Modeling the Recovery Rate in a Reduced Form Model

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by

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Introduction

Two approaches for pricing and hedging credit risk: *structural* (Merton (1974)) and *reduced form* (Jarrow and Turnbull (1992, 1995)).

An information based synthesis (Jarrow and Protter (2004)).

- *Structural models* (Management's perspective, complete information) continuous knowledge of the firm's asset value process. Default is the first hitting time of a barrier and for a continuous sample path process, default is a predictable stopping time.
- *Reduced form* (Market's perspective, incomplete information) discontinuous knowledge of the firm's asset value process. Default is the first jump time of a point process, an inaccessible stopping time.

Structural models - parameters can only be estimated implicitly via the use of market prices.

Reduced form models - by construction - are based on observables - the parameters can be estimated directly.

Focus on reduced form models...

Reduced form credit risk pricing of risky debt requires modelling two quantities:

(1) the *likelihood of default* (or the *default intensity*, if it exists),

(2) the *recovery rate* in the event of default.

Emphasis in the literature has been on modelling and estimating the default intensity.

The *recovery rate* usually assumes a simple form, either:

(1) a constant proportion of the firm's debt value at the instant before default (called the "recovery of market value"), or

(2) a constant proportion of an otherwise equivalent Treasury's value at default (called the "recovery of face value").

Existing models terminate if default occurs (default is an absorbing state).

This implies that the models only price risky debt *prior to* default.

But, markets exist for defaulted debt and the debt trades for a "long" time. And, prices of defaulted debt are the primary input to reported estimates of recovery rates (Moody's (1999), (2000), Altman, Brady, Resti, Sironi (2003), and Acharya, Bharath, Srinivasan (2004)).

Modelling defaulted debt prices, therefore, is important for computing realized recovery rates and the pricing of credit derivatives (e.g. CDS and recovery rate swaps (a possible new credit derivative)).

Exhibit 5

Descriptive Statistics for the Length of Time Spent in Bankruptcy

Bankruptcy Type	Count	Average (yrs)	Median (yrs)	Minimum (yrs)	Maximum (yrs)	Standard Deviation (yrs)
Chapter 11	116	1.68	1.43	0.19	7.01	1.20
Prepackaged Chapter 11	43	0.29	0.18	0.09	1.76	0.29
All	159	1.30	1.15	0.09	7.01	1.20

Moody's
Debt Recoveries for
Corporate
Bankruptcies
(June 1999)

Exhibit 21

Median & Average Defaulted Bond Prices, 1999 vs. Historical

	1999		1970-1998	
	Median	Average	Median	Average
Senior/Secured	$33.00	$43.08	$53.00	$52.31
Senior/Unsecured	$42.00	$46.72	$48.00	$48.84
Subordinated	$21.00	$30.34	$30.00	$33.17
Preferred Stock	$4.19	$10.92	$9.13	$11.06

Moodys
Historical Default
Rates of Corporate
Bond Issuers,
1920-1999
(Jan 2000)

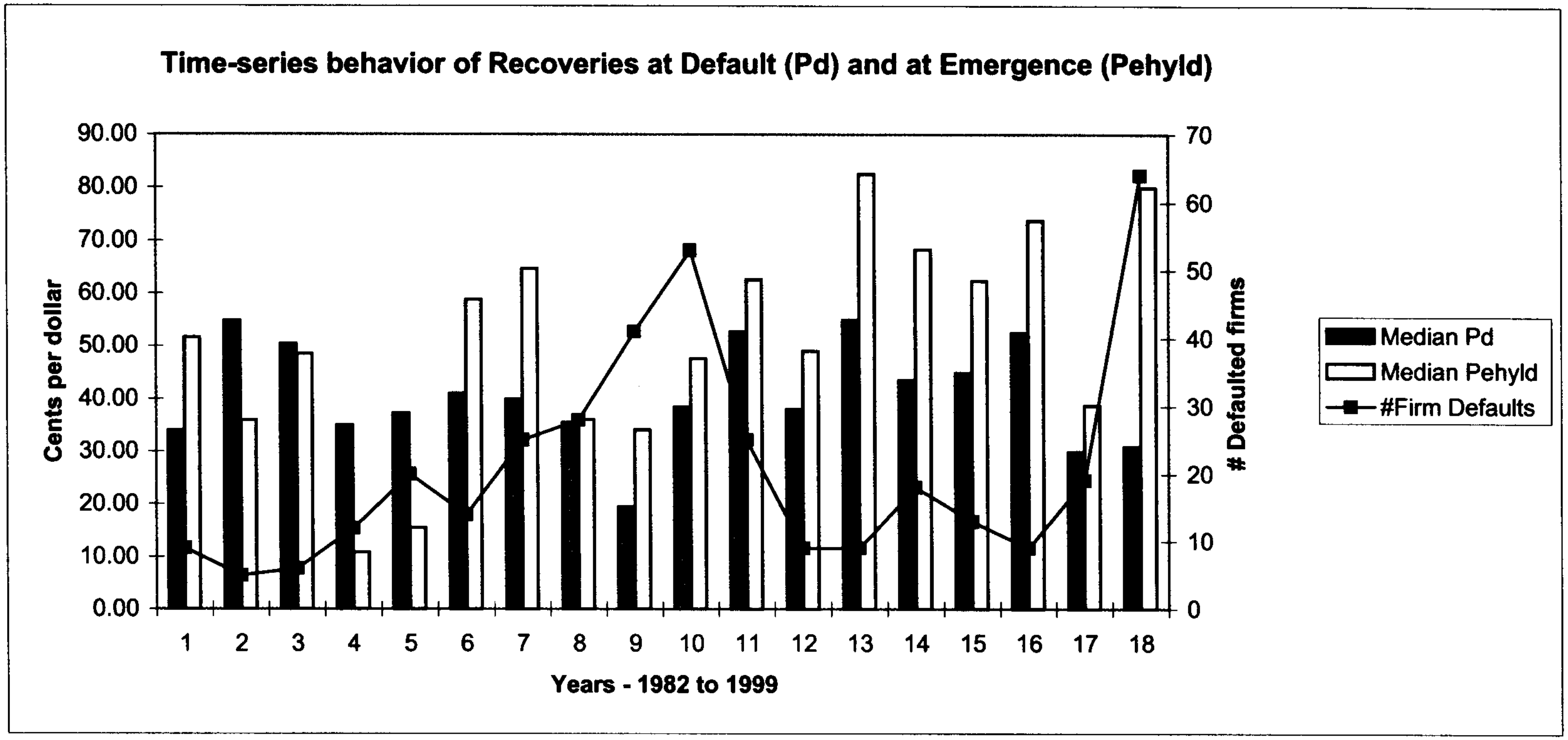


Figure 1: Time-series behavior of Recoveries at Default (Pd) and at Emergence (Pehyld)
This figure plots the time-series variation in the number of firm defaults (corresponding to Pd series), median recovery price at default (Pd) in each year, and median recovery price at emergence (Pehyld) in each year.

Acharya, Bharath, Srinivasan
Understanding the Recovery Rates on Defaulted Securities
2004

The purpose is to present a model of the firm's debt prices (before and *after* default) and the realized recovery rate in the context of a reduced form model.

We distinguish between default, insolvency, and bankruptcy (Jarrow and Purnanandam (2004)).

We use insights from the structural approach to model the realized recovery rate based on the firm's assets. To retain the reduced form structure, we employ the information reduction methodology of (Guo, Jarrow and Zeng (2005)).

We show that modelling the recovery rate process also has an important impact on the debt prices *before* default.

For clarity of presentation, we construct the model in its simplest form.

We choose processes so that "closed form" solutions are attainable. Closed form solutions facilitate intuition, understanding, and estimation.

Almost every aspect of the model’s structure is easily generalized, at the expense of obtaining more abstract representations, numerical approximations, and more difficult estimation procedures.

The General Framework

Traded are:

(1) a term structure of default free zero-coupon bonds and

(2) a firm's risky zero-coupon bond.

The firm's risky zero-coupon bond will represent a promised $\$1$ to be paid at some future date T.

If the firm defaults prior to time T, then there will be a recovery rate, between zero and one, paid per promised dollar, details later...

The Firm's Asset Value Process

Two representative processes for the firm's asset value $(X_t)_{t\geq 0}$.

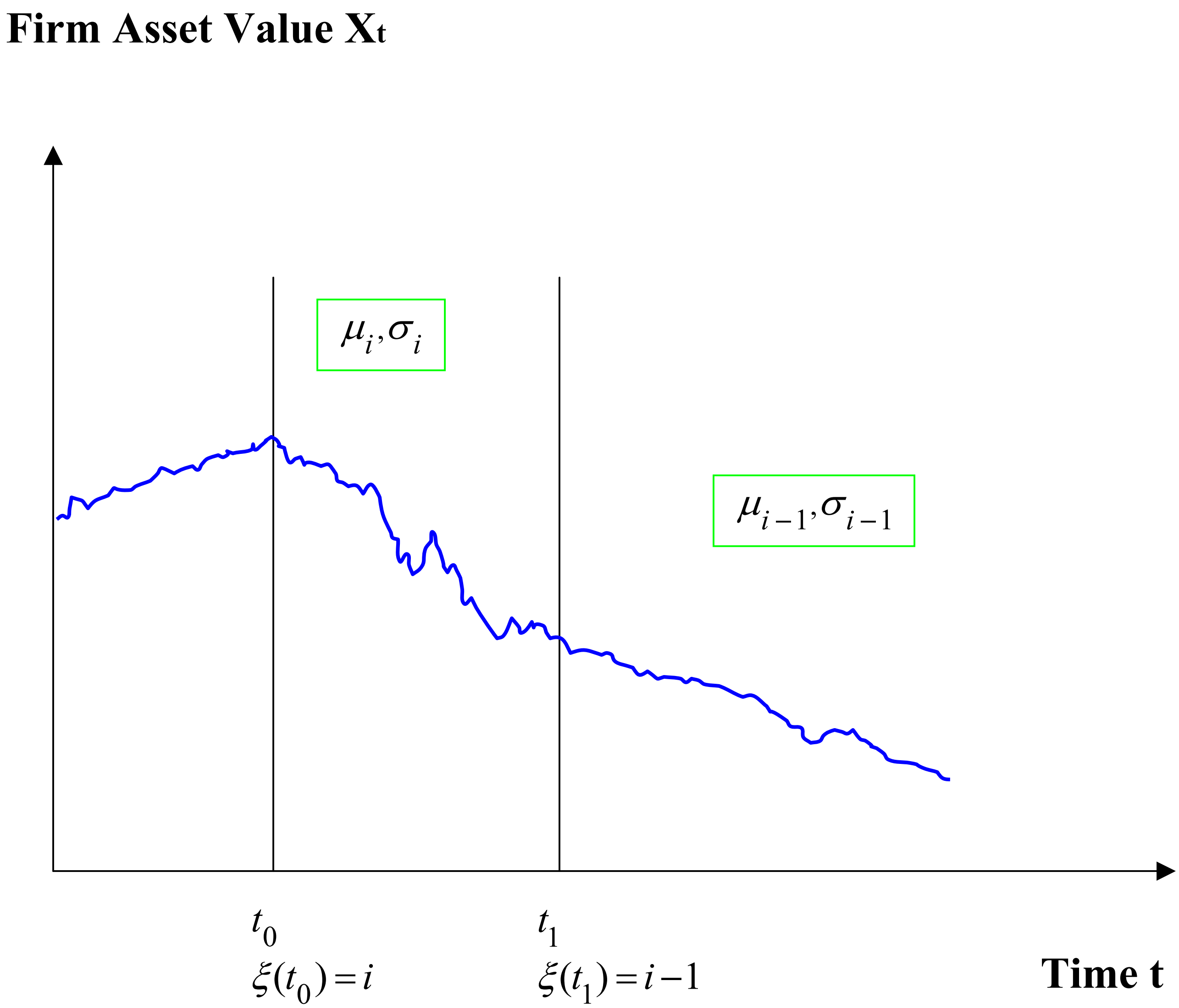
A Regime Switching Model

$$dX_t = X_t \mu_{\epsilon(t)} dt + X_t \sigma_{\epsilon(t)} dW$$

W is a standard one-dimensional Brownian motion

$(\epsilon(t))_{t\geq 0}$ is a finite-state continuous-time Markov chain, independent of W, taking values $0, 1, \cdots, S-1$ with a known generator $(q_{ij})_{S\times S}$. Let $q_i = \sum_{j\neq i} q_{ij}$.

The drift and volatility coefficients $\mu(\bullet), \sigma(\bullet)$ are functions of ϵ.



Regime Switching Model

Economic Interpretation

The state space $\epsilon(t)$ represents the firm's credit ratings, with $S - 1$ being the highest and 1 being the lowest rating. The last state 0, represents default.

$\epsilon(t)$ represents publicly available information.

Default is the random time τ given by

$$\tau = \inf\{t > 0 : \epsilon(t) = 0\}.$$

The default time has an intensity

$$\lambda_t = q_{\epsilon(t)0}.$$

Consistent with reduced form models.

The Jump-diffusion Model

$$X_t = X_0 e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t} \prod_{0 < s \le t, \Delta\epsilon(s) \ne 0} \xi_{\epsilon(s)}.$$

W and ϵ are as in regime switching model. Denote T_n the n-th jump time of ϵ.

ξ_i represents the jump amplitude in state i with distribution function F_i where $P(\xi_i = 1) = 0$.

Since 0 is the default state, we assume $P(\xi_0 \ge 1) = 0$.

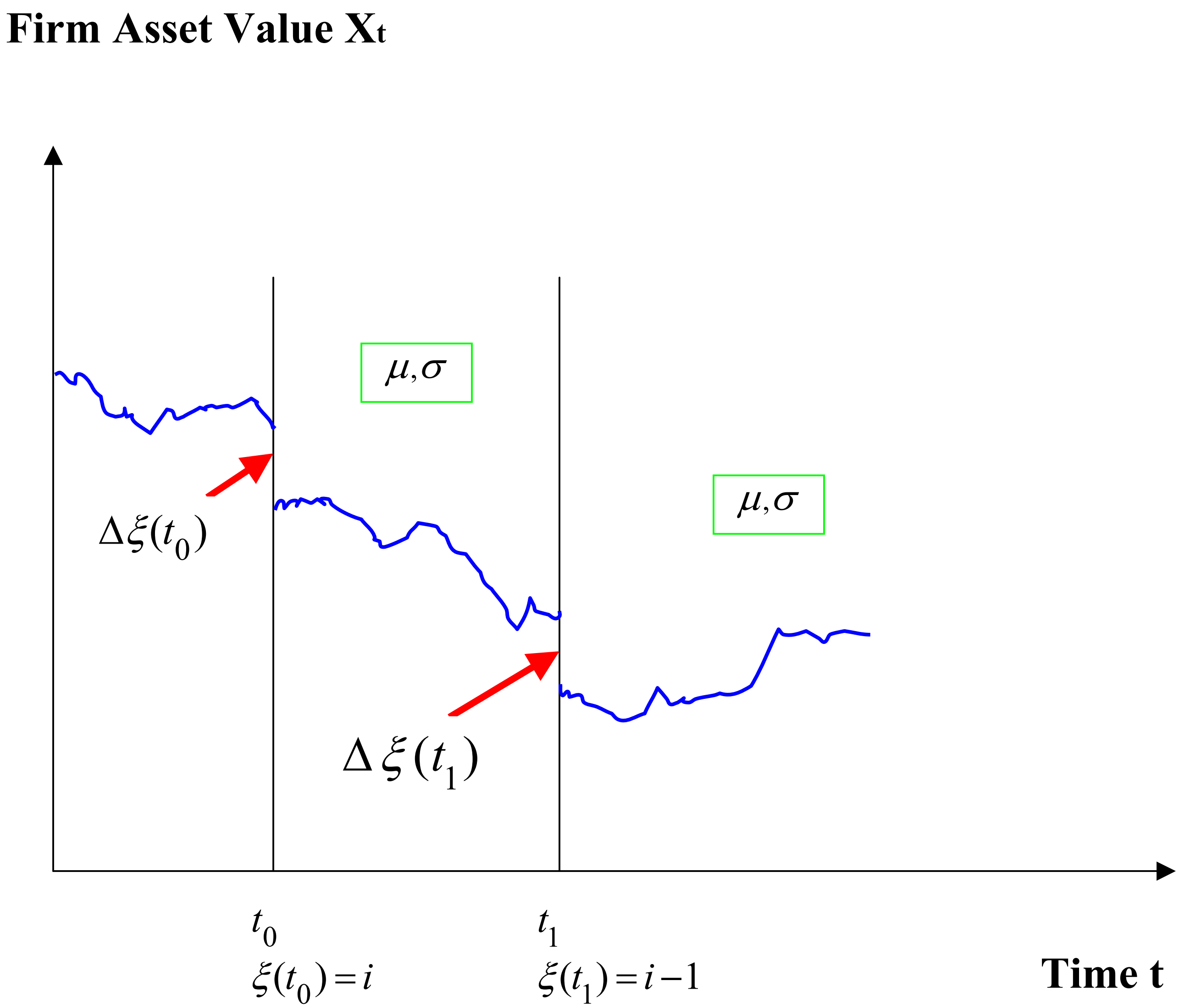
$(\xi_i)_{0 \le i \le S-1}$, ϵ and W are all independent.

Default is

$$\tau = \inf\{t \ge 0 : \epsilon(t) = 0\}$$

with intensity

$$\lambda_t = q_{\epsilon(t)0}.$$



Jump Diffusion Model

Default

Default occurs when a firm misses or delays a promised payment on one of its financial liabilities.

Default does not imply that the firm is insolvent (or enters bankruptcy - a legal condition), nor does it imply that all the firm's debt will not pay its promised payments.

In default, the firm faces deadweight losses due to monitoring by the firm's liability holders, 3rd party costs, and suboptimal investment decisions. The deadweight losses are reflected in:

(1) changed drift and volatility coefficients $\mu(\bullet)$ and $\sigma(\bullet)$ in regime switching,

(2) a sudden downward jump in the firm's asset value in the jump diffusion.

Insolvency and Bankruptcy

In default, there are two possible states of the firm:

(1) solvency $\{X_t \geq x\}$ and

(2) insolvency $\{X_t < x\}$.

Bankruptcy time

$$\tilde{\tau} = \inf\{t > 0 : X_t < x, \epsilon(t) = 0\}.$$

The insolvency barrier x is inclusive of all deadweight costs incurred immediately at the onset of default.

This implies that the insolvency does not induce bankruptcy prior to default.

LABELS	EVENTS	RECOVERY
no default	$\epsilon(t) \neq 0$	NA
default	$\epsilon(t) = 0$	NA
solvent	$X_t \geq x$	NA
insolvent	$X_t < x$	NA
not bankrupt	$\{\epsilon(t) \neq 0\} \cap \{X_t \geq x\}$	1
	$\{\epsilon(t) \neq 0\} \cap \{X_t < x\}$	1
	$\{\epsilon(t) = 0\} \cap \{X_t \geq x\}$	K
bankrupt	$\{\epsilon(t) = 0\} \cap \{X_t < x\}$	R

Table 1: **The Events and Recovery Rates of the Firm at time** t.

Bankruptcy occurs when the firm is in default and the firm becomes insolvent. Either default or insolvency alone does not induce bankruptcy. The parameters satsify $R \leq K \leq 1$.

Information

Investor A. - Management's perspective

The augmented natural filtration of (ϵ, X).

$\mathrm{F}_t^{\mathrm{A}} = \sigma\{X_s, \epsilon(s), 0 \leq s \leq t\} \vee \mathrm{N}$, where N is the collection of all negligible sets.

Investor B.- Market's perspective

ϵ and delayed information on X, observe only at a finite set of times $t_1, t_2, \ldots, t_k, \ldots$ and when the state of the firm changes $T_1, T_2, \ldots, T_n, \ldots$.

$\mathrm{F}_t^{\mathrm{B}}$ is the augmented minimal filtration generated by: $1_{\{\tau \leq t\}}$, $\sum_{n=0}^{\infty} X_{t_k} 1_{\{t_k \leq t\}}$, and $\sum_{n=0}^{\infty} X_{T_n} 1_{\{T_n \leq t\}}$ for the regime switching model, and $1_{\{\tau \leq t\}}$, $\sum_{n=0}^{\infty} X_{t_k} 1_{\{t_k \leq t\}}$, $\sum_{n=0}^{\infty} \xi_{\epsilon(T_n)} 1_{\{T_n \leq t\}}$ and $\sum_{n=0}^{\infty} X_{T_n} 1_{\{T_n \leq t\}}$ for the jump diffusion model.

The Regime Switching Model

Assume that the firm starts from a non-default state, i.e., $\epsilon(0) \neq 0$.

Theorem *(Investor A) The bankruptcy time* $\tilde{\tau} = \min(\tilde{\tau}_{\Lambda}, \tilde{\tau}_{\Lambda^C})$ *has a predictable component* $\tilde{\tau}_{\Lambda^C}$ *and a totally inaccessible component* $\tilde{\tau}_{\Lambda}$ *with intensity*

$$d_t^{R,A} = 1_{\{\tilde{\tau}>t,\ \epsilon(t)\neq 0\}} q_{\epsilon(t)0}\left(1_{\{X_t<x\}} + \frac{1}{2}1_{\{X_t=x\}}\right)$$

Remarks:

$\tilde{\tau}_{\Lambda^C}$: occurs when $\epsilon(t) = 0$ and $X_t \downarrow x$.

$\tilde{\tau}_{\Lambda}$: occurs when $X_t \leq x$ and $\epsilon(\bullet) \neq 0$ jumps to $\epsilon(t) = 0$.

(note: when $X_t > x$ and $\epsilon(\bullet) \neq 0$, $d_t^{R,A} = 0$)

Theorem *(Investor B) The bankruptcy time* $\tilde{\tau}$ *is totally inaccessible.*

If $t \in [t_k, t_{k+1})$ $T_n \leq t < T_{n+1}$, *then when* $\tilde{\tau} > t$, *the intensity is*

$$d_t^{R,B} = \begin{cases} -\dfrac{\psi_t(\theta_0, t - t_k \vee T_n, \frac{1}{\sigma_0} \log \frac{x}{X_{t_k \vee T_n}})}{\psi(\theta_0, t - t_k \vee T_n, \frac{1}{\sigma_0} \log \frac{x}{X_{t_k \vee T_n}})}, & \epsilon(t) = 0 \\ q_{\epsilon(t)0} \Phi\left(\dfrac{\frac{1}{\sigma_{\epsilon(t)}} \ln \frac{x}{X_{t_k \vee T_n}} - \theta_{\epsilon(t)}(t - T_n \vee t_k)}{\sqrt{t - T_n \vee t_k}} \right), & \epsilon(t) \neq 0 \end{cases}$$

where

$$\psi(\theta, t, y) = P\left(\inf_{0 \leq s \leq t} W_s + \theta s > y \right)$$

$$= 1 - \int_0^t \frac{|y|}{\sqrt{2\pi s^3}} e^{-\frac{(y - \theta s)^2}{2s}} ds \text{ for } y < 0$$

with $\theta_i = \frac{\mu_i}{\sigma_i} - \frac{\sigma_i}{2}$, Φ standard cum normal, and $\psi_t = \frac{\partial \psi}{\partial t}$.

The Jump Diffusion Model

Theorem *(Investor A) Let* $\epsilon(t) \neq 0$. *Bankruptcy* $\tilde{\tau} = \min(\tilde{\tau}_\Lambda, \tilde{\tau}_{\Lambda^C})$ *with* $\tilde{\tau}_\Lambda$ *predictable and* $\tilde{\tau}_\Lambda$ *totally inaccessible with intensity*

$$d_t^{J,A} = q_{\epsilon(t)0}\left[F_0\left(\frac{x}{X_t}-\right) + \frac{1}{2}\Delta F_0\left(\frac{x}{X_t}\right)\right]$$

where F_0 *is the distribution function of* ξ_0.

Theorem *(Investor B) The bankruptcy time* $\tilde{\tau}$ *is totally inaccessible. If* $\tilde{\tau} > t$, $t_k \leq t < t_{k+1}$, $T_n \leq t < T_{n+1}$, *the intensity is*

$$d_t^{J,B} = -\frac{\psi_t(\theta, t - t_k \vee T_n, \frac{1}{\sigma}\log\frac{x}{X_{t_k \vee T_n}})}{\psi(\theta, t - t_k \vee T_n, \frac{1}{\sigma}\log\frac{x}{X_{t_k \vee T_n}})}, \quad \epsilon(t) = 0$$

$$= q_{\epsilon(t)0}\int_0^1 F_0(dv)\Phi\left(\frac{\frac{1}{\sigma}\log\frac{x}{vX_{t_k\vee T_n}} - \theta(t - t_k \vee T_n)}{\sqrt{t - t_k \vee T_n}}\right),$$

$$\epsilon(t) \neq 0.$$

Remarks:

- The bankruptcy intensities depend on the typical independent variables used in empirical hazard rate estimation procedures (Chava and Jarrow (2004)).
- The bankruptcy intensity depends on the firm's health $\epsilon(t)$, the drift of the log(asset) price process $\mu_{\epsilon(t)} - \frac{\sigma^2_{\epsilon(t)}}{2}$, the volatility of the log(asset) price process $\sigma_{\epsilon(t)}$, and the firm's debt/asset value ratio $\frac{x}{X_t}$.
- As the state of the firm changes from healthy $\epsilon(t) \neq 0$ to default $\epsilon(t) = 0$, the bankruptcy intensity increases. As the drift of the asset price process $\mu_{\epsilon(t)} - \frac{\sigma^2_{\epsilon(t)}}{2}$ increases, the intensity decreases. As the volatility of the asset price process $\sigma_{\epsilon(t)}$ increases, the intensity increases. As the firm's debt/asset ratio increases, the firm's bankruptcy intensity increases.

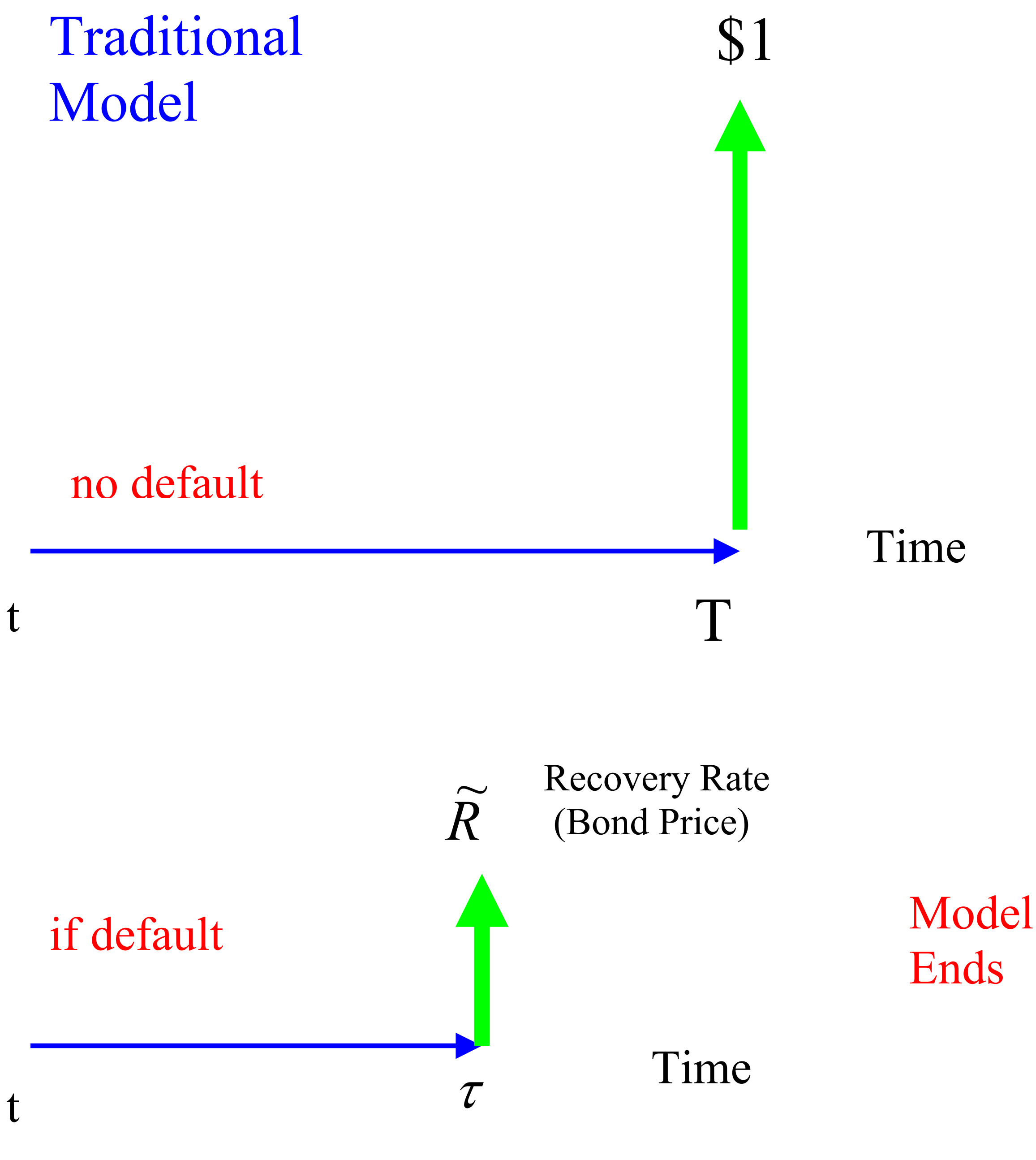
The Recovery Rate Process and Risky Debt Pricing

We assume the existence of an equivalent martingale measure making the discounted risky zero-coupon bond's price a martingale.

Equivalent to assuming no arbitrage. Markets may be incomplete. Fix a measure from the set of equivalent martingale measures (assume markets are in equilibrium).

For simplicity, we let the probability measure P underlying the regime switching model and the jump-diffusion model be this martingale measure.

Review: The Traditional Approach



where $\widetilde{R}$ is a constant.

Recall that τ is the default time.

Let the interest rate process be deterministic, then

$$V_C^i(t,T) = e^{-\int_t^T r(s)ds} E[\widetilde{R}1_{\{\tau \leq T\}} + 1_{\{\tau > T\}} \mid \mathrm{F}_t^i]$$

$$= e^{-\int_t^T r(s)ds}[1 - (1 - \widetilde{R})P(\tau \leq T \mid \mathrm{F}_t^i)] \text{ for } t \leq \tau$$

with $i \in \{\mathrm{A}, \mathrm{B}\}$.

The difference between investor A and B's prices is quantified by the difference between the conditional probabilities of A and B.

Proposition (A vs B's conditional probs)

Let $D^{\mathrm{A}}(t,T)$ and $D^{\mathrm{B}}(t,T)$ denote the conditional default probability before time T under the filtration $(\mathrm{F}_t^{\mathrm{A}})_{t\geq 0}$ and $(\mathrm{F}_t^{\mathrm{B}})_{t\geq 0}$, respectively.

Suppose at time t, on the event $\{\tau_0 > t\}$, $\sigma(X_s) \subset \mathrm{F}_t^{\mathrm{B}} \subset \mathrm{F}_s^{\mathrm{A}}$ where $s < t$ and $(X_t)_{t\geq 0}$ is the underlying Markov process,

then

$$D^{\mathrm{B}}(t,T)1_{\{\tau_0>t\}} = \frac{D^{\mathrm{A}}(s,T)}{D^{\mathrm{A}}(s,t)}1_{\{\tau_0>t\}}.$$

Remarks:

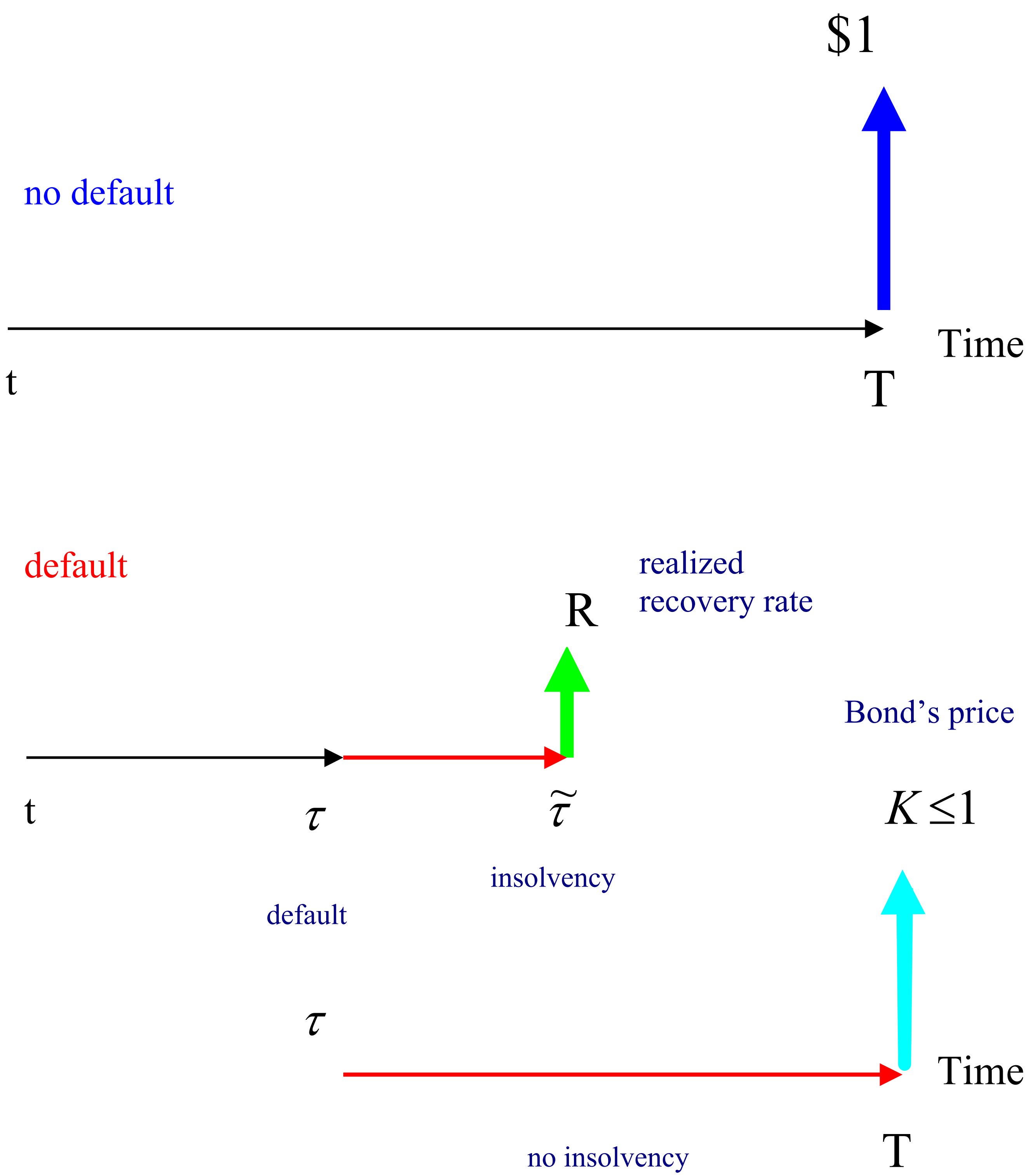
(1) $D^{\mathrm{B}}(0,T) = D^{\mathrm{A}}(0,T),$ because at time 0 both investors A and B have the same information.

(2) When $s \uparrow t$, $D^{\mathrm{B}}(s,T)$ and $D^{\mathrm{A}}(t,T)$ converge to the same value.

(3) One does not necessarily have $D^{\mathrm{A}}(t,T) > D^{\mathrm{B}}(t,T)$ or $D^{\mathrm{A}}(t,T) < D^{\mathrm{B}}(t,T)$.

The Extended Model

Extended Model



where K and R are constants (R<K).

Proposition (Debt Pricing)

(1) if default has not happened, $\tau > t$, then

$$V^i(t,T) = e^{-\int_t^T r(s)ds}[E_t^i[1_{\{\tau>T\}}] + (K-R)E_t^i[1_{\{\tilde{\tau}>T,\tau\le T\}}] + RE_t^i[1_{\{\tau\le T\}}]].$$

(2) if defaulted by time t, but still solvent, and assuming the default state is absorbing, then

$$V^i(t,T) = e^{-\int_t^T r(s)ds}[(K-R)P(\inf_{t\le u\le T} X_u > x|\mathrm{F}_t^i) + R]$$

where $E_t^i = E[\bullet|\mathrm{F}_t^i]$ is under the martingale measure with $i \in \{\mathrm{A},\mathrm{B}\}$.

Example: Jarrow-Lando-Turnbull

Use the regime switching model.

Let $\epsilon(t)$ follow the Markov chain as in Jarrow, Lando and Turnbull (1997) with default (state 0) an absorbing state.

The generator matrix $(q_{ij})_{S\times S}$ is of the form

$$Q = (q_{ij})_{S\times S}$$

$$= \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ q_{10} & -q_1 & q_{12} & \cdots & q_{1(S-1)} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ q_{(S-1)0} & q_{(S-1)1} & q_{(S-1)2} & \cdots & -q_{S-1} \end{pmatrix}$$

where $q_{ij} \geq 0$ for all i,j, and $q_i = \sum_{i\neq j} q_{ij}$.

Investor A

Default by time t

$$V^A(t,T) = \begin{cases} Re^{-\int_t^T r(s)ds} & X_t \leq x \\ e^{-\int_t^T r(s)ds}\left[(K-R)\psi\left(\theta_0, T-t, \frac{1}{\sigma_0}\log\frac{x}{X_t}\right) + R\right] & X_t > x \end{cases}$$

No default prior to time t

$$V^A(t,T) = e^{-\int_t^T r(s)ds}\Big[R + (1-R)P(\tau > T \mid \epsilon(t) \neq 0)$$

$$+(K-R)\int_t^T \int_x^\infty P(\tau \in ds, X_\tau \in dz \mid X_t, \epsilon(t) \neq 0)$$

$$\times\, \psi\left(\theta_0, T-s, \frac{1}{\sigma_0}\log\frac{x}{z}\right)\Big].$$

When $S = 2$, this expression is further simplified to

$$
\begin{aligned}
&V^{\mathrm{A}}(t,T) \\
&= e^{-\int_t^T r(s)ds}[R + (1-R)e^{-q_{10}(T-t)} \\
&\quad + (K-R)\int_t^T\int_x^\infty dzdsq_{10}e^{-q_{10}(s-t)}\phi(\frac{\frac{1}{\sigma_1}\log\frac{z}{X_t} - \theta_1(s-t)}{\sqrt{s-t}}) \\
&\quad \times \psi(\theta_0, T-s, \frac{1}{\sigma_0}\log\frac{x}{z})],
\end{aligned}
$$

where ϕ is the densityof the standard normal random variable.

Investor B

Define $u = t_k \vee T_n < t < t_{k+1} \wedge T_{n+1}$.

Default by time t

Assuming default is an absorbing state, here $\tau \le u < t$, and $\epsilon_u = \epsilon_t = 0$,

$$V^{\mathrm{B}}(t,T) = \begin{cases} Re^{-\int_t^T r(s)ds} \text{ if } X_u \le x \\ e^{-\int_t^T r(s)ds}[(K-R)\psi(\theta_0, T-u, \frac{1}{\sigma_0}\log\frac{x}{X_u}) + R] \\ \text{if } X_u > x \end{cases}$$

No default prior to time t

Here $u < t$, but $\epsilon_u = \epsilon_t$,

$$V^{\mathrm{B}}(t,T) = e^{-\int_t^T r(s)ds}[R + (1-R)P(\tau > T|\epsilon(t) \ne 0)$$

$$+(K-R)\int_t^T\int_x^\infty P(\tau \in ds, X_\tau \in dz|F_t^{\mathrm{B}})\psi(\theta_0, T-s, \frac{1}{\sigma_0}\log\frac{x}{z})].$$

When $S = 2$,

$$
\begin{aligned}
& V^{\mathrm{B}}(t,T) \\
= \; & e^{-\int_t^T r(s)ds}[R + (1-R)e^{-q_{10}(T-t)} \\
& +(K-R)\int_t^T \int_x^\infty dzdsq_{10}e^{-q_{10}(s-t)}\phi\Big(\frac{\frac{1}{\sigma_1}\log\frac{z}{X_u} - \theta_1(s-u)}{\sqrt{s-u}}\Big) \\
& \times \psi(\theta_0, T-s, \frac{1}{\sigma_0}\log\frac{x}{z})]
\end{aligned}
$$

A Comparison (Traditional vs Extended)

Traditional

$$V_C^i(t,T) = e^{-\int_t^T r(s)ds} E_t^i[1_{\{\tau>T\}} + \widetilde{R}1_{\{\tau\le T\}}] \text{ for } t \le \tau$$

Extended

$$V^i(t,T) = e^{-\int_t^T r(s)ds} E_t^i[1_{\{\tau>T\}} + R1_{\{\tau\le T\}} + (K-R)1_{\{\tau\le T,\tilde{\tau}>T\}}]$$

for $i \in \{\mathrm{A},\mathrm{B}\}$.

At Default

$$V_C^i(\tau, T) = \tilde{R} e^{-\int_\tau^T r(s)ds}$$

$$V^i(\tau, T) = R e^{-\int_\tau^T r(s)ds} \quad \textit{if} \text{ insolvent}$$

$$= e^{-\int_\tau^T r(s)ds}[(K - R)P(\inf_{\tau \le u \le T} X_u > x | F_\tau^i) + R] \quad \textit{otherwise}$$

for $i \in \{\mathrm{A}, \mathrm{B}\}$.

For calibration purposes, readily available are:

(1) the (average) market prices for defaulted debt at time of default τ, denoted M_τ, and

(2) the (average) market prices at the time of emergence from financial distress, denoted M_∞.

In the traditional model, calibrate $\tilde{R}$ by setting

$$M_\tau = \tilde{R}e^{-\int_\tau^T r(s)ds}.$$

For the extended model, set

$$R = M_\infty$$

and implicitly estimate K by solving, if solvent at τ,

$$M_\tau = e^{-\int_\tau^T r(s)ds}[(K - M_\infty)P(\inf_{\tau \le u \le T} X_u > x | \mathrm{F}_\tau^i) + M_\infty]$$

$$K = M_\infty + \frac{M_\tau e^{\int_\tau^T r(s)ds} - M_\infty}{P(\inf_{\tau \le u \le T} X_u > x | \mathrm{F}_\tau^i)}$$

Note that at τ, both A and B have identical information.

In terms of $\tilde{R}$ and R:

$$K = R + \frac{\tilde{R} - R}{P(\inf_{\tau \le u \le T} X_u > x | \mathrm{F}_\tau^i)}$$

In general, $\tilde{R}, R$ and K differ.

Prior to Default

$$V^i(t,T) - V_C^i(t,T) = e^{-\int_t^T r(s)ds}(R - \widetilde{R})P(\tau \le T|\mathrm{F}_t^i)$$
$$+ e^{-\int_t^T r(s)ds}(K - R)P(\tilde{\tau} \ge T, \tau \le T|\mathrm{F}_t^i)$$

for $i \in \{\mathrm{A}, \mathrm{B}\}$.

Using typical calibration methods, the random recovery rate process makes a difference to before default pricing.

Conclusion

- We constructed a model useful for pricing risky debt both before and **after** default.
- A random recovery rate process is important for pricing debt prior to default.
- This model should prove useful for estimating recovery rates and pricing credit derivatives.