Determinants of the Asset Correlations of German Corporations and Implications for Regulatory Capital

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Abstract

This empirical paper addresses the gap between the theoretically well-understood impact of systematic risk on the loss-distribution of a credit-risky loan portfolio and the lack of empirical estimates of the default correlation. To this purpose we start with a one-factor model in which the correlation with the systematic risk factor equals the asset correlation between two firms.

The asset correlation is estimated from time series of ten years with default histories of 53280 German companies. The sample is divided into categories that are homogenous with respect to default probability (PD) and firm size. In this way we can explore to what extent correlations depend on these two factors. Several economic explanations why asset correlation depends on size and PD are discussed.

The empirical analysis is motivated as well by current proposals for the internal ratings based approach of the new Basel Accord. They suggest that the asset correlation parameter in the risk–weight function depends on the PD and on the firm size of the borrower. Our empirical results are compared with this proposal.

Keywords: asset correlation, New Basel Accord, default correlation, firm size, single risk factor model

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1 Introduction

The objective of this paper is to estimate the asset correlation of German corporate borrowers and its dependency on two factors: the firm size and the probability of default (PD). The analysis of these two potential drivers of the asset correlation is inspired by a recent proposal of the Basel Committee for the corporate risk–weight function of the internal rating based (IRB) approach of the new Accord.[1] In this proposal a two-dimensional dependency of the parameter asset correlation on the PD and the size of the borrower is introduced.

Our analysis is based on the one–factor model that has been used to derive the IRB risk–weight function on which the regulatory capital charge for corporate loans is based in the new Accord.[2] We refer to this model because it facilitates the comparison of our empirical results with the calibration of the asset correlation parameter in the risk–weight function.

In the one–factor model there exists a one-to-one mapping between default correlation and asset correlation for a given probability of default. Hence, the analysis provides as well empirical results for the default correlation which is a key driver of credit risk in loan portfolios. Therefore, the results are relevant as well for credit risk modelling in general.

This paper makes the following four main contributions:

First, the asset correlation is estimated from default histories of German firms taken from a database that includes 53280 privately–owned or corporate companies. This database which is maintained by the Deutsche Bundesbank allows the calculation of default frequencies for ten years, from 1991 until 2000. The consideration of relatively small and non–public firms contrasts with earlier studies that have used default rates of firms with ratings from external rating agencies. Other studies have focused on the U.S.A.[3] but institutional differences between the U.S.A. and Germany may induce different levels of correlation and as well differences in the dependence on PD and firm size. A much weaker credit channel in Germany due to a strong house–bank relationship may, for instance, weaken the dependence of SMEs on the business cycle and, as a consequence, their asset correlation.

Second, we explore the sensitivity of the correlation estimates to the definition of default. Earlier studies have used default rates from insolvency statistics. This paper instead, controls for the effect of insolvency as a relatively late definition of default by using in addition default rates that have been calibrated to loan net provisions of German banks.

Third, a potential dependency of asset correlation on PD and *simultaneously* on firm size

[1] See Basel Committee on Banking Supervision (2003).

[2] See Gordy (2001).

[3] See e. g. Das et al. (2002) and Savigny and Renault (2002).

is explored. Earlier work by Dietsch and Petey (2002) has focused on the dependency of asset correlation solely on PD. Two recent papers by Lopez (2002) and Dietsch and Petey (2003) are to the best of our knowledge the only ones that consider the cumulative influence of PD and firm size on the asset correlation. However, both differ from our work in important ways.

The work by Lopez (2002) is based on a vendor model developed by Moody's KMV[4]. However, our approach of estimating the asset correlation from default data is more in line with the predominant book-value approach to the management of bank loans than an estimation from a structural model like the one by Moody's KMV that uses equity data. Furthermore, the work by Savigny and Renault (2002) casts some doubt about the precision of equity correlation as an indicator of default correlation.

Differences between our work and that by Dietsch and Petey (2003) exist in the use of a quite different sample of German corporates and in that our results are based on a longer time series of ten compared to four years that better covers a full business cycle.[5]

As a fourth contribution we provide tentative answers why dependencies of asset correlation on PD and firm size are observed. To this purpose we take into account earlier work in the literature as well as recent empirical results for German corporates.

Our paper is divided into seven major sections. In section 2 previous theoretical and empirical results on a potential PD- or size-dependency of asset correlation are reviewed. Section 3 describes the model framework and the estimators for the model parameters. Section 4 describes the data sample and explains how it is partitioned by size and by credit quality. The estimation results for the asset correlation are presented in section 5. Potential implications for the calibration of regulatory capital in the new Basel Accord are discussed in section 6. Section 7 summarizes and concludes.

2 Previous Theoretical and Empirical Results

In this section explanations from economic theory for a dependency of asset correlation on firm size and PD are reviewed, together with empirical results from earlier studies.

In the one-factor model asset correlation measures the exposure against systematic risk, broadly speaking against business cycle risk. The asset correlation is lower for small and medium enterprises (SMEs) than for large corporates if their systematic (idiosyncratic) risk is relatively smaller (higher). This may be the case if the size dependency conceals a dependency on the industrial sector. We call this the "business sector argument". Different sectors differ in their dependency on the business cycle and in their firm size distribution.

[4] See Crosbie (1999).

[5] Koopman et al. (2003) observe a business cycle with a period of 10 years in their empirical study with U.S. failure rates from 1927–1997.

Table 1: **Percentage of small and medium enterprises in Germany, 1997**

business sector	**percentage of small and medium enterprises**
manufacturing	15.6 %
construction	17.6 %
automotive	15.4 %
transport & communication services	31.7 %
health & financial services	27.4 %
other public & personal services	42.1 %

Therefore, if sectors which are highly cyclical are dominated by large firms whereas in less cyclical sectors SMEs prevail, then we expect to observe that systematic risk and asset correlation overall increase with firm size. In other words firm size would serve as a proxy for a business sector dependency of the asset correlation.

This hypothesis is consistent with the figures in table 1 which show the percentage of small and medium German companies in selected business sectors.[6] The first three sectors which are in general viewed as more cyclical possess a lower share of small and medium companies than the last three sectors which are in general considered as less cyclical. Considering the implication that the cyclical sectors have a relatively higher share of large companies higher asset correlation estimates for large companies may derive from this underlying sector dependency.

A second explanation for a higher asset correlation of large firms may be that they are better diversified than small firms. Because of their better diversification the idiosyncratic risk would be relatively smaller than for small firms and their correlation with the systematic risk factor relatively higher. However, empirical work by Roll (1988) casts some doubt on this hypothesis. He observed that the returns of size-matched portfolios of small firms are better explained by systematic risk factors (have a higher R^2) than the returns of large companies. This result suggests that in the contrary large firms tend to be less diversified than size–matched portfolios of small firms and, therefore, their asset correlation would in general be lower.

Whereas the previous two arguments support a positive dependency of the asset correlation on size, work by Bernanke and Gertler (1995) and Bernanke et al. (1996) suggests a converse relationship. These authors analyse adverse shocks to the economy which are amplified and propagated by changes in credit-market conditions. A key role plays the external finance premium which they define as the difference between the cost of funds

[6] The figures have been provided by the "Institut für Mittelstandsforschung" in Bonn and are based on data from the Federal Statistical Agency. They define small and medium companies as those with a yearly turnover of up to 50m EUR.

raised externally and internally. This premium arises from information asymmetries in the credit markets. It increases when the economic conditions deteriorate and collateral values decline. As a consequence firms have increasing difficulties to obtain funding even for profitable projects. This effect amplifies an economic downturn and is known in the literature as "financial accelerator".[7] The authors expect that the impact of a higher external finance premium will be stronger the more a corporate borrower has to rely on bank loans. Larger firms may to some extent be immune against this effect by tapping capital markets. However, small firms are expected to be more vulnerable against the financial accelerator because they are more dependent on bank loans. Therefore, macroeconomic shocks should have a stronger impact on SMEs which would imply a higher asset correlation.

In their empirical work with US data Bernanke et al. (1996) find, that in an economic downturn following tight money small–firm sales drop earlier and their short–term debt drops stronger compared with large firms.[8] This result still holds after controlling for industrial sector composition within size categories and suggests that asset correlation and size are negatively correlated.

The work by von Kalckreuth (2001) however suggests that the credit channel, although significant, plays only a secondary role in Germany which reduces the potential impact of the financial accelerator. A reason for this may be the traditionally strong house–bank relationship that influences the way how corporate loans are managed in German banks. This important factor ensures the availability of loans even in a downturn of the business cycle and mitigates the relevance of the financial accelerator.

In summary economic theory proposes two conflicting potential impacts on asset correlation: the business sector argument and the diversification argument suggest that asset correlation increases with firm size whereas the financial accelerator works in the opposite direction.

Compared with a potential dependence of asset correlation on firm size the theoretical arguments advocating that asset correlation decreases with PD are less developed. Two theoretical arguments for a PD-dependence are the following:

The first one is a time series argument: If the credit risk of a company increases, firm–specific risk factors become relatively more important than systematic risk and, therefore, the correlation with the systematic factor declines. This argument holds only if the deterioration in credit quality has not been initiated by the business cycle because otherwise it would have to be attributed to systematic risk. Instead, the argument holds if firm–specific events lower the credit quality and start a downward spiral.

The second argument is cross–sectional: firms that are more vulnerable to the business

[7] See Bernanke and Gertler (1995), p. 35.

[8] See Bernanke et al. (1996), p. 10-11.

cycle may choose a safer capital structure in order to account for this higher risk. Because of the more secure capital structure they have a lower probability of default.

Nickell et al. (2003) observe in their empirical work on the stability of ratings that the business cycle has a strong effect, particularly on the default rate and the volatility of rating migrations of non–investment grade bonds. They find that the volatility increases sharply in business cycle troughs for low–graded obligors which indicates a higher correlation with the systematic factor. Consistent with this result Savigny and Renault (2002) estimate higher asset correlations for non–investment grade than for investment grade companies. Their results are based on default rates of 21 years for US–companies rated by Standard and Poors.[9] However, how these results transfer to SME loans remains an open question.

Empirical work on the relation between asset correlation and PD for SME–borrowers has been carried out by Dietsch and Petey. They estimate the asset correlation in a very similar one-factor model for French corporates. They observe that for homogenous business sectors asset correlation increases with the risk of default with a noticable exception for the highest risk category.[10] A more recent study by the authors confirms these results.[11] They find that their estimates of the asset correlation increase with PD after controlling for size or for different business sectors. In summary, previous empirical results with exception of the work by Lopez (2002) contrast with the perception that asset correlation declines for higher PDs.

After this brief overview of theoretical explanations for a size- or PD-dependence of asset correlation we introduce in the following section the one-factor model and three estimation methods for the asset correlation which are afterwards used in the estimation of the asset correlation of German corporates.

3 One-Factor Model and Estimation Methodology

Default correlation and *asset correlation* are closely linked. Earlier studies have found strong evidence of correlation in the movements of the credit quality of different obligors.[12] If two obligors belong to a homogenous group sharing the same default correlation, its value can be determined from time series of defaulted and non-defaulted loans of this group without further assumptions.[13] Therefore, estimating correlation is not a problem of methodology. However, in practice we do not know firsthand which obligors build a homogenous group and, even if, estimates may be distorted by a small sample bias because regularly the available time series of default rates for loan portfolios are rather short with

[9] See Savigny and Renault (2002), table 3.

[10] See Dietsch and Petey (2002), p. 311–312.

[11] See Dietsch and Petey (2003).

[12] See Carty (1997).

[13] See Lucas (1995) where this methodology is applied.

no more than 10 yearly observations. The first problem of homogeneity will be accounted for by splitting our sample into groups according to their credit quality and firm size. The second problem of the small sample performance of the estimators has been discussed in the recent literature.[14]

A natural solution to solve the small sample problem is to pose parametric restrictions. Gordy and Heitfield (2002) improve on the efficiency of the estimation by working in a Merton–type firm value model. This framework is broadly consistent with the widely applied concept of CreditMetrics.[15] Instead of estimating default correlation first and deriving asset correlation thereafter, the authors estimate asset correlation directly from default data. If the model applies there is a one-to-one mapping, conditional on PD, between these two correlations.

The underlying one–factor model is as follows.[16] The firm value $A_{i,t}$ of obligor i follows a geometric Brownian motion. Under the usual assumptions its log-value at time t can be described as follows where μ denotes the drift rate and $Y_{i,t}$ the stochastic error term of obligor i:

$$log(A_{i,t}) = log(A_{i,0}) + \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma \sqrt{t}\, Y_{i,t}. \tag{1}$$

$Y_{i,t}$ follows a Gaussian distribution, i. e. a standard Normal distribution with mean 0 and variance 1. It is decomposed into the return of a systematic risk factor X_t and an idiosyncratic part $\epsilon_{i,t}$.

$$Y_{i,t} = \sqrt{\rho_i}\, X_t + \sqrt{1-\rho_i}\, \epsilon_{i,t}. \tag{2}$$

For every point in time t, X_t and $\epsilon_{i,t}$ are independent for every obligor i and follow a Gaussian distribution. The factor loading $\sqrt{\rho_i}$ of the systematic risk factor can be interpreted either as the sensitivity against systematic risk or as the square root of the asset correlation ρ_i of obligor i. As usual it is assumed that ρ_i does not vary over time.

Company i defaults if Y_i falls below the default threshold γ_i. The step-function $L_{i,t}$ describes if a credit event has occurred during the target horizon ($L_{i,t} = 1$) or not ($L_{i,t} = 0$) and follows a Bernoulli distribution.

In the following it is important to differentiate between the unconditional and the conditional default probability. The *unconditional* default probability of obligor i for the time span from 0 to t is defined as follows:

$$P(L_{i,t} = 1) = P(Y_{i,t} < \gamma_i) = \Phi(\gamma_i). \tag{3}$$

Let $g(x; \rho_i, \gamma_i)$ denote the default probability *conditional* on $X = x$ that is

$$g(x; \rho_i, \gamma_i) = P(L_i = 1 | X = x) = \Phi\left(\frac{\gamma_i - \sqrt{\rho_i}\, x}{\sqrt{1-\rho_i}}\right). \tag{4}$$

[14] See Gordy and Heitfield (2002) and Düllmann (2003).

[15] See Gupton et al. (1997).

[16] A more comprehensive description of the model can be found in Schönbucher (2000) and Höse and Huschens (2003).

Equation (4) provides as well the link to the proposed corporate risk–weight function of the IRB approaches in Basel II.[17] These risk weights are defined as the product of

- a factor of 12.5 (to compensate for the solvability coefficient of 0.08)
- the LGD (loss given default)
- the conditional PD given by (4) conditional on an adverse realization (99.9 % quantile) of X and
- an adjustment that accounts for the difference between the maturity of the exposure and the time horizon of one year.

The capital charge is determined by multiplying the exposure–at–default with its risk weight and the solvability coefficient of 0.08.

In order to estimate the model parameters in the one–factor model we apply three different estimation methods. The first two estimators are based on the maximum likelihood principle, whereas the other two estimators are based on the method–of–moments. The index i for a specific firm is dropped because we always refer to homogenous obligor buckets.

The first estimator of the model parameters ρ and γ is called $ML-$estimator. It uses the fact that the number of defaults D for a homogenous portfolio with n obligors for a certain time period is binomial distributed so that the conditional PD is defined as follows:

$$P(D = d|X = x) = \binom{n}{d} g(x; \rho, \gamma)^d \, (1 - g(x; \rho, \gamma))^{n-d}. \tag{5}$$

This $ML-$estimator has already been analysed by Gordy and Heitfield and involves maximizing the following log-likelihood-function $LL(\mathbf{n},\mathbf{d}; \rho, \gamma)$. Let $\mathbf{n}$ denote the $(T \times 1)-$vector of total numbers of obligors for T time periods and $\mathbf{d}$ the $(T \times 1)-$vector collecting the number of defaulted obligors.[18]

$$LL(\mathbf{n},\mathbf{d}; \rho, \gamma) = \sum_t \log(L_t(\mathbf{n},\mathbf{d}; \rho, \gamma)) \tag{6}$$

$$L_t(\mathbf{n},\mathbf{d}; \rho, \gamma) = \int_0^1 \binom{n_t}{d_t} g(\Phi^{-1}(x); \rho, \gamma)^{d_t} \, (1 - g(\Phi^{-1}(x); \rho, \gamma))^{n_t - d_t} \, d\,x. \tag{7}$$

The maximum of the log–likelihood function LL is determined numerically.

If the $ML-$estimator is applied to different size or rating buckets it is not fully efficient because it does not take into account that in a given time period the systematic factor is the same for all buckets. The second estimator takes this into account and estimates the

[17] See Basel Committee on Banking Supervision (2003).

[18] This is the estimator called *MLE 1* in Gordy and Heitfield (2002).

model parameters for all size and rating buckets simultaneously. For this property it is called full–information maximum likelihood estimator or, shortly, $FIML-$estimator. For J size or rating buckets the parameters $\rho_1, \ldots, \rho_J$ and $\gamma_1, \ldots, \gamma_J$ are estimated from the following log–likelihood function:

$$LL(\mathbf{n}_1, ...\mathbf{n}_J, \mathbf{d}_1, ..., \mathbf{d}_J; \rho_1, \ldots, \rho_J, \gamma_1, \ldots, \gamma_J) = \tag{8}$$

$$\sum_t \log \left(\int_0^1 \prod_{j=1}^J \binom{n_{j,t}}{d_{j,t}} g(\Phi^{-1}(x); \rho_j, \gamma_j)^{d_{j,t}} \, (1 - g(\Phi^{-1}(x); \rho, \gamma_))^{n_{j,t}-d_{j,t}} \, dx \right).$$

Again the maximum of the $LL-$function is determined numerically which is feasible because we use only a relative small number of three rating categories and three size buckets.

The third estimator is referred to as "method–of–moments"–estimator because the first and second moments of the conditional default probability $g(X)$ are matched with the moment estimates from the time series of default rates for $T \to \infty$. The *unconditional* PD is estimated by the average $\bar{p}$ of the time series of default rates.

$$\mathbb{E}\left[g(X)\right] = \bar{p}. \tag{9}$$

Let $\Phi_2(.)$ denote the cumulative bivariate Gaussian distribution. The following holds for the second moment:[19]

$$Var\left[g(X)\right] = \Phi_2\left(\Phi^{-1}(\bar{p}), \Phi^{-1}(\bar{p}), \rho\right) - \bar{p}^2. \tag{10}$$

The third estimator which we refer to as the "asymptotic moment estimator" (AMM) estimates $Var\left[g(X)\right]$ from the sample variance $\sigma^2_{\hat{p}}$ of the default frequencies.[20]

Note that the $ML-$ and the $MM-$estimator are applied to each rating category and size bucket separately. Only the $FIML-$estimator is fully efficient in the sense that it considers a common systematic factor.

Since all three estimators rely on asymptotic theory it is unclear which performs best in small samples. The impact of the estimation technique and the length of the time series is demonstrated in appendix 7.1 and in more detail in Düllmann (2003).

[19] See Gordy (2000), appendix C.

[20] See Blum et al. (2003), p. 118–119. A variant of this estimator has been suggested in Gordy (2000), p. 146–147. It adjusts the sample variance for the finite number of exposures in the sample from which the default frequencies are taken. In this study we use instead the AMM–estimator because a bias from the finite number is removed by calibrating the observed default rates to the whole German industry.

4 Classification into PD- and Size Categories

The Deutsche Bundesbank has granted us access to a database that contains the credit history of 53280 German firms from whom the Bundesbank has purchased fine–trade bills between 1987 and 2000.[21] Before the introduction of the Euro these fine bills were purchased at a special rate below the discount credit facility and this financing procedure has been much favored by German companies. The bank of the corporate obligor purchases a fine–trade bill from its client that is afterwards submitted to a branch office of the Bundesbank and included in the database. In this way there is no direct credit relationship between the Bundesbank and the firm. Therefore, contrary to the definition in the new Basel Accord, the default event is defined solely by legal insolvency.[22] Accordingly, the number of defaults is always conservative in the sense that it would be higher under the Basel definition of default.

The corporates included in the Bundesbank database are only a sample of all German corporates. Arguably this sample may be biased in the sense that the sector distribution in the database differs from the sector distribution of all German firms. Therefore, the default rates are calibrated in the first step to default frequencies which are representative for the German industry in order to remove a potential sector bias. A second calibration is carried out in order to explore the impact of the late definition of default on the estimates. In this second step the default rates are inferred from loan net provisions of German banks. In the following, both steps are explained in more detail.

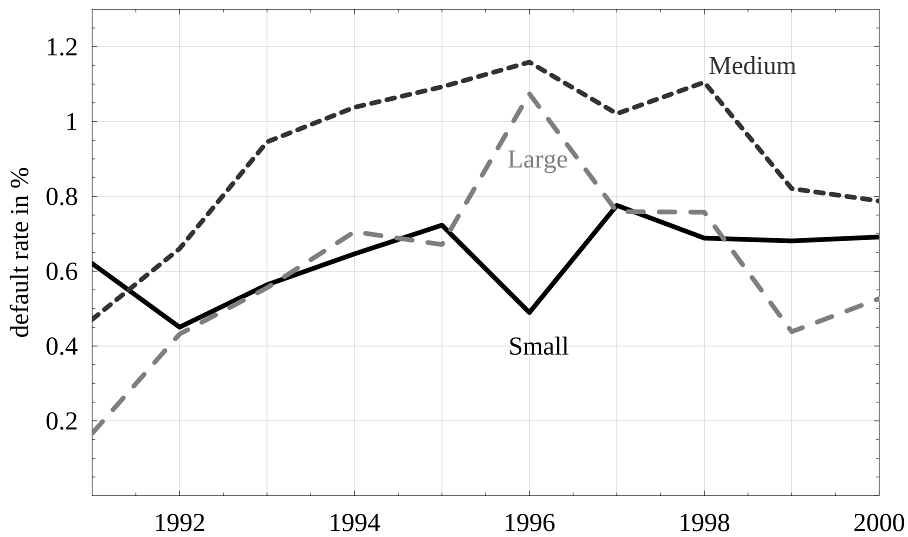
The first step in the calibration of the default rates uses the business–sector specific insolvency statistics of the Federal Statistical Office "Statistisches Bundesamt". A default rate is determined by dividing the number of defaulted obligors in year t by the total number of solvent obligors at the end of year $t-1$. The numerator of the default rate always refers to the number of all defaulted companies in the Bundesbank database. The number of firms in the denominator, however, is adjusted by the calibration to ensure that the ratio reflects the default rate from the insolvency statistics. This number of firms is then drawn from the Bundesbank database and used in the analysis of a PD- and/or size-dependency of the asset correlation. In this calibration step we account for different insolvency statistics of the three business sectors manufacturing, trade and a third sector that comprises the rest, namely firms in the service business.

The firms that are drawn from the Bundesbank database in the calibration are assigned to three different size categories. The boundaries are selected in order to ensure that the firms are relatively equally distributed among these categories. The respective intervals

[21] This is the same data base that is used in Hamerle et al. (2002). Due to data constraints and the need to have two years of balance sheet data for the calculation of a score value the time series of default rates only covers the years 1991–2000.

[22] This is laid down in the German Insolvency Code.

Figure 1: **Corporate default frequencies grouped by size between 1991 and 2000 in percent**



for size S, measured by the yearly turnover of the firm, are $0 < S \leq 5m$ EUR, $5m$ EUR$<$ $S \leq 20m$ EUR and $S > 20m$ EUR.

Figure 1 shows the time series of default rates in the three rating categories for small, medium and large firms.[23] The highest average default rate of 0.91 % (over time) is observed for medium size firms compared with 0.63 % for small firms and 0.61 % for large firms. This result is not fully consistent with other empirical evidence that the default probability increases with decreasing firm size. However, small and medium companies together have on average a higher default risk than large companies.

In order to control for a potential PD–dependency of asset correlation, we assign obligors with similar PDs to three different PD–categories which we refer to as *rating grades*. This allows estimating the asset correlation in the three categories small, medium and large firms conditional on their rating grade.

A time discrete probit–hazard model was estimated to assign the firm years into PD categories. The explanatory variables of this model are a constant, a firm specific credit score SC_{it} and a business climate index Z_t. The use of a macro–economic variable in addition to a firm–specific risk score is motivated by empirical studies concluding that such variables significantly improve the explanatory power of hazard rate models.[24]

The firm– and time–dependent hazard rate HR_{it} is determined as follows:

$$HR_{it} = \Phi(\beta_0 + \beta_1 \; SC_{it} + \beta_2 \; Z_t). \tag{11}$$

[23]See also tables 11 to 13 in appendix 7.2.
[24]See Hamerle et al. (2003).

Table 2: **Estimation results of the hazard rate model**

parameter	estimate	standard error	p–value
intercept	-1.763	0.133	$\lvert.\rvert < 10^{-4}$
credit score SC	-0.038	0.001	$\lvert.\rvert < 10^{-4}$
macro–variable Z	-0.005	0.002	0.001

Table 3: **Distribution of companies by size and sector in percent**

size	manufacturing	trade	services	total
small	31.6 %	30.3 %	38.1 %	100 %
medium	44.5 %	37.8 %	17.7 %	100 %
large	55.4 %	29.4 %	15.2 %	100 %
total	40.1 %	32.4 %	27.5 %	100 %

The estimation results for the regression coefficients, their standard errors, and their p–values are provided in table 2. All estimates are highly significant.

The boundaries of the rating categories are fixed to ensure that the defaults are relatively equally distributed among the three categories. The respective intervals of hazard rates for the three categories are $0 < HR \leq 0.01$ for grade A, $0.01 < HR \leq 0.015$ for grade B and $HR > 0.015$ for grade C. Rating grades from A to C are assigned to the PD–categories. Grade A, therefore, denotes the rating grade with the lowest and grade C the grade with the highest credit risk.

Table 3 summarizes the distribution of companies in the sample among three sectors: manufacturing, trade and a residual sector that mainly comprises of service companies. From small to large companies the percentage share of manufacturing increases from 31.6 % to 5.4 % and presumably the cyclicality. The contrary holds for the service sector whose weight in the sample decreases from small to large companies. The share of the trade sector shows no monotonic dependency on size.

The results in table 3 for the sector distribution would be consistent with the hypothesis that large companies have a higher asset correlation because of a bigger share of firms in the more cyclical manufacturing sector. If this hypothesis is supported by empirical results, is discussed in the following section.

5 Estimates of the Asset Correlation

The estimation of asset correlation is carried out in two calibration steps. In the first step the default rates of the sample from the Bundesbank database are calibrated to insolvency

Table 4: **Parameter estimates (with standard errors) from default rates calibrated to insolvency rates ($AMM-$ and $ML-$estimator)**

	$\hat{\rho}_{AMM}$			$\hat{\rho}_{ML}$		
rating	A	B	C	A	B	C
small	0.005	0.011	0.004	0.002	0.010	0.005
	(0.006)	(0.010)	(0.006)	(0.001)	(0.005)	(0.002)
medium	0.007	0.012	0.018	0.007	0.011	0.016
	(0.006)	(0.012)	(0.014)	(0.003)	(0.005)	(0.007)
large	0.021	0.015	0.064	0.013	0.016	0.045
	(0.011)	(0.021)	(0.036)	(0.006)	(0.007)	(0.020)

statistics. The calibrated default frequencies are listed for the size and rating grade buckets in tables 11 to 13 in appendix 7.2. From the estimation methods described in section 3 the $AMM-$ estimator and the $ML-$estimator are employed. The number of exposures for the $ML-$estimator is increased until the estimated values of ρ are stable. We think that these "asymptotic" estimators for an infinite or a "very large" number of exposures are more appropriate because the calibration to German insolvency statistics ensures that the default rates hold for the whole universe of German firms.

The estimates of ρ for the selected size– and PD–categories are summarized in table 4 for the $AMM-$ and the $ML-$estimator. The estimation results show a relatively low absolute level of the asset correlation. The strongest increase is observed for the lowest rating category C. This is consistent with estimation results for asset correlation in Dietsch and Petey (2002) for French corporates.[25]

The estimates of ρ in table 4 increase with size for all three rating grades and under both estimation methods. This result supports the hypothesis that asset correlation is positively correlated with firm size. It contrasts, however, with the observation in Dietsch and Petey (2003) that inside the SME–segment asset correlation decreases with firm size. Consistent with our results they observe the highest asset correlations for large corporates.

The standard errors in table 4 are determined by bootstrapping for the $AMM-$ and analytically for the $ML-$estimator. Especially for small correlation estimates the standard errors from bootstrapping are considerably higher than the asymptotic ones. This suggests that the asymptotic standard errors may be significantly underestimated. Due to the short time series of default data, the standard errors are relatively high, especially for the $AMM-$estimates. Although the ranking of the estimates for ρ in table 4 supports the hypothesis of a size-dependency, it is, therefore, difficult to prove this relation statistically.

In the second calibration step the default frequencies are inferred from loan net provisions

[25]See Dietsch and Petey (2002), p. 311–312.

of German banks. In this way we take into account a potential estimation bias that derives from the late legal definition of default and produces relatively low default rates.

The calibration to loan net provisions, however, cannot control for a potential bias that is induced by size-dependent differences in the distribution of default events. Legal default, for instance, is more common for small firms than for large companies.

The data on loan net provisions are extracted from the OECD-report on 'Bank Profitability - Financial Statements of Banks'. They are available for different banking groups from which we have selected the group of 'commercial banks'.

The transformation of the loan net provisions of year t into an inferred default-rate is carried out by the following formula in which LGD denotes the loss given default:

$$DR_{OECD}(t) = \frac{Loan\ Net\ Provisions(t)}{Loan\ volume(t-1) \times LGD}. \tag{12}$$

The calibrated default rate $DR^{cal}(b,g,t)$ for size bucket b and rating grade g in year t is determined from the default rates of all borrowers of this size bucket in that year, $DR(b,t)$, the obligors of this rating grade, $DR(b,g,t)$, and the default rate from the OECD-data, $DR_{OECD}(t)$:

$$DR^{cal}(b,g,t) = \frac{DR_{OECD}(t)}{DR(b,t)}\, DR(b,g,t). \tag{13}$$

The LGD is set to 50 % which equals the LGD–assumption of uncollateralised loans in the 2nd consultative paper of the new Basel accord. This assumption is made only to avoid severe empirical problems with estimating the LGD separately. Note that recent research suggests that the LGD should be modeled as a stochastic variable that is correlated with the PD.[26] In our case keeping LGD constant means that all variability in the loan provisions is transferred into the calibrated default rates. Therefore, the default rates behave more volatile than in the case of an LGD that is subject to systematic risk. This may distort the estimates of the asset correlation.

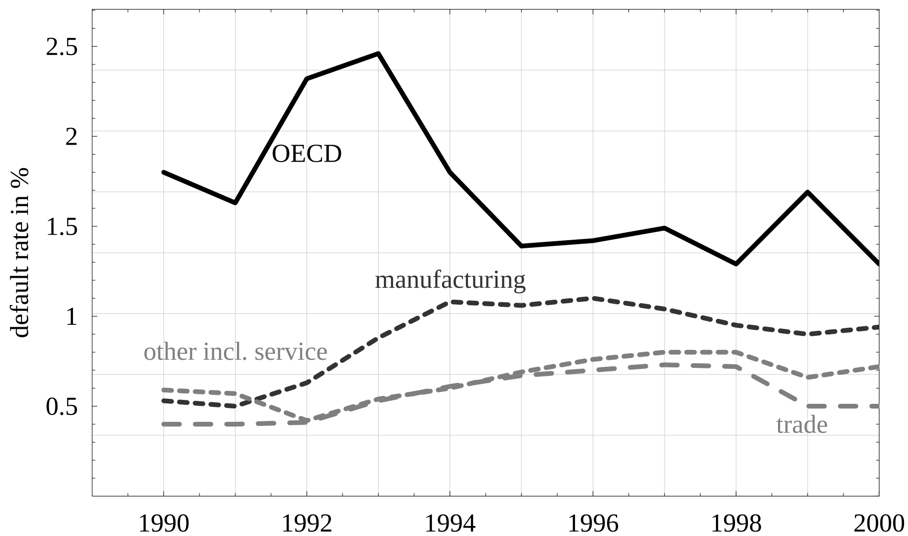
Figure 3 shows the default rates calibrated to insolvency statistics for the three business sectors manufacturing, trade and others (including services) and compares them with the default rates inferred from the loan net provisions. The following observations are noteworthy:

The default rates from loan net provisions are overall on a higher level. This may be explained by the fact that loan net provisions provide in general an "earlier" default criterion than insolvency and not all obligors for whom specific provisions have been made enter insolvency at a later stage.

The default rates determined by loan net provisions appear to be be more volatile. This observation may suggest that they are more sensitive to the business cycle and estimates of the asset correlation from these data may be higher than those from insolvency rates.

[26] See e. g. Hu and Perraudin (2002) or Altman et al. (2002) and further references given there.

Figure 2: **Corporate default frequencies calibrated to loan loss provisions and grouped by sector between 1991 and 2000 in percent**



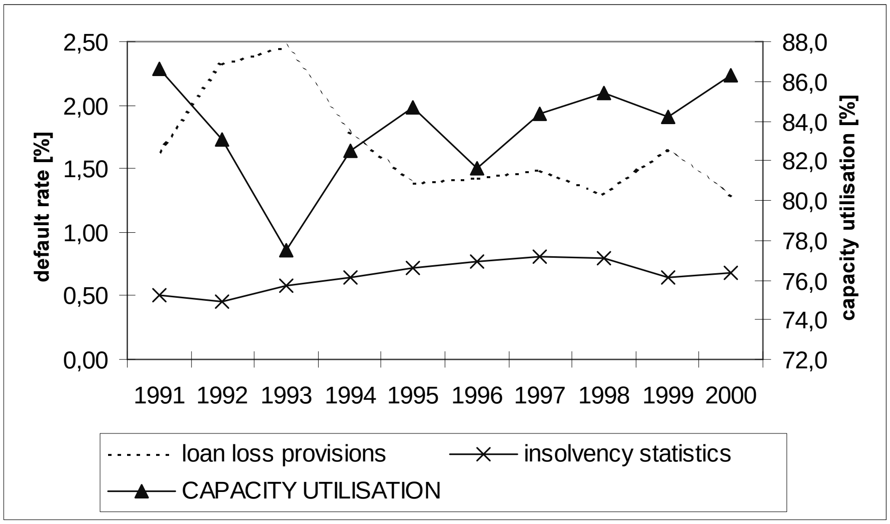
Furthermore, from the default rates of the three business sectors those for manufacturing are the most volatile. This observation is consistent with the perception that this sector is more cyclical than the others. A plausible reason may be a stronger cyclicality of this sector. Together with a relatively high share of large companies in this sector according to table 3 this may partly explain the empirical results indicating that asset correlation increases with firm size.

Based on the calibrated default rates $DR^{cal}(g,t)$ the asset correlation is estimated again with the $AMM-$estimator. The results are given in table 5. The level of the estimated asset correlation is higher for the calibrated data in table 5 than in table 4. The maximum is observed again for large corporates with the lowest rating grade but with 0.14 it is

Table 5: **Parameter estimates (with standard errors) from default rates calibrated to loan net provisions ($AMM-$estimator)**

	$\hat{\rho}_{AMM}$		
rating	A	B	C
small	0.012	0.046	0.032
	(0.008)	(0.028)	(0.020)
medium	0.016	0.033	0.075
	(0.010)	(0.024)	(0.043)
large	0.021	0.049	0.140
	(0.014)	(0.045)	(0.083)

Figure 3: **Corporate default frequencies calibrated to loan loss provisions and insolvency rates and, for comparison, the capacity utilisation in Germany between 1991 and 2000 in percent**



2.2times higher than in table 4. Again we observe that the asset correlation overall increases with size conditional on the rating grade. The only exception occurs for rating grade B where the estimated asset correlation of small companies is slightly higher than for medium size corporates. For the lowest rating grade C it increases with size quite strongly from 0.03 to 0.14.

Table 6 shows the estimates of the asset correlation by applying the $ML-$estimator. The estimates overall increase with firm size for all three PD-categories. The only exception occurs again for grade B in the transition from small to medium enterprises.

Moving the focus towards a potential PD-dependence of the asset correlation we observe in table 6 that the asset correlation increases with PD for medium and for large enterprises. Both findings reflect previous results from the $AMM-$estimates in table 5. Note, that although both estimators provide similar results as to the ranking of asset correlations of PD- and size categories the level of the estimates differs, particularly for large companies with high PDs (0.094 for $ML-$ vs. 0.140 for $AMM-$estimates). In order to examine if this difference can arise from estimation error we simulate both estimators with a given parameter set of $\rho = 0.1$ and $PD = 0.1$. Whereas the mean estimates (0.094 for $ML-$ and 0.099 for the $AMM-$estimator) are close to the true values, the mean squared error in both cases is with 0.054 and 0.048 rather high. This result indicates that the differences between table 5 and table 6 may be caused by estimation error.

In the one-factor model and in a homogenous portfolio an estimate $\hat{\rho}^{def}$ of the default correlation between two firms can be determined directly from the estimates of the PD

Table 6: **Estimates of asset correlation and default correlation from default rates calibrated to loan loss provisions ($ML-$estimator)**

	$\hat{\rho}_{ML}$			$\hat{\rho}_{ML}^{def}$		
rating	A	B	C	A	B	C
small	0.009	0.040	0.025	$\lvert\hat{\rho}\rvert < 10^{-3}$	0.007	0.006
	(0.004)	(0.017)	(0.011)			
medium	0.012	0.036	0.057	$\lvert\hat{\rho}\rvert < 10^{-3}$	0.007	0.018
	(0.005)	(0.016)	(0.024)			
large	0.016	0.053	0.094	$\lvert\hat{\rho}\rvert < 10^{-3}$	0.012	0.033
	(0.007)	(0.022)	(0.040)			

Table 7: **Estimates of asset correlation and default correlation from default rates calibrated to loan loss provisions ($FIML-$estimator)**

	$\hat{\rho}_{FML}$			$\hat{\rho}_{FML}^{def}$		
rating	A	B	C	A	B	C
small	0.009	0.032	0.023	0.001	0.006	0.006
	(0.006)	(0.015)	(0.010)			
medium	0.009	0.022	0.057	0.001	0.005	0.019
	(0.005)	(0.011)	(0.020)			
large	0.011	0.013	0.068	0.001	0.003	0.026
	(0.007)	(0.016)	(0.035)			

and the asset correlation.

$$\hat{\rho}^{def} = \frac{\Phi\left(\Phi^{-1}(\hat{p}), \Phi^{-1}(\hat{p}), \hat{\rho}\right) - \hat{p}^2}{\hat{p}\,(1-\hat{p})}. \tag{14}$$

The estimates of $\hat{\rho}^{def}$ are provided in the right section of table 6. Due to the low level of the estimates of the asset correlation the default correlation between firms in the best rating class is less than 0.1 %. In the rating classes B and C we observe that default correlations increase with size. The highest estimate of 3.3 % is observed for large firms in rating category C.

Table 7 provides estimation results for the asset correlation in all nine size and rating buckets if the $FIML-$estimator is applied. This estimator is more efficient than the $ML-$estimator if the systematic risk is driven by a common systematic factor. The results notably differ from those with the $ML-$estimator in table 6 in two cases: The asset correlation estimates for large corporates are smaller in the rating classes B and C. Contrary to previous results where the parameters were estimated bucket for bucket, a declining asset correlation for larger firms is no longer observed for rating category B. However, the size–dependence still holds in rating categories A and C.

The joint ML–estimation is more efficient only if the one–factor assumption holds that imposes a strong restriction on the model. The Vuong test is applicable as a test of model specification to nested as well as non–nested models. The Vuong–statistic τ_{vuong} asymptotically follows a Gaussian distribution:

$$\tau_{vuong} = \frac{LL_{unrestricted} - LL_{one\ factor\ assumtion}}{\sqrt{N\omega^2}} \tag{15}$$

ω^2 is defined as the sample variance of the differences between the log–likelihood contributions and N as the sample size. The null hypothesis that both models are equivalent has to be rejected on a 99% confidence level. This implies that even if the systematic risk is adequately modelled by a single factor, this factor is not the same for all size– and PD–categories. Therefore, the estimation results for the $ML-$estimator may be more robust because they do not depend on the additional assumption of a *common* factor for *all* buckets. In the following we use the $ML-$estimates as reference.

Envoking the Wald principle we apply a statistical test in order to answer the question if the observed differences in the estimates of ρ for portfolios of different firm size are statistically significant. If this is the case we should at least observe a significant difference between the estimates of asset correlation of large and of small firms. The following test statistic

$$\frac{\hat{\rho}^{large} - \hat{\rho}^{small}}{\sqrt{(\hat{\sigma}^{large})^2 + (\hat{\sigma}^{small})^2}} \tag{16}$$

asymptotically follows a Gaussian distribution.[27] We test the null hypothesis, that the estimates of ρ are the same for the categories of large and small firms in every rating class. The p–values are given in table 8.

We can reject the null hypothesis on a confidence level of 95% for three out of four estimates in the lowest rating category. For the other two rating categories it is mostly not possible to reject the null hypothesis.

Three out of four rejections of the null hypothesis in table 8 occur for the ML-estimator. The standard errors for this estimator are overall smaller than for the $AMM-$estimates because they are asymptotic values, determined analytically. This may be the reason that the null hypothesis is more often rejected for the $ML-$estimates.

We conclude that although the estimates indicate that asset correlation depends on size, this relation is only in some cases statistically significant on the usual confidence levels. The latter result may be driven by a relatively high estimation error that is due to the short time series of data.

[27]Note, that this test implicitly takes into account that the systematic risk factors of the two buckets are different. However, it does not account for a correlation between these factors

Table 8: **P-values for mean-test of differences between estimates of ρ for large and small firms**

estimator	**default rates** calibrated to ...	**rating** A	B	C
AMM	insolvency rates	0.114	0.432	*0.049*
	loss Provisions	0.292	0.473	0.103
ML	insolvency rates	*0.035*	0.243	*0.023*
	loss Provisions	0.193	0.320	*0.048*

6 Comparison of Empirical Estimates with Supervisory Values

Next we compare these empirical results with the prospective IRB risk weights of the new Basel Accord. Since the publication of the second consultative document in January 2001 two notable changes have been made to the risk weights for corporate obligors from which the capital charge can be determined. These two changes involve the parameter asset correlation that was originally fixed at 0.2. The asset correlation parameter in the risk–weight function of the corporate portfolio, proposed in the third consultative document from April 2003 varies between 0.12 and 0.24, decreasing with the obligor's PD. Firms in the corporate portfolio with a yearly turnover below 50 m EUR are treated as SMEs. They receive a capital relief dependent on their firm size which is measured by yearly turnover. As a consequence of the combined dependency on PD and firm size, the asset correlation for SMEs varies between 0.08 and 0.24. Subject to a "use test" small companies with an exposure of less than 1 m EUR can be assigned to the retail portfolio. The corresponding asset correlation is lower than in the corporate portfolio and varies between 0.02 and 0.17 depending on the PD of the borrower but not on its turnover.[28]

The first modification in the third consultative paper, which assumes that asset correlation declines with PD, is not supported by our empirical results. Whereas the results are mixed for the default rates calibrated to the insolvency statistics the results for the calibration to loan loss provisions suggest that asset correlation increases monotonicly with higher PDs in two out of three size classes. This increase is stronger for larger corporates.

The second modification that asset correlation increases with size is qualitatively backed by the correlation estimates in tables 4, 5, 6 and for two out of three rating categories in table 7. It holds, therefore, independent of the calibration and of the estimation method. The increase is strongest for the category with the highest credit risk. This result may derive from the distribution of companies among business sectors, e. g. that the share of

[28]See Basel Committee on Banking Supervision (2003).

firms in cyclical sectors increases with firm size.

Turning to correlation estimates based on the calibration to loan loss provisions we find again that asset correlation overall increases with firm size. Summarizing, our results support a size-dependent capital relief for SMEs in the IRB risk weights.

If we compare the level of the asset correlation parameters in the risk–weight function of the corporate portfolio with our empirical estimates, we find that the estimates are lower than the values set by supervisors. Our estimates are more in line with the supervisory values for the retail portfolio. But even for retail firms with a yearly turnover of up to 5 m EUR[29] the highest estimate of 0.05 occurs in the risk–weight function only for PDs beyond 5% which is well beyond an average PD for obligors in this size class.

In their work on the asset correlation of SMEs Dietsch and Petey (2003) observe as well this discrepancy between supervisory values and their empirical estimates. They put forward two explanations. First the time series of observations may be too short so that default rates appear to be more stable than they are over the full cycle. However, our time series of default rates starts in 1991 instead of 1997 and includes several years with high default rates so that this reason may not provide a sufficient explanation.

Second, in both studies asset correlations have been estimated from relatively large, quasi–exhaustive samples of businesses. Therefore, the estimated asset correlations are broadly representative for the German economy, but a single bank will usually have a much smaller loan portfolio and as a consequence observe higher asset correlations.

In addition to these arguments, the relatively high standard errors provide additional support for setting more conservative values of the asset correlations. Given the data limitations the empirical lessons about the direction, how asset correlation depends on PD and firm size may be more robust than the level of the correlation estimates.

7 Conclusion

In this paper the asset correlation as a measure of systematic credit risk is estimated from a database of balance sheet information and ten years of default rates of German corporates that is maintained at the Bundesbank. The relatively high standard errors provide a general caveat for all empirical analyses of the asset correlation based on small samples.

The design of the estimation procedure is focused on exploring a potential dependence of asset correlation on PD and firm size. The results from default rates, calibrated to German insolvency statistics, show that aggregated over all rating categories as well as for single

[29] This limit for firms in the retail portfolio has been applied on a national basis in Germany for the third quantitative impact study from October, 2002 but is not included in the third consultative document.

rating grades the asset correlation increases with size. However, we do not observe an unambiguous dependence on PD. The absolute level of the asset correlation is relatively low (between 0.002 and 0.06). The same estimation carried out with default rates implied by net loan loss provisions of German banks show higher asset correlations (between 0.01 and 0.14). Again we find that asset correlation overall increases with firm size but we do not observe an unambiguous relation between asset correlation and PD. The results are robust with respect to the applied estimator.

Comparing the results with the currently proposed calibration of the IRB–risk weights for corporate loans in the new Basel Accord needs a word of caution. These risk weights are calibrated from a macro–prudential as well as a micro–prudential perspective. The decision to have an asset correlation parameter declining with PD, for instance, is justifiable by the desire to reduce pro-cyclical effects of the New Accord. In this paper instead, we deal only with the micro-perspective and in this case our estimation results indicate a converse but more ambiguous relationship between asset correlation and PD. The modification that asset correlation increases with size which has been introduced into the risk weights but is restricted to SMEs is overall corroborated by our estimates. This relation exists in all of the obligors PD–categories and seems to be stronger for obligors with higher PDs. The empirical results for a size- and a $PD-$dependence of asset correlation are relatively robust with respect to the chosen estimation technique. The size–dependence seems to be weaker for the (joint) $FIML-$estimates that take into account a common systematic factor for all size– and PD–buckets. However, the Vuong test as a test of model specification rejects the hypothesis of a common systematic factor for all buckets. Therefore, we rate the ML-estimates without this restriction, which show a strong size–dependence, as more meaningful.

Considering the conflicting economic effects that may cause a size–dependency, more work is necessary to better understand which factors determine the size–dependency that we have observed. There is tentative evidence that differences in the sector distribution of the size–categories contribute to higher asset correlations for larger firms. According to this "business sector argument" larger companies are more common in cyclical sectors whereas small and medium companies are more concentrated in the less cyclical service sector.

Our results suggest that further research is warranted for the estimation of asset correlations. Several studies have recently addressed this issue, motivated by its importance for portfolio models of credit risk as well as by the Basel II model that underlies future minimum capital requirements for banks. Notable differences have occurred between the empirical results of these studies. This diversity underlines not only the importance of further empirical research but even more advocates a stronger focus on the economic factors that can explain the observed differences.

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Appendix

7.1 Small sample properties of the $ML-$ and $AMM-$estimator for PD and asset correlation

Since the small sample performance of the estimators presented in section 3 is unknown Monte Carlo simulations of the one–factor model are carried out to provide a guideline which estimation technique is more accurate for a relevant small sample size. In addition to the $ML-$ and the $AMM-$ estimator from section 3 we apply a method–of-moments estimator with the small sample correction proposed by Gordy[30]. In order to differentiate between the two method–of–moments estimators it is called (finite) $FMM-$estimator.

For the MC simulations the number of issuers N is set to 1000, the number of simulation runs S is set to 1500 and the number of time periods T varies between 5 and 31 periods. In order to base the DGP for the simulation runs on estimates from a realistic sample we select default histories of bond data, rated by Moody's and aggregated over all speculative grade rating classes.[31] The $ML-$estimates $\rho_0 = 0.098$ and $\gamma_0 = -1.805$ (corresponds to an unconditional PD of 3.55 %) serve as starting values for the simulation runs.

The relevant statistics of the estimation error are given in table 9. The bias is defined as the difference between the mean of the simulation estimates and the true value ρ (here 0.098). A positive value indicates, therefore, an upward bias of the estimator. In general the figure of 31 observation periods is far beyond the sample size that is typically available for bank loan loss data. To explore how shorter time series of default frequencies affect the estimates of ρ we gradually reduce T from 31 to 20, to 10 and finally to 5 observations. The figure of 5 observations is consistent with the number of years of data that will ultimately be required by banks following the IRB approach of the new Basel accord.[32]

Table 9: **Sample statistics of $ML-$, $AMM-$ and $FMM-$estimates of the asset correlation ρ (true value: 0.09)**

sample size	**bias**			**standard error**		
	ML	AMM	FMM	ML	AMM	FMM
5	-0.020	-0.012	-0.017	0.056	0.055	0.056
10	-0.010	-0.006	-0.011	0.042	0.044	0.044
20	-0.004	-0.002	-0.006	0.032	0.035	0.035
31	-0.003	< 0.000	-0.004	0.025	0.029	0.029

[30]See Gordy (2000), p. 146–147.
[31]See Hamilton et al. (2001), p. 45–46.
[32]For a transition period a shorter time series of 2 years will be deemed sufficient.

Table 10: **Sample statistics of $ML-$ and $AMM-$ estimates of the default probability PD (true value: 0.0355)**

sample size	**bias**		**standard error**	
	ML	AMM	ML	AMM
5	-0.0009	< -0.0011	0.013	0.013
10	0.0005	0.0001	0.009	0.009
20	0.0005	< 0.000	0.007	0.006
31	0.0002	< 0.000	0.006	0.005

Table 9 shows how the accuracy of the estimates increases with sample size. For the $ML-$estimates with 31 observations the standard error is 2.5 bp compared with 5.6 bp for 5 observations in time. This means that for a Basel II-compliant period of 5 years the standard error is as high as 57 % of the true value of $\rho_0 = 0.098$. These numbers reveal the wide margin of error if asset correlation has to be estimated from small samples.

Table 10 presents the PD–estimates for the $ML-$ and the $AMM-$estimation method. The bias and the standard errors of the estimates for a sample size of 10 and higher are below 1 bp. Only for a time series of 5 years the standard error increases to 1.3 bp for both estimation methods. Considering the absolute level of the unconditional PD which is 3.55 % the bias is rather small. However, for time series up to five years the standard error is relatively high as can be observed from the 95 % confidence interval from 0.004 to 0.065. The relatively wide confidence interval shows the difficulty in estimating correlated default probabilities from short time series.

7.2 Default Rates calibrated to Insolvency Statistics

Table 11: **Default frequencies of small firms for different rating grades between 1991 and 2000 in percent**

year	all	grade A	grade B	grade C
1991	0.62	0.34	1.47	2.52
1992	0.45	0.25	1.18	1.72
1993	0.56	0.26	1.45	2.06
1994	0.65	0.28	1.43	1.90
1995	0.72	0.33	1.11	2.12
1996	0.49	0.28	0.69	1.31
1997	0.78	0.41	1.11	2.17
1998	0.69	0.33	0.83	2.06
1999	0.68	0.27	1.09	2.16
2000	0.69	0.27	1.76	2.04
Average	0.63	0.30	1.21	2.00

Table 12: **Default Frequencies of medium firms for different rating grades between 1991 and 2000 in percent**

year	all	grade A	grade B	grade C
1991	0.47	0.25	1.72	3.31
1992	0.66	0.40	1.78	4.74
1993	0.95	0.54	2.76	5.83
1994	1.04	0.47	3.20	3.62
1995	1.09	0.40	3.19	5.03
1996	1.16	0.55	1.72	6.25
1997	1.02	0.55	2.62	3.53
1998	1.10	0.59	1.83	4.57
1999	0.82	0.39	2.56	2.83
2000	0.79	0.46	1.82	2.81
Average	0.91	0.46	2.32	4.25

Table 13: **Default Frequencies of large firms for different rating grades between 1991 and 2000 in percent**

year	all	grade A	grade B	grade C
1991	0.17	0.15	0.82	0.00
1992	0.43	0.26	1.94	4.55
1993	0.56	0.39	1.50	3.55
1994	0.71	0.30	1.71	4.72
1995	0.67	0.31	1.43	5.52
1996	1.07	0.52	1.30	7.50
1997	0.76	0.38	2.17	4.64
1998	0.76	0.45	2.61	2.82
1999	0.44	0.20	1.79	2.55
2000	0.53	0.35	2.32	1.31
Average	0.61	0.33	1.76	3.72