Overconfidence in the Continuous–Time Principal–Agent Problem*

Karl Ludwig Keiber†

February, 2003

*Comments are welcome. Any errors are my responsibility.

†Address: WHU Otto Beisheim Graduate School of Management, Chair of Finance, Burgplatz 2, D–56179 Vallendar, Germany. Phone: +49–261–6509–422. Fax: +49–261–6509–409. E–mail: kkeiber@whu.edu. Karl Ludwig Keiber is a lecturer of Finance at the WHU Otto Beisheim Graduate School of Management.

Overconfidence in the Continuous–Time Principal–Agent Problem

Abstract

In this paper we analyze the impact of overconfidence on the continuous–time principal–agent problem when both the risk neutral principal and the risk averse agent are assumed to be subject to this psychological bias. The first–best and second–best sharing rules as well as the agency costs are derived when the outcome process which is controlled privately by the agent is not observable directly by the two parties to the contract but a common signal on the outcome process is available. Both the first–best contract and the first–best control are reported to be independent of the parties' overconfidence. In contrast the second–best contract and the second–best control, which is always less than the first–best control, as well as the agency costs depend on the degree of overconfidence. The comparative static results document that the second–best control decreases but the agency costs increase with the parties' overconfidence. The various components of the second–best sharing rule exhibit mixed comparative static results with respect to the degree of overconfidence.

JEL classification: D82, G34, J33.

1 Introduction

As paradigm of corporate finance the principal–agent theory is concerned mainly with the analysis of the agency relationship between shareholders and managers. The principal–agent theory provides the answer how compensation contracts have to be designed efficiently. Here efficiency means to provide the manager — henceforth called agent — with incentives such that he exerts an effort in the best interest of the shareholder — referred to as principal below — when the principal cannot monitor the agent's effort perfectly. The lack of perfect monitoring generates discretion on the effort on the part of the agent who makes use of that discretion in his own interest. Thus a moral hazard problem arises.

The notion of DeBondt and Thaler (1995) that overconfidence is perhaps the most robust finding in the psychology of judgement in combination with the evidence on the overconfidence of entrepreneurs collected in Cooper, Dunkelberg and Woo (1988), the overconfidence of managers reported in Russo and Schoemaker (1992), and Malmendier and Tate (2002), as well as Busenitz and Barney's (1997) findings on the overconfidence of both groups gives immediately rise to the questions if and how overconfidence affects the principal–agent problem. These issues are addressed in this paper.

There already exists a number of papers that analyze behavioral aspects in corporate finance. Shefrin (2001*a*) argues that overconfidence cannot be overcome by incentive compatibility. Managerial optimism delivers explanations for various corporate finance phenomena in Heaton (2002). The case for the increase of a firm's value by overconfidence and optimism is made by Gervais, Heaton and Odean (2000). Goel and Thakor (2000) argue that competition among managers for leadership can be taken as source of managerial overconfidence. The persistence of entrepreneurial overconfidence is discussed in Bernardo and Welch (2001). Most closely related to the present paper is Bolton, Scheinkman and Xiong's (2002) piece. It provides a rationale for managers exerting short–term efforts in a two–period principal–agent model when shareholders effect a speculative bubble in the firm's value due to overconfidence with respect to the precision of a public signal on the value of a new short-term project.

Besides these treatments of overconfidence in corporate finance there exists a body of literature that deals with the impact of overconfidence on financial markets. The works of Kyle and Wang (1997), Daniel, Hirshleifer and Subrahmanyam (1998), Odean (1998), Hirshleifer and Luo (2001), Gervais and Odean (2001), Daniel, Hirshleifer and Subrahmanyam (2001), Shefrin (2001*b*), Scheinkman and Xiong (2002), and Caballé and Sákovics (2003) all belong to this strand of the literature. The overviews of Hirshleifer (2001), Daniel, Hirshleifer and Teoh (2002), and Barberis and Thaler (2001) try to consolidate the behavioral biases which affect financial markets as well as the various findings and approaches of behavioral finance. Remarkably only the last few pages of the most recent survey of Barberis and Thaler (2001) are devoted to corporate finance what might indicate that behavioral corporate finance is still in its infancy.

Since this paper merges the principal–agent paradigm of corporate finance with an empirically well–documented aspect of human behavior — namely overconfidence — it adds to the behavioral corporate finance literature. This is achieved by relying on the continuous–time approach to the principal–agent problem. The continuous–time principal–agent model was presented by Holmström and Milgrom (1987) initially. Chronologically, further treatments of the continuous–time approach to the principal–agent problem can be found for example in Schättler and Sung (1993), Sung (1995), Schättler and Sung (1997), Müller (1998), Sung (2001*b*), and Sung (2001*a*). Most recently, an application of the continuous–time principal–agent model to delegated portfolio management is due to Ou-Yang (2003).[1]

The continuous–time principal–agent model presented in this paper differs in two major aspects from the aforementioned approaches to the principal–agent problem. First, we assume that the outcome process whose drift is controlled privately by the agent is not observable directly by the two parties to the agency relationship but that a common signal on the outcome process is available.[2] Second, we assume less than full rationality on the part of both

[1]Early discrete–time discussions of the principal–agent problem can be found in Ross (1973), Holmström (1979), and Grossman and Hart (1983) among others. For a recent textbook treatment of the principal–agent problem we refer to Laffont and Martimort (2002).

[2]Usually in principal–agent theory it is assumed that solely the agent's control is beyond

the principal and the agent. In essence we assume that both parties are overconfident. Note that this assumption is in concordance with the evidence on entrepreneurial and managerial overconfidence stated previously. In particular the principal and the agent are assumed to be subject to the same degree of overconfidence.[3] Here being overconfident means that the precision of the common signal on the outcome process is overestimated. Consequently the signal is taken to be more accurate than it really is.[4] Finally, since both parties are affected by the overconfidence bias symmetrically they agree on the outcome process conditionally on the common signal. It is exactly that filtered outcome process which the labor contract is written upon.[5]

We determine the labor contract which solves the principal–agent problem according to the separation principle. Thus our approach requires first the solution of a filtering problem and second the solution of a stochastic control problem.[6] Since both parties to the principal–agent problem are subject to the overconfidence bias with respect to the common signal on the outcome process the solution of the filtering problem results in a filtered outcome

observability on the part of the principal but that the outcome process can be observed mutually. Note that since here the outcome process is assumed not to be observable it cannot be contracted upon.

[3]This assumption is not as restrictive as it may seem since it is not very likely that the principal hires an agent who disagrees substantially on the assessment of firm characteristics of paramount importance. Loosely, this assumption captures the notion of the common saying *"Birds of a feather flock together."*

[4]Usually overconfidence is captured in this way in behavioral finance models. Cf. for example Kyle and Wang (1997), Daniel, Hirshleifer and Subrahmanyam (1998), Odean (1998), and Caballé and Sákovics (2003).

[5]Usually it is assumed that the outcome process itself is observable and hence can be contracted upon. For example Holmström and Milgrom (1987) suggest that the outcome process can be thought of as an accounting measure which is observable indeed. In contrast, in our approach the parties agree on the filtered outcome process to contract upon. Thus the filtering procedure describes the creation of a performance measure from the common signal on the unobservable outcome process. From this perspective we extend the principal–agent relationship by an additional stage that allows to study the impact of the overconfidence bias with respect to the informational content of the common signal for the outcome process.

[6]Cf. for example Fleming and Rishel (1975) for the separation principle in stochastic control theory.

process which depends to some extent on the degree of overconfidence.

Our analysis provides us with a variety of insights. We report that overconfidence does affect neither the first–best contract nor the first–best control. In the second–best case we find that the compensation contract, the agent's control, and the agency costs all depend on the degree of overconfidence. The comparative static analysis yields that the second–best control decreases but the agency costs increase the more the severity of the parties' overconfidence bias is pronounced. Finally the individual components of the sharing rule exhibit mixed comparative static results. For example the risk premium offered to the agent increases or decreases with the parties' overconfidence depending on the severity of the overconfidence bias.

The remainder of the paper proceeds as follows. Following this introduction section 2 delivers the basic structure of the continuous–time principal–agent model which underlies the analysis. Next the filtering problem is discussed in section 3. In section 4 the first–best and second–best compensation contracts as well as the agency costs are derived. The comparative static analysis is provided in this section too. Section 5 concludes and outlines further research avenues. All proofs are moved to the appendix.

2 Agency relationship

This section provides a description of the agency relationship which is to be analyzed in the remainder of the paper. The setup conforms to the standard exhibition of the principal–agent problem. Hence our exposition is easily accessible and comparable to the related literature.

The principal and the agent are assumed to be expected utility maximizers. The principal is supposed to be risk neutral. In contrast, the agent's preferences are represented by a negative exponential utility function. Thus the agent is risk averse and $a > 0$ denotes the constant absolute risk aversion coefficient. Both utility functions are defined over final wealth. The agent's reservation level of final wealth amounts to W_0.

After agreeing on a sharing rule at time $t = 0$ the agent controls the drift of the outcome process X_t during the unit time interval $[0; 1]$. The outcome process is described by an Itô process of the form

$$dX_t = f(u_t)dt + \sigma_x dB_{x,t}, \tag{1}$$

where u_t denotes the agent's instantaneous control, $f(u_t)$ and σ_x^2 represent the instantaneous drift rate and instantaneous variance rate respectively, and $B_{x,t}$ is a standard Wiener process. The function f is assumed to be linear. In particular $f'(\cdot) > 0$ and $f''(\cdot) = 0$. For exerting the effort u_t the agent incurs instantaneous costs $c(u_t)$. We confine our analysis to quadratic cost functions. The quadratic function c is assumed to be increasing and convex that is $c'(\cdot) > 0$ and $c''(\cdot) > 0$.[7]

The agency relationship differs from the standard approach by the fact that the outcome process X_t in addition to the agent's control u_t is beyond observability. But, both parties to the contract observe the common signal Y_t on the outcome process. The signal process Y_t is an Itô process defined by

$$dY_t = X_t dt + \sigma_y dB_{y,t}, \tag{2}$$

where σ_y^2 denotes the instantaneous variance rate and $B_{y,t}$ is a standard Wiener process which is uncorrelated with the Wiener process $B_{x,t}$. Note that the signal process Y_t is informative for the outcome process X_t by construction since the unobservable state X_t of the outcome process represents the drift rate. However, due to the Wiener process $B_{y,t}$ the signal Y_t does not provide perfect information on the state X_t. Hence the signal Y_t is noisy.

The parties' overconfidence bias with respect to the quality of the signal is quantified by the coefficient $0 < \kappa < 1$. Being overconfident with coefficient κ means that the variance rate of the signal process is taken to be $\kappa\sigma_y^2$ instead of σ_y^2. Hence the overconfidence bias implies that the signal noise is perceived to be less dispersed. Consequently, subject to the overconfidence bias the signal process is taken to be more precise or more informative than it really is.[8] Note that $\kappa = 1$ corresponds to a rational unbiased assessment of the of

[7]The assumptions on the shapes of the functions f and c ensure that we do not have to worry about the implementability issue. Cf. Sung (1997) and Sung (2001*b*). Thus, in what follows, we can focus on the analysis of the impact of the overconfidence bias on the agency relationship.

[8]Scheinkman and Xiong (2002) capture the overconfidence bias similarly. Their argu-

the signal process. Finally, a more severe overconfidence bias is induced by a lower coefficient of overconfidence.

Since the outcome process X_t is solely observable indirectly through the signal process Y_t the parties to the contract estimate the state of the outcome process by extracting continuously the information from the common signal process Y_t. This filtering procedure results in the filtered outcome process which is denoted by $\hat{X}_t$. As $\hat{X}_t$ represents the common best estimate of the outcome process X_t the parties' agree to share $\hat{X}_1$ at the end of the unit time interval that is at time $t = 1$. Hence the sharing rule is some function of the final filtered outcome $\hat{X}_1$.[9] Additionally, by learning the state of the outcome process from the signal process the agent revises his control continuously according to the filtered outcome process $\hat{X}_t$. Consequently the agent's control in its most general form is some function of the filtered outcome process $\hat{X}_t$ that is $u_t \equiv u(\hat{X}_t, t)$.

3 Filtering problem

The solution of the parties' filtering problem is provided in this section. The filtered outcome process $\hat{X}_t$ subject to the overconfidence bias is specified in proposition 1.

Proposition 1 *The filtered outcome process $\hat{X}_t$ is an Itô process given by*

$$d\hat{X}_t = f(u_t)dt + \frac{\sigma_x}{\sqrt{\kappa}\sigma_y} d\hat{B}_t,$$

where $d\hat{B}_t \equiv dY_t - \hat{X}_t dt$ and $\int_0^t \frac{d\hat{B}_s}{\sqrt{\kappa}\sigma_y}$ is a standard Wiener process, and κ denotes the parties' coefficient of overconfidence.

Note that proposition 1 characterizes the stationary solution of a generalized Kalman–Bucy filter. The effect of the overconfidence bias is straightforward. The more severe the overconfidence bias is — that means the lower

ment that the principal and the agent are less than fully rational in the sense that they do not infer the correct precision from the signals applies.

[9]Note that even an accounting measure which is contracted upon in standard approaches to the principal–agent problem is to some extent an estimate. Our approach makes the construction of the relevant performance measure explicit and thus allows to inspect the impact of the overconfidence bias.

the coefficient κ of overconfidence is — the greater is the filtered outcome process' variance rate $\sigma_x^2(\kappa\sigma_y^2)^{-1}$. Consequently the signal innovations dY_t translate into greater adjustments $d\hat{X}_t$ immediately. Thus the filtered process $\hat{X}_t$ reacts more sensitively towards a signal innovation dY_t the more pronounced the overconfidence bias is. This exactly captures the impact of the overconfidence bias on the filtering problem. The overconfident parties' to the contract put too much weight on the signal compared to a rational assessment of the signal process' quality.

4 Sharing rules and agency costs

In this section we determine the first–best and second–best sharing rule as well as the agency costs induced by moral hazard. According to the separation principle of stochastic control theory we now solve the stochastic control problems associated with the agency relationship after having solved the filtering problem in the section 3. Finally we perform a comparative static analysis with respect to the overconfidence bias the parties to the agency relationship are subject to.

The first–best compensation contract $s_1(\hat{X}_1)$ is specified in proposition 2.

Proposition 2 *The first–best sharing rule is*

$$s_1(\hat{X}_1) = W_0 + c(u_1),$$

where u_1 represents the constant first–best control.

Hence the first–best sharing rule compensates the agent for the cost of exerting continuously the constant first–best control u_1 during the unit time interval and guarantees the reservation level of final wealth. Note that the parties' overconfidence bias does not affect the first–best sharing rule.

The second–best compensation contract $s_2(\hat{X}_1)$ is given in proposition 3.

Proposition 3 *The second–best sharing rule is*

$$s_2(\hat{X}_1) = W_0 + c(u_2) + \frac{a}{2}\frac{\sigma_x^2}{\kappa\sigma_y^2}\frac{c'(u_2)^2}{f'(u_2)^2} + \frac{c'(u_2)}{f'(u_2)}\hat{X}_1 - \frac{c'(u_2)}{f'(u_2)}f(u_2),$$

where u_2 represents the constant second–best control.

Similarly to the first–best compensation contract the second–best sharing rule compensates the agent for the cost of the permanent constant effort u_2 during the time of the agency relationship and gives the reservation level of final wealth as first component. Note that the agent's compensation is a function of the final filtered outcome $\hat{X}_1$ which is random ultimately. Thus the agent bears compensation risk. Consequently, the risk averse agent is paid a risk premium — the second–best sharing rule's third term — additionally. On the other hand, the dependency on the final filtered outcome $\hat{X}_1$ can be interpreted as providing the agent with the incentive to exert a higher effort permanently as the agent seems to participate directly at first sight. But this is only half the truth since the last term of the second–best sharing rule in turn destroys that incentive. This can be seen from integrating the filtered outcome process in proposition 1 over the unit time interval and restating the second–best sharing rule

$$s_2(\hat{X}_1) = W_0 + c(u_2) + \frac{a}{2}\frac{\sigma_x^2}{\kappa\sigma_y^2}\frac{c'(u_2)^2}{f'(u_2)^2} + \frac{c'(u_2)}{f'(u_2)}\frac{\sigma_x}{\sqrt{\kappa}\sigma_y}\hat{B}_1 \tag{3}$$

alternatively. Hence, the agent's second–best compensation $s_2(\hat{X}_1)$ is random due to the randomness of $\hat{B}_1$. Note that it is exactly the risk — more precisely the variance — of the second–best sharing rule $s_2(\hat{X}_1)$ which is compensated by the risk premium. This is due to the fact that $\hat{B}_1$ has unit variance. So the argument of incentive provision does not apply to the representation (3) of the second–best sharing rule anymore whereas the compensation risk argument becomes more striking.

Finally, proposition 3 compares favorably to Holmström and Milgrom's (1987) and Schättler and Sung's (1993) results as regards the various components of the second–best sharing rule as well as their interpretation. The insight that the overconfidence bias affects the agency relationship is delivered by straightforward inspection of the second–best sharing rule in proposition 3 and in representation (3) respectively.

The stochastic control problems underlying the propositions 2 and 3 allow to derive corollary 1.

Corollary 1 *The first–best control u_1 is independent of the overconfidence bias but the second–best control u_2 depends on the coefficient of overconfi-*

dence. The second–best control u_2 is always less than the first–best control u_1 that is the inequality $u_2 < u_1$ holds generally.

Once the solutions to the first–best and second–best stochastic control problems are at hand the agency costs implied by moral hazard can be ascertained. The agency costs are given in proposition 4.

Proposition 4 *The agency costs amount to*

$$f(u_1) - f(u_2) - \big(c(u_1) - c(u_2)\big) + \frac{a}{2}\frac{\sigma_x^2}{\kappa\sigma_y^2}\frac{c'(u_2)^2}{f'(u_2)^2}.$$

Expectedly, the agency costs depend on the parties' degree of overconfidence. But note that the impact of the overconfidence bias is not obvious at first glance since the second–best control varies with the coefficient of overconfidence too. Immediately having the agency costs specified in proposition 4 at hand it is straightforward to obtain corollary 2.

Corollary 2 *The agency costs given in proposition 4 are always positive.*

Note that corollary 2 just confirms a well–known result of the principal–agent literature and thus serves plainly as consistency check. In the second–best stochastic control problem the principal suffers an expected utility loss due to the moral hazard problem.

After having solved both the first–best and second–best stochastic control problems we now turn to the comparative static analysis that is we study the impact of the overconfidence bias on the agency relationship. Obviously it makes sense to apply the comparative static analysis solely to those results for which we have reported a dependency on the coefficient of overconfidence. These are the second–best control, the agency costs, and the various components of the sharing rule. The comparative static results are collected in proposition 5.

Proposition 5 *Ceteris paribus a more severe overconfidence bias (a.) reduces the second–best control, (b.) increases the agency costs, and (c.) decreases the agent's compensation of the cost from exerting the second–best control during the unit time interval. A stronger overconfidence bias (d.)*

decreases both the sensitivity to the unexpected outcome $\hat{B}_1$ and the risk premium if $\kappa_0 \geq 1$ or $\kappa \in (0, \kappa_0]$ but (e.) increases both the sensitivity to the unexpected outcome $\hat{B}_1$ and the risk premium if $\kappa \in [\kappa_0, 1)$. The coefficient of overconfidence κ_0 is defined in the proof.

The most clear cut comparative static results which are reported in proposition 5 are those concerning the second–best control, the agency costs, and the compensation of the agent's effort costs. In any case a stronger overconfidence bias worsens the moral hazard problem that is the principal suffers a bigger loss in terms of expected utility accompanied by a lower effort on the part of the agent who consequently incurs lower effort costs to be compensated.

As proposition 5 reports, the effect of a stronger overconfidence bias on both the sharing rule's sensitivity to the unexpected outcome $\hat{B}_1$ and the risk premium depends on the characteristics of the agency relationship. This insight results from thorough inspection of the threshold κ_0 which is determined by various parameters of the agency problem. For $\kappa_0 \in (0, 1)$ the impact of a more pronounced overconfidence bias differs depending on the absolute degree κ of overconfidence.

Concerning the impact of a more severe overconfidence bias on the risk premium and the sharing rule's sensitivity to the unexpected outcome $\hat{B}_1$ there are two effects at work simultaneously. Obviously, the first effect comes from the coefficient of overconfidence directly whereas the second effect is related to the change of the second–best control u_2. Note that due to the convexity of the function c the marginal costs $c'(u_2)$ become smaller as does the coefficient of overconfidence κ if the severity of the overconfidence bias increases.

In the case of an initial strong overconfidence bias — $\kappa \in (0, \kappa_0]$ — the reduction of the coefficient of overconfidence decreases the squared marginal costs $c'(u_2)^2$ more than proportionately what in turn yields the comparative static results as reported in part (d.) of proposition 5. In contrast, in the case of a minor overconfidence bias — $\kappa \in [\kappa_0, 1)$ — initially a more pronounced overconfidence bias reduces the squared marginal costs $c'(u_2)^2$ less than proportionately what delivers the remaining comparative static result in the last part of proposition 5. Finally, the degree of overconfidence κ_0

determines exactly the threshold where both effects are the same in magnitude. In that case the changes of the coefficient of overconfidence and of the squared marginal costs $c'(u_2)^2$ are proportionate.[10]

However, the sharing rule's sensitivity to the unexpected outcome and the risk premium respond to changes in the degree of overconfidence in parallel consistently. An increased (decreased) sensitivity to the unexpected outcome comes along with a higher (lower) risk premium generally.

5 Conclusion

This paper provided an analysis of the principal–agent relationship in continuous–time. Compared to existing continuous–time principal–agent models two major modifications were implemented. First, the outcome process whose drift is controlled privately by the agent is not observable directly but both the principal and the agent observe a common noisy signal on the outcome process. Second, we assumed in conformity with the empirical evidence that both the principal and the agent are overconfident of the same degree with respect to the common signal's quality. These two modifications enable us to analyze the impact of overconfidence on the agency relationship thoroughly.

The first–best and second–best sharing rules were derived according to the separation principle of the stochastic control theory by first tackling the principal's and agent's filtering problem and solving the stochastic control problem subsequently. Our results can be put in line smoothly with existing treatments of the continuous–time principal–agent problem but allow to study the impact of the overconfidence bias on the agency relationship additionally.

The most striking result of the comparative static analysis is that the overconfidence bias worsens the moral hazard problem which is implied by imperfect monitoring of the agent's effort on the part of the principal. From the principal's perspective the overconfidence bias is value — more precisely expected utility — destructing. Thus the principal's and agent's tendency to

[10]These insights are delivered by inspection of the elasticity $\eta(\kappa) \equiv \frac{dc'(u_2)^2}{c'(u_2)^2} / \frac{d\kappa}{\kappa}$, where we find $\eta(\kappa) > 1$ if $\kappa \in (0, \kappa_0)$, $\eta(\kappa) < 1$ if $\kappa \in (\kappa_0, 1)$, and $\eta(\kappa_0) = 1$.

overweight firm specific information affects the agency relationship adversely. Even incentive compatibility cannot remedy the harmful impact of the overconfidence bias. In this respect our model allows to confirm Shefrin's (2001*a*) notion formally.

The explicit analysis of the common filtering problem allows to exemplify two issues. First, the filtering problem demonstrates how to construct a performance measure — here $\hat{X}_t$ — which is contracted upon from noisy firm specific information — here Y_t. Second, the filtering problem shows how a biased assessment of the firm specific information's quality affects the construction of that performance measure. Taking the agency costs as measure of the agent's performance implies a straightforward policy implication. The principal — that is the shareholders of a publicly held company — and the agent — that is the hired management — must ensure a rational assessment of firm specific information as unbiased as possible thus minimizing the agency costs.

The second–best sharing rule documents the dependency of the compensation contract on the coefficient of overconfidence. Besides the reservation level of wealth the fixed compensation consists of two components which depend on the severity of the overconfidence bias. These are the compensation of the agent's effort costs and the risk premium. The variable compensation is determined by the second–best sharing rule's sensitivity to the unexpected outcome. Consequently the overconfidence bias might serve as explanation for different compensation schemes applied to the same agency relationship ceteris paribus. Stated differently, the overconfidence bias can be viewed as source of the variety of compensation arrangements within the same industry where companies delegate similar tasks to the management. Alternatively, our analysis yields the insight that the assessment of the firm specific information's quality — that is the belief in that information's sharpness — is an issue that cannot be neglected in the analysis of agency relationships.

In the second–best case, the dependency of both the risk premium and the sharing rule's sensitivity to the unexpected outcome on the degree of overconfidence was documented to be non–monotonic if an agency relationship is described by $\kappa_0 \in (0, 1)$. In that case — even if two agency relationships have the same characteristics except for κ — it may happen that one cannot judge from the inspection of those two components of the second–best sharing rules in which agency relationship the overconfidence bias is more severe since the direct effect of the overconfidence bias and the indirect effect through the second–best control offset. Ultimately the second–best control allows to differentiate between the two agency relationships as regards the severity of the overconfidence bias. In contrast, if $\kappa_0 \geq 1$ then, due to the monotonicity results concerning the risk premium and the sensitivity to the unexpected outcome, one can decide in which of the two agency relationships the overconfidence bias is more pronounced without looking at the second–best control.

Finally, the comparative static analysis provided us with the insight that the sensitivity to the unexpected outcome which determines the exposure to the compensation risk and the risk premium evolve together in parallel as concerns the monotonicity.

To finish the conclusions we point out that as a variant of the presented approach to the principal–agent problem one might study the case where the agent controls a multidimensional outcome process by choosing a multidimensional control and having more than an unique Wiener process as source of the outcome process' uncertainty. Although the notation employs vectors and matrices in that case, the analysis is along the same two step procedure. Alternatively, a multidimensional analysis allows to discuss how a multidimensional signal on the outcome process — that is various signals — boils down to a single performance measure which is used in contracting. Additionally, the filtering problem in the multidimensional case permits the discussion of the impact of both correlated signals and diverse overconfidence biases with respect to different signals. These issues are left for future research.

References

Barberis, N. and Thaler, R. (2001), A survey of behavioral finance. *Working Paper*, University of Chicago.

Bernardo, A. E. and Welch, I. (2001), 'On the evolution of overconfidence and entrepreneurs', *Journal of Economics and Management Strategy* **10**(3), 301–330.

Bolton, P., Scheinkman, J. and Xiong, W. (2002), Executive compensation and short–termist behavior in speculative markets. *Working Paper*, Princeton University.

Busenitz, L. W. and Barney, J. B. (1997), 'Differences between entrepreneurs and managers in large organizations: biases and heuristics in strategic decision–making', *Journal of Business Venturing* **12**(1), 9–30.

Caballé, J. and Sákovics, J. (2003), 'Speculating against an overconfident market', *Journal of Financial Markets* **6**(2), 199–225.

Cooper, A. C., Dunkelberg, W. C. and Woo, C. Y. (1988), 'Entrepreneurs' perceived chances for success', *Journal of Business Venturing* **3**(2), 97–108.

Daniel, K., Hirshleifer, D. and Subrahmanyam, A. (1998), 'Investor psychology and security market under– and overreaction', *Journal of Finance* **53**(6), 1839–1885.

Daniel, K., Hirshleifer, D. and Subrahmanyam, A. (2001), 'Overconfidence, arbitrage, and equilibrium asset pricing', *Journal of Finance* **56**(3), 921–965.

Daniel, K., Hirshleifer, D. and Teoh, S. H. (2002), 'Investor pschology in capital markets: evidence and policy implications', *Journal of Monetary Economics* **49**(1), 139–209.

DeBondt, W. and Thaler, R. (1995), Financial decision–making in markets and firms: a behavioral perspective, *in* R. A. Jarrow, V. Maksimovic and W. T. Ziemba, eds, 'Finance – Handbooks in Operations Research and Management Science', Vol. 9, Elsevier, New York, pp. 385–410.

Fleming, W. H. and Rishel, R. W. (1975), *Deterministic and stochastic control*, Springer, Berlin.

Gervais, S., Heaton, J. B. and Odean, T. (2000), Capital budgeting in the presence of managerial overconfidence and optimism. *Working Paper*, University of Pennsylvania.

Gervais, S. and Odean, T. (2001), 'Learning to be overconfident', *Review of Financial Studies* **14**(1), 1–27.

Goel, A. M. and Thakor, A. V. (2000), Rationality, overconfidence and leadership. *Working Paper*, University of Michigan.

Grossman, S. J. and Hart, O. D. (1983), 'An analysis of the principal–agent problem', *Econometrica* **51**(1), 7–45.

Heaton, J. B. (2002), 'Managerial optimism and corporate finance', *Financial Management* **31**(2), 33–45.

Hirshleifer, D. (2001), 'Investor psychology and asset prices', *Journal of Finance* **56**(4), 1533–1597.

Hirshleifer, D. and Luo, G. Y. (2001), 'On the survival of overconfident traders in a competitive securities market', *Journal of Financial Markets* **4**(1), 73–84.

Holmström, B. (1979), 'Moral hazard and observability', *Bell Journal of Economics* **10**(1), 74–91.

Holmström, B. and Milgrom, P. (1987), 'Aggregation and linearity in the provision of intertemporal incentives', *Econometrica* **55**(2), 303–328.

Kyle, A. S. and Wang, F. A. (1997), 'Speculation duopoly with agreement to disagree: can overconfidence survive the market test', *Journal of Finance* **52**(5), 2073–2090.

Laffont, J.-J. and Martimort, D. (2002), *The theory of incentives*, Princeton University Press, Princeton.

Liptser, R. S. and Shiryaev, A. N. (2001), *Statistics of random processes I: General theory*, 2 edn, Springer–Verlag, Berlin.

Malmendier, U. and Tate, G. (2002), CEO overconfidence and corporate investment. *Working Paper*, Stanford University.

Müller, H. M. (1998), 'The first–best sharing rule in the continuous–time principal–agent problem with exponential utility', *Journal of Economic Theory* **79**, 276–280.

Odean, T. (1998), 'Volume, volatility, price, and profit when all traders are above average', *Journal of Finance* **53**(6), 1887–1934.

Ou-Yang, H. (2003), 'Optimal contracts in a continuous–time delegated portfolio management problem', *Review of Financial Studies* **16**(1), 173–208.

Ross, S. A. (1973), 'The economic theory of agency: the principal's problem', *American Economic Review* **63**(2), 134–139.

Russo, J. E. and Schoemaker, P. J. H. (1992), 'Managing overconfidence', *Sloan Management Review* **33**(2), 7–17.

Scheinkman, J. and Xiong, W. (2002), Overconfidence, short–sale constraints, and bubbles. *Working Paper*, Princeton University.

Schättler, H. and Sung, J. (1993), 'The first–order approach to the continuous–time principal–agent problem with exponential utility', *Journal of Economic Theory* **61**, 331–371.

Schättler, H. and Sung, J. (1997), 'On optimal sharing rules in discrete– and continuous–time principal–agent problem with exponential utility', *Journal of Economic Dynamics and Control* **21**, 551–574.

Shefrin, H. (2001*a*), 'Behavioral corporate finance', *Journal of Applied Corporate Finance* **14**(3), 113–124.

Shefrin, H. (2001*b*), On kernels and sentiment. *Working Paper*, Santa Clara University.

Sung, J. (1995), 'Linearity with project selection and controllable diffusion rate in continuous–time principal–agent problems', *RAND Journal of Economics* **26**(4), 720–743.

Sung, J. (1997), 'Corporate insurance and managerial incentives', *Journal of Economic Theory* **74**, 297–332.

Sung, J. (2001*a*), Lectures on the theory of contracts in corporate finance: from discrete–time to continuous–time models. *Lecture Notes*, University of Illinois, Chicago.

Sung, J. (2001*b*), Optimal contracts under moral hazard and adverse selection: a continuous–time approach. *Working Paper*, University of Illinois, Chicago.

A Appendix

Lemma 1 *Let the unobservable state X_t of a system and the signal Y_t on the state of the system be given by*

$$dX_t = \left[a(t)X_t + c_0(t) + c_1(t)g(t, Y_0^t)\right] dt + b_1(t)dW_1(t) + b_2(t)dW_2(t) \quad (4)$$

and

$$dY_t = \left[A(t)X_t + C_0(t) + C_1(t)h(t, Y_0^t)\right] dt + B(t)dW_2(t), \quad (5)$$

where $W_1(t)$ and $W_2(t)$ are independent standard Wiener processes, $a(t)$, $A(t)$, $b_i(t)$, $B(t)$, $c_j(t)$ and $C_j(t)$ are continuous deterministic functions ($i = 1, 2; j = 0, 1$), and $g(t, Y_0^t)$ and $h(t, Y_0^t)$ are functions of time t and signals $Y_0^t \equiv \{Y_s | 0 \leq s \leq t\}$ such that equations (4) and (5) obey the unique solution and

$$\int_0^T \mathrm{E}[g^2(t, Y_0^t) + h^2(t, Y_0^t)]dt < \infty. \quad (6)$$

The optimal filtering estimate $\hat{X}_t$ and the filtering error $P(t) = \mathrm{E}[(X_t - \hat{X}_t)^2]$ are defined by the generalized Kalman filter

$$d\hat{X}_t = [a(t)\hat{X}_t + c_0(t) + c_1(t)g(t, Y_0^t)]dt + \frac{b_2(t)B(t) + P(t)A(t)}{B^2(t)} \cdot \left(dY_t - [A(t)\hat{X}_t + C_0(t) + C_1(t)h(t, Y_0^t)]dt\right) \quad (7)$$

and

$$P'(t) = 2a(t)P(t) + b_1^2(t) + b_2^2(t) - \frac{(b_2(t)B(t) + P(t)A(t))^2}{B^2(t)} \quad (8)$$

subject to the initial conditions

$$\hat{X}(0) = \mathrm{E}[X(0)|Y(0)] \quad (9)$$

and

$$P(0) = \mathrm{E}[(X(0) - \hat{X}(0))^2]. \quad (10)$$

Proof. Cf. Liptser and Shiryaev (2001). □

A.1 Proof of proposition 1

Proof. The proof is a straightforward application of lemma 1. First, define $a(t) = 0$, $c_0(t) = 0$, $c_1(t) = 1$, $g(t, Y_0^t) = f(u_t)$, $b_1(t) = \sigma_x$, $b_2(t) = 0$, $A(t) = 1$, $C_0(t) = 0$, $C_1(t) = 0$ and $B(t) = \sigma_y$. Second, determine the stationary solution for $P(t)$ from the Riccati differential equation (8) by claiming $P'(t) = 0$. This yields $P(t) = \sigma_x \sigma_y$. Last, plug the stationary solution for $P(t)$ into (7) and then replace σ_y by $\sqrt{\kappa}\sigma_y$ according to the overconfidence bias. This completes the proof. □

A.2 Proof of proposition 2

Proof. We conjecture the general shape

$$s(\hat{X}_1) = S(\hat{X}_1) + \int_0^1 \alpha(t, \hat{X}_t)dt + \int_0^1 \beta(t, \hat{X}_t)d\hat{X}_t \tag{11}$$

for the sharing rule, where α and β are bounded and time t predictable functions and $S(\hat{X}_1)$ is a time 1 measurable random variable. Note, that the normality and the independence of the increments $d\hat{X}_t$ implies the normality of $s(\hat{X}_1)$. The first–best sharing rule $s_1(\hat{X}_1)$ is the solution to the first–best program which due to the principal's risk neutrality becomes

$$\max_{\boldsymbol{u}, s_1(\hat{X}_1)} \quad \mathrm{E}\left[\hat{X}_1 - s_1(\hat{X}_1)\right] \tag{12}$$

$$\text{s.t.} \quad d\hat{X}_t = f(u_t)dt + \frac{\sigma_x}{\sqrt{\kappa}\sigma_y} d\hat{B}_t \tag{13}$$

$$\mathrm{E}\left[-\exp\left(-a\left(s_1(\hat{X}_1) - \int_0^1 c(u_t)dt\right)\right)\right] \geq -\exp(-aW_0), \tag{14}$$

where $\boldsymbol{u}$ comprises $\forall t$ the time t controls u_t and $s_1(\hat{X}_1)$ has shape (11). Note that (14) represents the agent's participation constraint. Let $\hat{X}_0 = 0$. Then integrating (13) yields

$$\hat{X}_1 = \int_0^1 f(u_t)dt + \frac{\sigma_x}{\sqrt{\kappa}\sigma_y} \hat{B}_1. \tag{15}$$

Define

$$\tilde{z} \equiv s_1(\hat{X}_1) - \int_0^1 c(u_t)dt, \tag{16}$$

which has law $\mathcal{N}(\mu_z, \sigma_z^2)$. Applying (15) and (16) to (12)–(14) and exploiting $\mathrm{E}[\hat{B}_1] = 0$ results in an equivalent program

$$\max_{\boldsymbol{u},\tilde{z}} \quad \mathrm{E}\left[-\tilde{z} + \int_0^1 (f(u_t) - c(u_t))dt\right] \tag{17}$$

$$\text{s.t.} \quad \mathrm{E}\left[-e^{-a\tilde{z}}\right] \geq -e^{-aW_0}, \tag{18}$$

which can be split in two independent optimization problems. First,

$$\max_{\tilde{z}} \quad -\mu_z \tag{19}$$

$$\text{s.t.} \quad \mathrm{E}\left[-e^{-a\tilde{z}}\right] \geq -e^{-aW_0}, \tag{20}$$

and second,

$$\max_{\boldsymbol{u}} \quad \mathrm{E}\left[\int_0^1 (f(u_t) - c(u_t))dt\right]. \tag{21}$$

Note that the choice of $\tilde{z}$ in the first optimization problem boils down to choosing the mean μ_z and the variance σ_z^2. Since $\mathrm{E}\left[-e^{-a\tilde{z}}\right] = -e^{-a(\mu_z - \frac{1}{2}a\sigma_z^2)}$, the constraint (20) is equivalent to

$$\mu_z - \frac{1}{2}a\sigma_z^2 \geq W_0. \tag{22}$$

Hence, at least $\mu_z = W_0 + \frac{1}{2}a\sigma_z^2$ in order to meet the constraint (22). Note that $\sigma_z^2 = 0$ gives the minimum lower bound $\mu_z = W_0$ that fulfills (22) and thus maximizes $-\mu_z$. Consequently, the optimal choices are $\mu_z = W_0$ and $\sigma_z^2 = 0$, what yields $\tilde{z} = W_0$ immediately. Next, pointwise optimization of (21) yields

$$f'(u_t) = c'(u_t) \quad \forall t, \tag{23}$$

which due to the linearity of f yields $c'(u_{t_1}) = c'(u_{t_2})$, $\forall t_1, t_2$. Thus $u_{t_1} = u_{t_2}$, $\forall t_1, t_2$. Finally, $u_1 = u_t$, $\forall t$ represents the constant first–best control. Apply $u_t = u_1$ and $\tilde{z} = W_0$ to (16). Integrating and rearranging yields the proposition. This completes the proof. □

A.3 Proof of proposition 3

Proof. The second–best sharing rule $s_2(\hat{X}_1)$ is the solution to the second–best program which according to the principal's risk neutrality is

$$\max_{s_2(\hat{X}_1)} \quad \mathrm{E}\left[\hat{X}_1 - s_2(\hat{X}_1)\right] \tag{24}$$

$$\text{s.t.} \quad d\hat{X}_t = f(u_t)dt + \frac{\sigma_x}{\sqrt{\kappa}\sigma_y} d\hat{B}_t \tag{25}$$

$$\max_{\boldsymbol{u}} \ \mathrm{E}\left[-\exp\left(-a\left(s_2(\hat{X}_1) - \int_0^1 c(u_t)dt\right)\right)\right] \tag{26}$$

$$\mathrm{E}\left[-\exp\left(-a\left(s_2(\hat{X}_1) - \int_0^1 c(u_t)dt\right)\right)\right] \geq -\exp(-aW_0), \tag{27}$$

where $\boldsymbol{u}$ comprises $\forall t$ the time t controls u_t and $s_2(\hat{X}_1)$ has shape (11). Here (26) represents the incentive compatibility constraint. Replacing the second–best sharing rule $s_2(\hat{X}_1)$ with the sharing rule conjecture (11) allows to restate the agent's problem

$$\max_{\boldsymbol{u}} \quad \mathrm{E}\left[-e^{-a\left(S(\hat{X}_1)+\int_0^1 \alpha(t,\hat{X}_t)dt+\int_0^1 \beta(t,\hat{X}_t)d\hat{X}_t-\int_0^1 c(u_t)dt\right)}\right] \tag{28}$$

$$\text{s.t.} \quad d\hat{X}_t = f(u_t)dt + \frac{\sigma_x}{\sqrt{\kappa}\sigma_y} d\hat{B}_t \tag{29}$$

alternatively. Plugging (29) into (28) and rearranging terms in the exponent yields

$$\max_{\boldsymbol{u}} \quad \mathrm{E}\left[-e^{-a\left(S(\hat{X}_1)+\int_0^1\left[\alpha(t,\hat{X}_t)+\beta(t,\hat{X}_t)f(u_t)-c(u_t)\right]dt+\int_0^1 \beta(t,\hat{X}_t)\frac{\sigma_x}{\sqrt{\kappa}\sigma_y}d\hat{B}_t\right)}\right] \tag{30}$$

for the agent's problem. Next define the value function

$$V(t,\hat{X}_t) = \mathrm{E}\left[-e^{-a\left(S(\hat{X}_1)+\int_t^1\left[\alpha(s,\hat{X}_s)+\beta(s,\hat{X}_s)f(u_s)-c(u_s)\right]ds+\int_t^1 \beta(s,\hat{X}_s)\frac{\sigma_x}{\sqrt{\kappa}\sigma_y}d\hat{B}_s\right)}\right], \tag{31}$$

where u_s denotes the optimal time s control $\forall s \in [t,1]$. Thus $V(t,\hat{X}_t)$ gives the maximum expected utility that the agent achieves from pursuing an optimal strategy $\{u_s\}_{t\leq s\leq 1}$ during the time interval $[t,1]$. According to Sung (2001*a*) or Schättler and Sung (1993) the value function obeys the partial differential equation

$$0 = \frac{\partial V(t,\hat{X}_t)}{\partial t} + \frac{1}{2}\frac{\partial^2 V(t,\hat{X}_t)}{\partial \hat{X}_t^2}\frac{\sigma_x^2}{\kappa\sigma_y^2} + \max h(t,\hat{X}_t), \tag{32}$$

where

$$h(t,\hat{X}_t) \equiv \frac{\partial V(t,\hat{X}_t)}{\partial \hat{X}_t}\left(f(u_t) - a\beta(t,\hat{X}_t)\frac{\sigma_x^2}{\kappa\sigma_y^2}\right) + aV(t,\hat{X}_t)\left(c(u_t) - \alpha(t,\hat{X}_t) - \beta(t,\hat{X}_t)f(u_t) + \frac{1}{2}a\beta(t,\hat{X}_t)^2\frac{\sigma_x^2}{\kappa\sigma_y^2}\right). \tag{33}$$

Hence, an optimal time t control u_t must maximize $h(t,\hat{X}_t)$. Therefore, the first order condition for an optimal time t control u_t is

$$\frac{dh(t,\hat{X}_t)}{du_t} = \frac{\partial V(t,\hat{X}_t)}{\partial \hat{X}_t}f'(u_t) + aV(t,\hat{X}_t)\left(c'(u_t) - \beta(t,\hat{X}_t)f'(u_t)\right) \overset{!}{=} 0, \tag{34}$$

which can be stated equivalently as

$$\frac{\partial V(t,\hat{X}_t)}{\partial \hat{X}_t} = -aV(t,\hat{X}_t)\left(\frac{c'(u_t)}{f'(u_t)} - \beta(t,\hat{X}_t)\right). \tag{35}$$

After applying (35) to (33) the partial differential equation (32) becomes

$$0 = \frac{\partial V(t,\hat{X}_t)}{\partial t} + \frac{1}{2}\frac{\partial^2 V(t,\hat{X}_t)}{\partial \hat{X}_t^2}\frac{\sigma_x^2}{\kappa\sigma_y^2} + aV(t,\hat{X}_t)H(t,\hat{X}_t), \tag{36}$$

where

$$H(t,\hat{X}_t) \equiv -\left(\frac{c'(u_t)}{f'(u_t)} - \beta(t,\hat{X}_t)\right)\left(f(u_t) - a\beta(t,\hat{X}_t)\frac{\sigma_x^2}{\kappa\sigma_y^2}\right) + c(u_t) - \alpha(t,\hat{X}_t) - \beta(t,\hat{X}_t)f(u_t) + \frac{1}{2}a\beta(t,\hat{X}_t)^2\frac{\sigma_x^2}{\kappa\sigma_y^2}. \tag{37}$$

Having defined the agent's value function $V(t,\hat{X}_t)$ allows to characterize the agent's certainty equivalent wealth process W_t. Note that the utility of the wealth W_t at time t has to equal the agent's maximum expected utility $V(t,\hat{X}_t)$. Formally,

$$V(t,\hat{X}_t) = -e^{-aW_t}, \tag{38}$$

which is equivalent to

$$W_t = -\frac{1}{a}\ln\left(-V(t,\hat{X}_t)\right). \tag{39}$$

Since $\hat{X}_t$ is stochastic the differential dW_t according to Itô's lemma is

$$dW_t = -\frac{1}{aV(t,\hat{X}_t)}dV(t,\hat{X}_t) + \frac{1}{2}\frac{1}{aV(t,\hat{X}_t)^2}dV(t,\hat{X}_t)^2 \tag{40}$$

and thus

$$W_\tau = W_0 + \int_0^\tau dW_t \tag{41}$$

$$= W_0 - \int_0^\tau \frac{1}{aV(t,\hat{X}_t)}dV(t,\hat{X}_t) + \frac{1}{2}\int_0^\tau \frac{1}{aV(t,\hat{X}_t)^2}dV(t,\hat{X}_t)^2. \tag{42}$$

Again, by Itô's lemma the differential $dV(t,\hat{X}_t)$ is

$$dV(t,\hat{X}_t) = \frac{\partial V(t,\hat{X}_t)}{\partial t}dt + \frac{\partial V(t,\hat{X}_t)}{\partial \hat{X}_t}d\hat{X}_t + \frac{1}{2}\frac{\partial^2 V(t,\hat{X}_t)}{\partial \hat{X}_t^2}d\hat{X}_t^2. \tag{43}$$

Note that from (29) we calculate

$$d\hat{X}_t^2 = \frac{\sigma_x^2}{\kappa\sigma_y^2}dt. \tag{44}$$

Hence using (44) the differential $dV(t,\hat{X}_t)$ in (43) becomes

$$dV(t,\hat{X}_t) = \left(\frac{\partial V(t,\hat{X}_t)}{\partial t} + \frac{1}{2}\frac{\partial^2 V(t,\hat{X}_t)}{\partial \hat{X}_t^2}\frac{\sigma_x^2}{\kappa\sigma_y^2}\right)dt + \frac{\partial V(t,\hat{X}_t)}{\partial \hat{X}_t}d\hat{X}_t. \tag{45}$$

Applying (36) and (35) to (45) we obtain

$$dV(t,\hat{X}_t) = \\ - aV(t,\hat{X}_t)H(t,\hat{X}_t)dt - aV(t,\hat{X}_t)\left(\frac{c'(u_t)}{f'(u_t)} - \beta(t,\hat{X}_t)\right)d\hat{X}_t, \tag{46}$$

which in turn implies

$$dV(t,\hat{X}_t)^2 = a^2V(t,\hat{X}_t)^2\left(\frac{c'(u_t)}{f'(u_t)} - \beta(t,\hat{X}_t)\right)^2\frac{\sigma_x^2}{\kappa\sigma_y^2}dt \tag{47}$$

immediately. Plugging (46) and (47) into (42) yields

$$W_\tau = W_0 + \int_0^\tau \left[H(t,\hat{X}_t) + \frac{1}{2} a \left(\frac{c'(u_t)}{f'(u_t)} - \beta(t,\hat{X}_t) \right)^2 \frac{\sigma_x^2}{\kappa\sigma_y^2} \right] dt + \int_0^\tau \left(\frac{c'(u_t)}{f'(u_t)} - \beta(t,\hat{X}_t) \right) d\hat{X}_t. \quad (48)$$

The definition of the value function in (31) implies $V(1,\hat{X}_1) = -e^{-aS(\hat{X}_1)}$. On the other hand the definition of the certainty equivalent wealth process in (38) requires $V(1,\hat{X}_1) = -e^{-aW_1}$. Thus

$$S(\hat{X}_1) = W_1 \quad (49)$$

necessarily. Using (49) the second–best sharing rule results as

$$s_2(\hat{X}_1) = W_1 + \int_0^1 \alpha(t,\hat{X}_t)dt + \int_0^1 \beta(t,\hat{X}_t)d\hat{X}_t, \quad (50)$$

which by application of (48) for $\tau = 1$ becomes

$$s_2(\hat{X}_1) = W_0 + \int_0^1 \left[H(t,\hat{X}_t) + \frac{1}{2} a \left(\frac{c'(u_t)}{f'(u_t)} - \beta(t,\hat{X}_t) \right)^2 \frac{\sigma_x^2}{\kappa\sigma_y^2} + \alpha(t,\hat{X}_t) \right] dt + \int_0^1 \frac{c'(u_t)}{f'(u_t)} d\hat{X}_t. \quad (51)$$

Plugging (37) into (51) and cancelling terms yields

$$s_2(\hat{X}_1) = W_0 + \int_0^1 \left[c(u_t) + \frac{a}{2} \frac{c'(u_t)^2}{f'(u_t)^2} \frac{\sigma_x^2}{\kappa\sigma_y^2} - \frac{c'(u_t)}{f'(u_t)} f(u_t) \right] dt + \int_0^1 \frac{c'(u_t)}{f'(u_t)} d\hat{X}_t. \quad (52)$$

Note that the agent's time t controls u_t in the sharing rule (52) are optimal since the derivation imposed the first order condition (35). Furthermore, the sharing rule (52) maximizes the agent's expected utility and thus solves the agent's problem. Therefore the sharing rule (52) is called admissible.

The principal's choice of a sharing rule is restricted to an admissible sharing rule since the incentive compatibility constraint must be met. Straightforward inspection of (52) yields that the choice of an admissible sharing rule boils down to the choice of a control $\boldsymbol{u}$. If the control $\boldsymbol{u}$ chosen by the principal is optimal for the agent too then that control $\boldsymbol{u}$ is called implementable. Since we assumed the function f to be linear and the function c to be quadratic any control $\boldsymbol{u}$ chosen by the principal is implementable. The implementability issue is discussed by Sung (1997) at length. Ultimately, the principal's problem is to choose a control $\boldsymbol{u}$ that maximizes his expected utility and to restrict to an admissible sharing rule. Thus, the principal's problem becomes

$$\max_{\boldsymbol{u}} \quad \mathrm{E}\left[\hat{X}_1 - W_0 - \int_0^1 \Gamma_1(u_t)dt - \int_0^1 \Gamma_2(u_t)d\hat{X}_t\right] \tag{53}$$

$$\text{s.t.} \quad d\hat{X}_t = f(u_t)dt + \frac{\sigma_x}{\sqrt{\kappa}\sigma_y}d\hat{B}_t, \tag{54}$$

where

$$\Gamma_1(u_t) = c(u_t) + \frac{a}{2}\frac{c'(u_t)^2}{f'(u_t)^2}\frac{\sigma_x^2}{\kappa\sigma_y^2} - \frac{c'(u_t)}{f'(u_t)}f(u_t) \quad \text{and} \tag{55}$$

$$\Gamma_2(u_t) = \frac{c'(u_t)}{f'(u_t)}. \tag{56}$$

Both integrating and plugging (54) into (53) yields

$$\max_{\boldsymbol{u}} \quad \mathrm{E}\left[-W_0 + \int_0^1 \Gamma_3(u_t)dt - \frac{\sigma_x}{\sqrt{\kappa}\sigma_y}\int_0^1 \Gamma_4(u_t)d\hat{B}_t\right] \tag{57}$$

for the principal's problem, where

$$\begin{aligned}\Gamma_3(u_t) &= f(u_t) - \Gamma_1(u_t) - \Gamma_2(u_t)f(u_t)\\ &= f(u_t) - c(u_t) - \frac{a}{2}\frac{c'(u_t)^2}{f'(u_t)^2}\frac{\sigma_x^2}{\kappa\sigma_y^2} \quad \text{and}\end{aligned} \tag{58}$$

$$\Gamma_4(u_t) = \Gamma_2(u_t) - 1. \tag{59}$$

Evaluating the expectation operator allows the principal's problem to be restated as

$$\max_{\boldsymbol{u}} \quad -W_0 + \int_0^1 \Gamma_3(u_t)dt, \tag{60}$$

which is equivalent to

$$\max_{\boldsymbol{u}} \quad \int_0^1 \left[f(u_t) - c(u_t) - \frac{a}{2}\frac{c'(u_t)^2}{f'(u_t)^2}\frac{\sigma_x^2}{\kappa\sigma_y^2}\right]dt. \tag{61}$$

Pointwise optimization requires the solution of

$$\max_{u_t} \quad f(u_t) - c(u_t) - \frac{a}{2}\frac{c'(u_t)^2}{f'(u_t)^2}\frac{\sigma_x^2}{\kappa\sigma_y^2} \quad \forall t. \tag{62}$$

Since the optimization problem (62) is the same for any u_t the first order conditions yield identical optimal time t controls u_t, $\forall t$. Thus, $u_2 = u_t$, $\forall t$ represents the constant second–best control. Applying $u_t = u_2$ to (52) yields

$$s_2(\hat{X}_1) = \\ W_0 + \int_0^1 \left[c(u_2) + \frac{a}{2}\frac{c'(u_2)^2}{f'(u_2)^2}\frac{\sigma_x^2}{\kappa\sigma_y^2} - \frac{c'(u_2)}{f'(u_2)} f(u_2) \right] dt + \int_0^1 \frac{c'(u_2)}{f'(u_2)} d\hat{X}_t, \tag{63}$$

which after integration gives the sharing rule as given in the proposition. This completes the proof. □

A.4 Proof of corollary 1

Proof. According to the linearity assumption as regards the function f we define $f'(u) \equiv \phi > 0$. Since the function c is assumed to be quadratic we define $c'(u) \equiv \gamma_1 u + \gamma_0$. Note that $c''(u) = \gamma_1 > 0$ by the convexity of the function c. The first–best control u_1 has to meet the first order condition (23). Consequently, $f'(u_1) = c'(u_1)$. Alternatively, $\phi = \gamma_1 u_1 + \gamma_0$. Hence,

$$u_1 = \frac{1}{\gamma_1}(\phi - \gamma_0). \tag{64}$$

Thus the first–best control u_1 is independent of κ. The second–best control u_2 solves the optimization problem (62). Consequently, the second–best control u_2 has to meet

$$\frac{d}{du_2}\left(f(u_2) - c(u_2) - \frac{a}{2}\frac{c'(u_2)^2}{f'(u_2)^2}\frac{\sigma_x^2}{\kappa\sigma_y^2} \right) \stackrel{!}{=} 0, \tag{65}$$

which after differentiation and by exploiting $f''(u_2) = 0$ becomes

$$f'(u_2) - c'(u_2)\left(1 + \frac{a\sigma_x^2}{\kappa\sigma_y^2}\frac{c''(u_2)}{f'(u_2)^2}\right) = 0. \tag{66}$$

Replacing $f'(u_2) = \phi$ and $c'(u_2) = \gamma_1 u_2 + \gamma_0$ results in

$$\phi - (\gamma_1 u_2 + \gamma_0)\left(1 + \frac{a\sigma_x^2}{\kappa\sigma_y^2}\frac{\gamma_1}{\phi^2}\right) = 0, \tag{67}$$

which can be solved for

$$u_2 = \frac{1}{\gamma_1}\left(\frac{\phi}{1+\frac{a\sigma_x^2}{\kappa\sigma_y^2}\frac{\gamma_1}{\phi^2}} - \gamma_0\right). \tag{68}$$

Hence the second–best control depends on κ. Note that

$$\frac{a\sigma_x^2}{\kappa\sigma_y^2}\frac{\gamma_1}{\phi^2} > 0 \tag{69}$$

implies $u_2 < u_1$ immediately. This completes the proof. □

A.5 Proof of proposition 4

Proof. The principal's expected utility in the first–best case amounts to

$$\mathrm{E}\left[\hat{X}_1 - s_1(\hat{X}_1)\right] = \mathrm{E}\left[\int_0^1 f(u_1)dt + \frac{\sigma_x}{\sqrt{\kappa}\sigma_y}\int_0^1 d\hat{B}_t - W_0 - c(u_1)\right], \tag{70}$$

where we replaced the integrated filtered outcome process $\hat{X}_1 = \int_0^1 d\hat{X}_t$ as well as the first–best sharing rule $s_1(\hat{X}_1)$ from proposition 2 and applied the first–best control u_1. Integrating and evaluating the expectation operator yields the principal's first–best expected utility

$$-W_0 + f(u_1) - c(u_1). \tag{71}$$

In the second–best case the principal's expected utility is

$$\mathrm{E}\left[\hat{X}_1 - s_2(\hat{X}_1)\right] = \\ \mathrm{E}\left[\left(1-\Gamma_2(u_2)\right)\left(\int_0^1 f(u_2)dt + \frac{\sigma_x}{\sqrt{\kappa}\sigma_y}\int_0^1 d\hat{B}_t\right) - W_0 - \Gamma_1(u_2)\right], \tag{72}$$

where again we replaced the integrated filtered outcome process as well as the second–best sharing rule $s_2(\hat{X}_1)$ from proposition 3 and applied the second–best control u_2. Note that the functions Γ_1 and Γ_2 are defined in (55) and (56) respectively. Integration and evaluation of the expectation operator results in

$$\big(1-\Gamma_2(u_2)\big)f(u_2) - W_0 - \Gamma_1(u_2) = \\ -W_0 + f(u_2) - (\Gamma_1(u_2) + \Gamma_2(u_2)f(u_2)) = \\ -W_0 + f(u_2) - c(u_2) - \frac{a}{2}\frac{\sigma_x^2}{\kappa\sigma_y^2}\frac{c'(u_2)^2}{f'(u_2)^2} \tag{73}$$

for the principal's expected utility in the second–best case. Subtracting the expected utility in the second–best case (73) from the expected utility in the first–best case (71) yields the agency costs as given in the proposition. This completes the proof. □

A.6 Proof of corollary 2

Proof. The agency costs from proposition 4 amount to

$$f(u_1) - f(u_2) - \big(c(u_1) - c(u_2)\big) + \frac{a}{2}\frac{\sigma_x^2}{\kappa\sigma_y^2}\frac{c'(u_2)^2}{f'(u_2)^2}. \tag{74}$$

Obviously, by definition

$$\frac{a}{2}\frac{\sigma_x^2}{\kappa\sigma_y^2}\frac{c'(u_2)^2}{f'(u_2)^2} > 0. \tag{75}$$

The linearity of the function f yields

$$f(u_2) = f(u_1) + f'(u_1)\cdot(u_2 - u_1). \tag{76}$$

The second order Taylor series expansion of the quadratic function c around u_1 gives

$$c(u_2) = c(u_1) + c'(u_1)\cdot(u_2 - u_1) + \frac{1}{2}c''(u_1)\cdot(u_2 - u_1)^2. \tag{77}$$

Now we obtain for the first part of the agency costs

$$\begin{aligned} f(u_1) - f(u_2) - \big(c(u_1) - c(u_2)\big) &= \\ - f'(u_1)\cdot(u_2 - u_1) + c'(u_1)\cdot(u_2 - u_1) + \frac{1}{2}c''(u_1)\cdot(u_2 - u_1)^2 &> \\ - f'(u_1)\cdot(u_2 - u_1) + c'(u_1)\cdot(u_2 - u_1) &= \\ \big(c'(u_1) - f'(u_1)\big)\cdot(u_2 - u_1) &= 0, \end{aligned} \tag{78}$$

which yields that the first part of the agency costs is also positive generally. Note that the first equation in (78) comes from application of both (76) and (77). The inequality is true since the function c is convex that is $c''(u_2) > 0$. The last equation holds since the first–best control u_1 meets the first order condition (23). This completes the proof. □

A.7 Proof of proposition 5

Proof. From the first order condition (66) we define the function

$$F(u_2,\kappa) \equiv f'(u_2) - c'(u_2)\left(1 + \frac{a\sigma_x^2}{\kappa\sigma_y^2}\frac{c''(u_2)}{f'(u_2)^2}\right). \tag{79}$$

At an optimum the total derivative of the function F has to be zero. Hence,

$$dF(u_2,\kappa) = \frac{\partial F(u_2,\kappa)}{\partial u_2}du_2 + \frac{\partial F(u_2,\kappa)}{\partial \kappa}d\kappa \stackrel{!}{=} 0 \tag{80}$$

and consequently

$$\frac{du_2}{d\kappa} = -\frac{\frac{\partial F(u_2,\kappa)}{\partial \kappa}}{\frac{\partial F(u_2,\kappa)}{\partial u_2}}. \tag{81}$$

Since $c''(u_2) = \gamma_1$ and $f'(u_2) = \phi$, and thus constant, we calculate

$$\frac{\partial F(u_2,\kappa)}{\partial u_2} = -c''(u_2)\left(1 + \frac{a\sigma_x^2}{\kappa\sigma_y^2}\frac{c''(u_2)}{f'(u_2)^2}\right) \tag{82}$$

and

$$\frac{\partial F(u_2,\kappa)}{\partial \kappa} = \frac{a\sigma_x^2}{\kappa^2\sigma_y^2}\frac{c'(u_2)c''(u_2)}{f'(u_2)^2}. \tag{83}$$

Finally,

$$\frac{du_2}{d\kappa} = -\frac{\frac{a\sigma_x^2}{\kappa^2\sigma_y^2}\frac{c'(u_2)c''(u_2)}{f'(u_2)^2}}{-c''(u_2)\left(1 + \frac{a\sigma_x^2}{\kappa\sigma_y^2}\frac{c''(u_2)}{f'(u_2)^2}\right)} \tag{84}$$

$$= \frac{1}{\kappa}\cdot\frac{c'(u_2)}{\frac{\kappa\sigma_y^2}{a\sigma_x^2}f'(u_2)^2 + c''(u_2)} > 0, \tag{85}$$

what gives part (a.) of the proposition. The first derivative of the agency costs with respect to κ is

$$\frac{d}{d\kappa}\left(f(u_1) - f(u_2) - \big(c(u_1) - c(u_2)\big) + \frac{a}{2}\frac{\sigma_x^2}{\kappa\sigma_y^2}\frac{c'(u_2)^2}{f'(u_2)^2}\right) =$$

$$\big(c'(u_2) - f'(u_2)\big)\frac{du_2}{d\kappa} + \frac{a}{2}\frac{\sigma_x^2}{\sigma_y^2}\frac{1}{f'(u_2)^2}\frac{1}{\kappa^2}\left(2c''(u_2)\frac{du_2}{d\kappa}\kappa - c'(u_2)\right)c'(u_2), \tag{86}$$

which after applying (85) becomes

$$\frac{1}{\kappa}\cdot\frac{\big(c'(u_2)-f'(u_2)\big)c'(u_2)}{\frac{\kappa\sigma_y^2}{a\sigma_x^2}f'(u_2)^2+c''(u_2)} \\ +\frac{1}{2\kappa}\frac{a\sigma_x^2}{\kappa\sigma_y^2}\frac{c'(u_2)}{f'(u_2)^2}\left(\frac{2c''(u_2)c'(u_2)}{\frac{\kappa\sigma_y^2}{a\sigma_x^2}f'(u_2)^2+c''(u_2)}-c'(u_2)\right). \quad (87)$$

Simplifying (87) yields

$$\frac{1}{2\kappa}\frac{a\sigma_x^2}{\kappa\sigma_y^2}\frac{c'(u_2)}{f'(u_2)^2}\left(c'(u_2)-\frac{2f'(u_2)}{1+\frac{a\sigma_x^2}{\kappa\sigma_y^2}\frac{c''(u_2)}{f'(u_2)^2}}\right) \quad (88)$$

for the agency costs' first derivative with respect to κ. Plugging (68) into $c'(u_2)=\gamma_1 u_2+\gamma_0$ gives

$$c'(u_2)=\frac{f'(u_2)}{1+\frac{a\sigma_x^2}{\kappa\sigma_y^2}\frac{c''(u_2)}{f'(u_2)^2}}, \quad (89)$$

which applied to the term in parenthesis of (88) results in

$$-\frac{1}{2\kappa}\frac{a\sigma_x^2}{\kappa\sigma_y^2}\frac{c'(u_2)}{f'(u_2)^2}\frac{f'(u_2)}{1+\frac{a\sigma_x^2}{\kappa\sigma_y^2}\frac{c''(u_2)}{f'(u_2)^2}}<0 \quad (90)$$

for the first derivative of the agency costs with respect to κ. This yields part (b.) of the proposition. Part (c.) of the proposition is delivered by

$$\frac{dc(u_2)}{d\kappa}=c'(u_2)\cdot\frac{du_2}{d\kappa}>0. \quad (91)$$

The first derivative of the second–best sharing rule's risk premium with respect to κ is

$$\Delta_1\equiv\frac{d}{d\kappa}\left(\frac{a}{2}\frac{\sigma_x^2}{\kappa\sigma_y^2}\frac{c'(u_2)^2}{f'(u_2)^2}\right) \\ =\frac{a}{2}\frac{\sigma_x^2}{\sigma_y^2}\frac{1}{f'(u_2)^2}\frac{1}{\kappa^2}\left(2c''(u_2)\frac{du_2}{d\kappa}\kappa-c'(u_2)\right)c'(u_2), \quad (92)$$

which after plugging in (85) becomes

$$\frac{a}{2}\frac{\sigma_x^2}{\sigma_y^2}\frac{1}{f'(u_2)^2}\frac{1}{\kappa^2}\left(\frac{2c''(u_2)c'(u_2)}{\frac{\kappa\sigma_y^2}{a\sigma_x^2}f'(u_2)^2+c''(u_2)}-c'(u_2)\right)c'(u_2) \quad (93)$$

or equivalently

$$\frac{a}{2}\frac{\sigma_x^2}{\sigma_y^2}\frac{c'(u_2)^2}{f'(u_2)^2}\frac{1}{\kappa^2}\left(\frac{2}{\frac{\kappa\sigma_y^2}{a\sigma_x^2}\frac{f'(u_2)^2}{c''(u_2)}+1}-1\right). \tag{94}$$

Now define

$$\kappa_0 \equiv \frac{a\sigma_x^2}{\sigma_y^2}\frac{c''(u_2)}{f'(u_2)^2}. \tag{95}$$

From (94) we conclude that

$$\Delta_1 \begin{cases} >0 & \text{, if} \quad \kappa < \kappa_0 \\ <0 & \text{, if} \quad \kappa > \kappa_0. \end{cases} \tag{96}$$

Finally, the first derivative of the second–best sharing rule's sensitivity to the unexpected outcome with respect to κ is

$$\begin{aligned} \Delta_2 \equiv \frac{d}{d\kappa}\left(\frac{\sigma_x}{\sqrt{\kappa}\sigma_y}\frac{c'(u_2)}{f'(u_2)}\right) &= \sqrt{\frac{2}{a}}\cdot\frac{d}{d\kappa}\sqrt{\frac{a}{2}\frac{\sigma_x^2}{\kappa\sigma_y^2}\frac{c'(u_2)^2}{f'(u_2)^2}} \\ &= \sqrt{\frac{2}{a}}\cdot\frac{1}{2}\cdot\left(\frac{a}{2}\frac{\sigma_x^2}{\kappa\sigma_y^2}\frac{c'(u_2)^2}{f'(u_2)^2}\right)^{-\frac{1}{2}}\cdot\frac{d}{d\kappa}\left(\frac{a}{2}\frac{\sigma_x^2}{\kappa\sigma_y^2}\frac{c'(u_2)^2}{f'(u_2)^2}\right) \\ &= \sqrt{\frac{2}{a}}\cdot\frac{1}{2}\cdot\left(\frac{a}{2}\frac{\sigma_x^2}{\kappa\sigma_y^2}\frac{c'(u_2)^2}{f'(u_2)^2}\right)^{-\frac{1}{2}}\cdot\Delta_1, \end{aligned} \tag{97}$$

what implies immediately

$$\Delta_2 \begin{cases} >0 & \text{, if} \quad \kappa < \kappa_0 \\ <0 & \text{, if} \quad \kappa > \kappa_0. \end{cases} \tag{98}$$

Note that (96) and (98) in conjunction with $\kappa \in (0,1)$ yield the parts (d.) and (e.) of the proposition. This completes the proof. □