0.066	-0.033
$R_r(t, 8)$	4.831	0.252	-0.348				0.939	0.890	0.483	-0.110	0.005
$R_r(t, 10)$	4.923	0.210	-0.612				0.910	0.845	0.305	-0.146	0.048
$S_1(t, 2)$	0.398	0.255	1.315				0.951	0.914	0.461	-0.111	0.062
$S_1(t, 4)$	0.729	0.249	1.325				0.957	0.907	0.455	0.072	0.031
$S_1(t, 6)$	0.826	0.151	0.993	0.447	-0.090	0.852	0.916	0.831	0.203	0.010	-0.061
$S_1(t, 8)$	0.848	0.097	0.431				0.828	0.677	-0.208	-0.044	-0.138
$S_1(t, 10)$	0.845	0.100	-0.176				0.820	0.678	-0.038	-0.092	-0.161
$S_2(t, 2)$	0.531	0.454	1.518				0.936	0.918	0.479	-0.362	0.002
$S_2(t, 4)$	1.035	0.357	1.152				0.912	0.869	0.581	-0.259	-0.060
$S_2(t, 6)$	1.169	0.225	1.066	0.702	-0.200	1.386	0.904	0.842	0.451	-0.182	-0.055
$S_2(t, 8)$	1.215	0.161	0.831				0.895	0.827	0.200	-0.174	-0.060
$S_2(t, 10)$	1.233	0.166	0.356				0.771	0.687	0.114	-0.318	-0.154
$S_3(t, 2)$	1.003	0.374	1.612				0.934	0.886	0.439	-0.144	-0.070
$S_3(t, 4)$	1.244	0.237	1.534				0.931	0.883	0.350	-0.160	-0.0074
$S_3(t, 6)$	1.306	0.145	0.908	0.330	-0.987	0.775	0.839	0.755	0.120	-0.212	-0.076
$S_3(t, 8)$	1.326	0.127	0.650				0.684	0.538	-0.094	-0.213	-0.056
$S_3(t, 10)$	1.332	0.151	0.775				0.656	0.495	-0.179	-0.219	-0.004

Table 4: Dependence structure of systematic spot rate components. Correlation coefficients between time series of corresponding maturities and OLS-estimates of the coefficients of pairwise linear regressions are given. For regressions between systematic spot spreads of the same maturity class, we choose the time series of the riskier class to be the dependent variable. For the comparison of risk-free rates with systematic spreads, we choose the spot spread of the corresponding maturity to be the dependent variable.
**: significant at 1%-level.

τ	$S_1(t,\tau), S_2(t,\tau)$	$S_1(t,\tau), S_3(t,\tau)$	$S_2(t,\tau), S_3(t,\tau)$	$R_r(t,\tau), S_1(t,\tau)$	$R_r(t,\tau), S_2(t,\tau)$	$R_r(t,\tau), S_3(t,\tau)$
	Dependence between time series of levels: correlation coefficient / linear regression coefficient					
2	0.936 / 1.665**	0.851 / 1.249**	0.921 / 0.760**	-0.723 / -0.305**	-0.825 / -0.620**	-0.869 / -0.538**
4	0.932 / 1334**	0.843 / 0.801**	0.889 / 0.589**	-0.886 / -0.480**	-0.893 / -0.694**	-0.827 / -0.423**
6	0.896 / 1.335**	0.878 / 0.844**	0.902 / 0.582**	-0.851 / -0.388**	-0.849 / -0.577**	-0.758 / -0.332**
8	0.802 / 1.324**	0.763 / 0.997**	0.620 / 0.491**	-0.630 / -0.243**	-0.730 / -0.465**	-0.445 / -0.224**
10	0.679 / 1.130**	0.395 / 0.599**	0.205 / 0.186	-0.315 / -0.150**	-0.557 / -0.440**	-0.103 / -0.074
τ	Dependence between time series of first differences: correlation coefficient / linear regression coefficient					
2	0.394 / 0.807**	0.300 / 0.513**	0.480 / 0.401**	-0.483 / -0.199**	-0.345 / -0.230	-0.243 / -0.146
4	0.542 / 1.098**	0.584 / 0.695**	0.503 / 0.295**	-0.632 / -0.451**	-0.433 / -0.628**	-0.564 / -0.480**
6	0.636 / 1.009**	0.617 / 0.804**	0.580 / 0.476**	-0.633 / -0.422**	-0.487 / -0.515**	-0.575 / -0.499**
8	0.666 / 0.864**	0.575 / 0.979**	0.409 / 0.537**	-0.607 / -0.391**	-0.584 / -0.487**	-0.432 / -0.473**
10	0.418 / 0.794**	0.544 / 1.099**	0.142 / 0.151	-0.604 / -0.405**	-0.438 / -0.557**	-0.338 / -0-458**

Table 5: Summary statistics on pricing errors of individual coupon bonds.

Pricing errors of coupon bonds (%)	MAD	RMSE	Mean	Median	5%-quantile	95%-quantile	MYE [bps]
German Bundesanleihen	0.221	0.308	0.196	0.164	-0.091	0.595	-6.6
Aaa/Aa corporate class (risk=1)	1.310	2.147	0.111	0.218	-3.355	2.572	-5.5
A corporate class (risk=2)	1.480	2.324	0.316	0.356	-2.763	3.484	-9.5
Baa corporate class (risk=3)	1.583	2.398	0.097	0.292	-4.439	3.384	-7.8

The median yield errors (MYE, last column) for the subsamples are calculated for a bond with the median maturity $\tau_{N,0.5}$ ($\tau_{0.5}$ denotes the corresponding payment dates) and coupon $CF_{0.5}$ of the respective subsample as given in table 1. We choose the implied yield to maturity of the "median" bond of each subsample $\widehat{ytm}_{sub}$ to solve

$$V_{0.5}(sub, \tau_{0.5}) - \widehat{V}(sub, \tau_{0.5}) = \varepsilon_{0.5}(sub, \tau_{0.5}),$$

where $\varepsilon_{0.5}(sub, \tau_{0.5})$ denotes the median pricing error, as given in column 5.

$$V_{0.5}(sub, \tau_{0.5}) = \sum_{\tau=\tau_{1,0.5}}^{\tau_{N,0.5}} e^{-ytm_{sub,0.5}\tau} \cdot CF_{0.5}(\tau)$$

$$\widehat{V}(sub, \tau_{0.5}) = \sum_{\tau=\tau_{1,0.5}}^{\tau_{N,0.5}} e^{-\widehat{ytm}_{sub}\tau} \cdot CF_{0.5}(\tau).$$

$ytm_{sub,0.5}$ denotes the median yield to maturity of the considered subsample and $V_{0.5}(sub, \tau_{0.5})$ the hypothetical price of a bond characterized by all median characteristics. $ytm_{sub,0.5} - \widehat{ytm}_{sub}$ $[bps]$ is given in the last column.

Table 6: Explanation of pricing residuals by individual differences in liquidity and sector affiliation of the issuer. We enquire the relevance of the liquidity-related variables 'issue size' (Mio. EUR), 'age of issue' (years) and 'average weekly price changes' (a ratio between zero and one). With respect to sector affiliation, the basic industry dummy is included in the intercept. Time to maturity is a control variable.
*: Significance at 5%-level, **: Significance at 1%-level

Dependent variable: pricing errors all corporates

R^2	0.115	
Variable	Coeff	Std Error
Constant	0.595**	0.137
Issue size	0.252**	0.032
Age of issue	-0.004	0.008
Avg. weekly price changes	-0.644**	0.130
Time to maturity	-0.014*	0.007
Sector dummies:		
Services, cyclical	-0.540**	0.089
Services, noncyclical	-1.570**	0.055
Consumption, cyclical	0.163**	0.049
Consumption, noncyclical	0.581**	0.058
Financials	0.777**	0.050
Utilities	1.033**	0.058

Dependent variable: pricing errors risk=2 bonds

R^2	0.126	
Variable	Coeff	Std Error
Constant	0.459	0.310
Issue size	-0.038	0.061
Age of issue	-0.004	0.015
Avg. weekly price changes	-0.317	0.305
Time to maturity	-0.036**	0.011
Sector dummies:		
Services, cyclical	0.850**	0.259
Services, noncyclical	-0.730**	0.091
Consumption, cyclical	0.328**	0.071
Consumption, noncyclical	0.419**	0.108
Financials	3.190**	0.125
Utilities	1.029**	0.095

Dependent variable: pricing errors risk=1 bonds

R^2	0.325	
Variable	Coeff	Std Error
Constant	-0.492*	0.204
Issue size	0.078*	0.034
Age of issue	0.037**	0.009
Avg. weekly price changes	0.876**	0.201
Time to maturity	0.027**	0.009
Sector dummies:		
Services, cyclical	0.000	0.000
Services, noncyclical	-3.243**	0.071
Consumption, cyclical	-2.272**	0.291
Consumption, noncyclical	0.388*	0.180
Financials	-0.116*	0.057
Utilities	0.441**	0.075

Dependent variable: pricing errors risk=3 bonds

R^2	0.163	
Variable	Coeff	Std Error
Constant	-0.257	0.252
Issue size	3.098**	0.144
Age of issue	0.206**	0.023
Avg. weekly price changes	-1.908**	0.212
Time to maturity	-0.046**	0.017
Sector dummies:		
Services, cyclical	-0.268*	0.110
Services, noncyclical	-0.057	0.147
Consumption, cyclical	0.274*	0.109
Consumption, noncyclical	1.036**	0.086
Financials	1.301**	0.310
Utilities	2.180**	0.157

Table 7: Parameter estimates for the factor model and the standard deviations of measurement noise. Heteroscedasticity-consistent White-estimators of the asymptotic standard errors of estimates are given in parentheses.

	Processes under physical measure					Processes under EEM		
	Parameter estimates				Uncond.	Drift parameters		Uncond.
Factor	κ_k^*	θ_k^*	σ_k	λ_k	std deviation	κ_k	θ_k	std deviation
$f_r(t)$	0.4912	0.0373	0.0705	-0.1498	0.0137	0.3414	0.0536	0.0198
	(0.07515)	(0.00569)	(0.00245)	(0.07484)				
$f_c(t)$	1.5023	0.0016	0.1639	-1.2335	0.0037	0.2688	0.0088	0.0209
	(0.06009)	(0.00006)	(0.00241)	(0.06025)				
$f_1(t)$	2.0658	0.0012	0.1569	-1.1848	0.0027	0.8811	0.0028	0.0063
	(0.11372)	(0.00006)	(0.00215)	(0.11738)				
$f_2(t)$	1.5435	0.0022	0.1427	-1.1202	0.0038	0.4234	0.0080	0.0139
	(0.13551)	(0.00019)	(0.00169)	(0.13517)		for CDO-valuation:	0.00525	0.0124
$f_3(t)$	1.5173	0.0039	0.1961	-0.7459	0.0071	0.7714	0.0077	0.0139
	(0.19598)	(0.00052)	(0.00327)	(0.19690)				

	Std deviations of measurement noise for maturitiy classes τ				
Observables	2	4	6	8	10
risk-free	0.00236	0.00078	0.00011	0.00054	0.00093
	(0.000206)	(0.000062)	(0.000003)	(0.000110)	(0.000297)
class 1	0.00206	0.00096	0.00047	0.00003	0.00039
	(0.000039)	(0.000045)	(0.000042)	(0.000001)	(0.000030)
class 2	0.00282	0.00070	0.00030	0.00052	0.00102
	(0.000070)	(0.000016)	(0.000005)	(0.000046)	(0.000182)
class 3	0.00135	0.00069	0.00045	0.00131	0.00184
	(0.000026)	(0.000020)	(0.000022)	(0.000848)	(0.001229)

Table 8: Summary statistics on residuals [bps] of factor model for observable risk-free spot rates and spot spreads of the three corporate risk classes.

	Statistics on residuals [bps]						Autocorrelation		
Observables	MAD	Mean	Std deviation	Median	5%-quantile	95%-quantile	Lag=1	Lag=2	Lag=10
risk-free									
2	17.44	11.59	14.30	9.61	-16.22	45.87	0.910	0.833	0.402
4	7.05	0.43	3.78	1.74	-12.72	11.49	0.930	0.865	0.486
6	0.23	-0.13	0.19	-0.12	-0.56	0.36	0.481	0.401	0.118
8	4.36	1.25	3.28	-0.51	-4.89	11.61	0.900	0.835	0.408
10	7.36	2.88	6.10	-0.10	-7.47	19.71	0.894	0.830	0.373
class 1									
2	17.32	-14.69	16.22	-11.95	-39.00	9.34	0.712	0.574	0.067
4	8.34	-2.87	10.07	-4.88	-13.82	20.25	0.818	0.718	0.285
6	3.67	0.03	5.12	-0.78	-5.37	10.99	0.825	0.721	0.380
8	0.67	-0.61	2.18	0.02	-2.88	0.11	0.116	0.231	-0.055
10	3.29	-2.12	4.12	-1.49	-9.86	3.43	0.681	0.613	0.179
class 2									
2	24.67	-23.11	16.90	-23.78	-48.28	1.09	0.453	0.212	0.174
4	6.35	-1.55	8.08	-0.73	-12.01	8.72	0.343	0.243	0.032
6	2.43	0.19	4.41	1.31	-3.77	3.37	0.106	0.137	0.005
8	4.43	-1.06	5.19	-1.78	-8.45	7.90	0.350	0.254	-0.032
10	8.23	-2.64	9.53	-3.95	-14.54	13.54	0.335	0.167	-0.078
class 3									
2	10.21	-3.40	12.20	-5.10	-21.71	15.64	0.579	0.402	0.023
4	5.35	0.17	7.27	1.59	-13.45	10.48	0.741	0.602	-0.107
6	3.27	-0.14	5.41	0.92	-6.75	4.55	0.239	0.169	-0.183
8	8.67	-0.76	12.04	0.41	-19.18	19.24	0.729	0.531	-0.101
10	13.88	-1.25	18.72	-1.09	-28.97	34.40	0.807	0.629	-0.062

Table 9: Within-sample correlations between estimated factor series.

	$f_r(t)$	$f_c(t)$	$f_1(t)$	$f_2(t)$	$f_3(t)$
$f_r(t)$	1	-0.677	-0.279	-0.738	-0.416
$f_c(t)$		1	0.242	0.382	-0.092
$f_1(t)$			1	0.226	0.186
$f_2(t)$				1	0.748
$f_3(t)$					1

Table 10: Residual correlation between term structures of spot spreads.

τ	class 1, class 2	class 1, class 3	class 2, class 3
2	0.583	0.230	0.232
4	0.356	0.498	0.260
6	0.250	-0.012	0.429
8	0.258	0.364	0.121
10	0.310	0.433	0.012

Table 11: Pairwise correlations between risk-neutral expected loss rates of corporate claims of different classes. The main diagonal displays the correlations between loss rates of two obligors of the same risk level, but differing in sector affiliation. The off-diagonal correlations apply to obligors of different risk levels. For CDO valuation, the mean parameter is reset to $\theta_2 = (\theta_1 + \theta_3)/2$. The resulting dependencies are given in the lower half.

Correlations between risk-neutral expected loss rates

	for estimated parameters		
	class 1	class 2	class 3
class 1	0.9180	0.7982	0.7983
class 2		0.6941	0.6942
class 3			0.6942

	for θ_2 used for CDO valuation		
	class 1	class 2	class 3
class 1	0.9180	0.8440	0.7983
class 2		0.7760	0.7339
class 3			0.6942

Table 12: Comparison of risk-neutral loss probability distributions for portfolios varying in diversification among sectors.

	Diversification of reference portfolio					
	minimal		sectors		maximal	
	PV(loss)	#defaults	PV(loss)	#defaults	PV(loss)	#defaults
Mean	5.978	6.92	5.902	6.81	5.812	6.83
Std dev	5.073	6.23	4.790	5.95	4.558	5.89
Median	4.556	5	4.589	5	4.532	5
Quantiles:						
5%-Quantile	0.814	1	0.891	1	0.958	1
10%-Quantile	1.002	1	1.668	2	1.748	2
90%-Quantile	12.492	15	12.004	14	11.607	14
95%-Quantile	16.347	20	15.825	19	15.197	19
Quantile ranges:						
95%-5%	3.742	4	3.698	4	3.574	4
90%-10%	11.490	14	10.336	12	9.859	12
75%-25%	5.296	6	4.780	6	4.459	6

Table 13: Valuation of basket credit default swaps referring to losses in specified ranges of the reference portfolio. The structure mimics typical default risk securitizations (CDOs), with a senior and mezzanine tranche issued as CLNs bearing a spread claim for portfolio default risk compensation and a retained junior tranche bearing first losses. On the side of the reference portfolio, three diversification scenarios are examined, on the side of the contingent claims, two overcollateralization (OC) scenarios are assumed.

		minimal			sectors			maximal		
	Low OC		$PV(CF_{tr})$			$PV(CF_{tr})$			$PV(CF_{tr})$	
Tranches	$P_{tr}(0)$	s_{tr}[%]	Mean	Std Dev	s_{tr}[%]	Mean	Std Dev	s_{tr}[%]	Mean	Std Dev
Senior	90	0.26	0	2.95	0.23	0	2.78	0.22	0	2.75
Mezzanine	8	11.42	0	3.43	11.33	0	3.29	11.33	0	3.23
Junior	2	0	-1.73	0.43	0	-1.77	0.36	0	-1.80	0.30

		minimal			sectors			maximal		
	High OC		$PV(CF_{tr})$			$PV(CF_{tr})$			$PV(CF_{tr})$	
Tranches	$P_{tr}(0)$	s_{tr}[%]	Mean	Std Dev	s_{tr}[%]	Mean	Std Dev	s_{tr}[%]	Mean	Std Dev
Senior	84	0.10	0	1.76	0.09	0	1.63	0.08	0	1.62
Mezzanine	10	4.05	0	2.93	3.72	0	2.80	3.59	0	2.78
Junior	6	0	-3.91	1.74	0	-4.00	1.65	0	-4.07	1.55

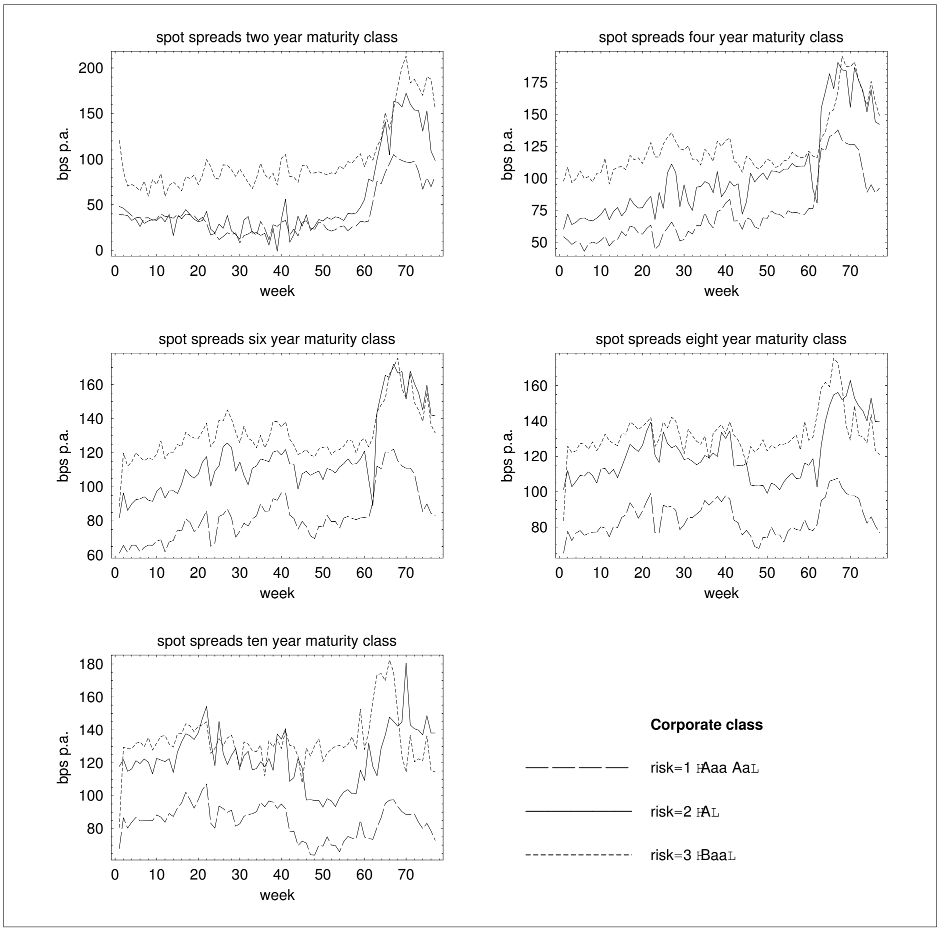

Figure 1: Evolution of corporate spot spreads as implied in coupon bond prices for different levels of default risk.

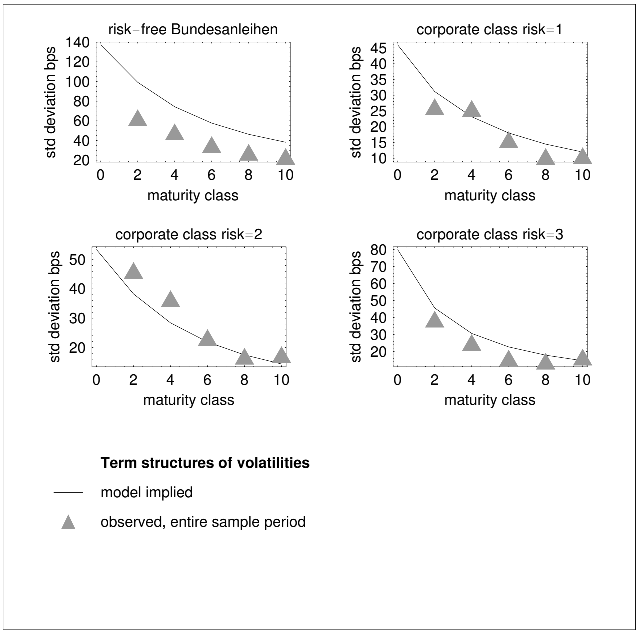

Figure 2: Term structures of volatility of risk-free rates and corporate spreads of different risk levels.

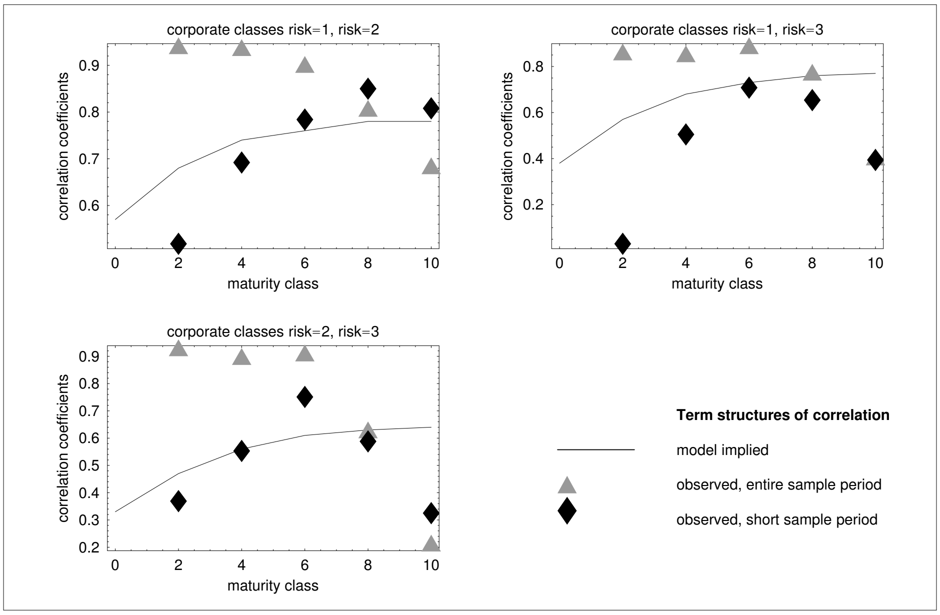

Figure 3: Term structures of correlation between corporate spreads of different risk levels.

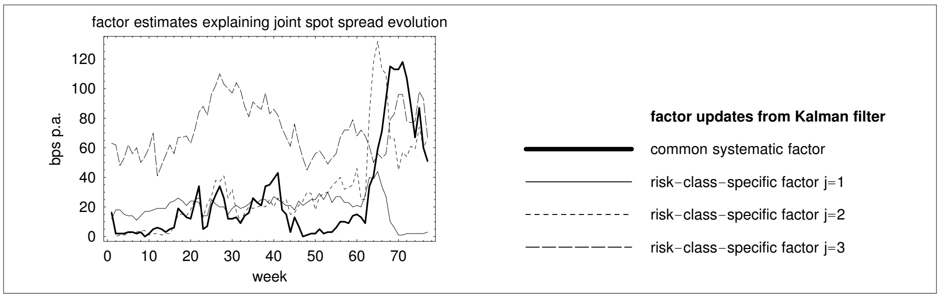

Figure 4: Estimated dynamics of factors determining joint spread evolution.

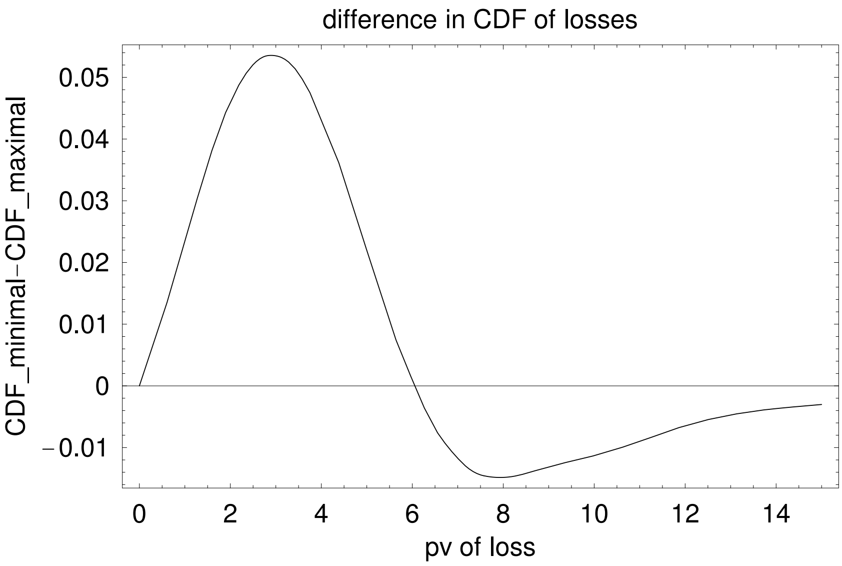

Figure 5: Difference between cumulative distribution functions of diversification scenarios *minimal* and *maximal*.