Risk Capital Allocation using Value at Risk Limits:

Incorporating unpredictable Correlations between Traders' Exposures[1]

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ABSTRACT

Value at risk of multiple assets is calculated by taking into account correlations between these assets. In order to guarantee full usage of value at risk limits (risk capital) and thereby to maximize bank's profit, top down capital allocation also has to account for correlations. With business units deciding more or less independently about size and direction (long or short) of their investments correlations between business units' exposures cannot be determined ex ante.

Within a simulation model we develop a value at risk limit system that guarantees no overshooting of total limit and sub-limits as well as full usage of total risk capital considering all diversification effects.

We first model the capital budgeting procedure mainly used by banks today and show that the average usage of total risk capital hereby is only 31.38 %. The effect of great parts of risk capital not being utilized can be avoided by implementing a treasurer. Although the treasurer has no forecasting skills the profit of the whole trading division increases essentially.

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1 INTRODUCTION

In January 1996 the Basle Committee on Banking Supervision launched the amendment to the capital accord to incorporate market risks. According to the qualitative standards of part B "use of internal models to measure market risks" (section B.2) banks are required to install bank-wide value at risk limits for controlling traders' risk exposures. A common practice in banks using the internal models approach for regulatory capital calculation is that value at risk limits (risk capital)[1] are allocated in a top down process to business units. Because value at risk of multiple assets is determined by incorporating the correlations between these assets, top down capital allocation has to account for correlation effects, too. As a result of having asset correlations smaller than one, the sum of value at risk limits of the business units exceeds the total value at risk limit of the bank.

Banks hire traders and other financial specialists in order to use their experience and forecasting skills to earn profits. If traders choose their exposures more or less independently, diversification and the resulting total value at risk of a bank cannot be predicted, even if ex ante the trading universe is known and correlations between all risk factors are stationary. Because of this diversification effect a significant part of risk capital is not used to generate profits. Perold (2001) reports for a New York based investment bank that the fraction of unused risk capital is more than 70 % of total risk capital.

Of course, a shareholder of a bank as well as bank's management wants all business units to maximize profit of given risk capital. An allocation approach that enables traders to always use total risk capital should increase the probability of higher profits. On the other hand, traders' risk exposure should be restricted because of regulatory and

[1] For further definition see chapter 2.

internal requirements. It is a well known fact that bank capital is the “rare and costly resource” in banking and therefore has to be used reasonably.[2]

Because of risk interdependencies between traders’ positions and to attain optimal investment decisions these decisions have to be centralized. However in trading decisions have to be made within high frequencies, and as a result, the costs of centralization and the delays associated when transmitting new information to headquarters each time an investment is made, may be prohibitive.[3] Obviously, centralization does not work for trading businesses.

To conclude, a risk capital allocation approach has to prohibit ex ante overshooting of total value at risk limits and sub-limits and in order to make optimal decisions total risk capital should always be used totally. Last but not least, the capital budgeting approach should also be of practical use for real trading situations. In this paper we want to address this problem using simulation technique.

The reminder is organized as follows: In chapter 2 we review the literature about value at risk and risk capital, discuss open questions of risk capital allocation and sum up the main features of performance measurement and risk capital allocation. Our simulation study is presented in chapter 3. We first adopt the commonly used limit allocation system by banks today (basic model) (3.1), develop a benchmark model (3.2) and a more practical approach (3.3), which we call the treasurer model. The three alternatives are compared and evaluated in 3.4. We summarize our results in chapter 4.

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[2] See for a detailed discussion Berger / Herring / Szegö (1995).

[3] See Froot / Stein (1998), p. 67.

2 VALUE AT RISK AND CAPITAL ALLOCATION: A REVIEW OF LITERATURE

In our study we consider a bank that uses value at risk limits for controlling market risk exposure of its trading division. Value at risk is defined to be the €-amount of loss which is exceeded by actual losses of the trading position or portfolio with small probability p, e.g. 1 %, at the end of a short holding period H, e.g. one day. Accordingly, we define risk capital as the ex ante assigned value at risk limit.

The concept of value at risk was born in practice when investment banks asked for an easy way to manage the market risk of their trading books and especially their derivatives positions.[4] Consequently, it is not surprising that value at risk is an insufficient risk measure at least from a theoretical perspective. Value at risk does neither measure coherent risk nor increasing risk[5]

In our understanding value at risk does not measure risk but bank risk capital. Risk capital does not necessarily measure the magnitude of loss. Instead it restricts the probability of actual losses being larger than the value at risk limit to p.[6] Risk capital might be an adequate tool for restricting the insolvency probability to a desired level.[7]

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[4] See Global Derivatives Study Group (1993).

[5] See Artzner / Delbean / Eber /Heath (1999) and Rothschild / Stiglitz (1970). As an example, consider an out of the money short put a trader has written. The put expires in ten days which equals the value at risk holding period H. Assume that this position leads to a loss with a probability of only 4 %, meaning that in all other cases the trader will gain the option price. Of course the value at risk at 95 % confidence level (p=5%) is 0. Artzner / Delbaen / Eber / Heath (1997) use this example to show that value at risk is not a coherent risk measure. It is interesting to note that one can show with the same example that value at risk fails to measure increasing risk.

[6] In this respect value at risk is a constraint. Adding the restriction of not overshooting value at risk limits for example to the standard Markowitz's portfolio optimisation problem results in the well known ap-

It is reasonable to distinguish "allocated" risk capital from "used" risk capital. Bank's management allocates risk capital in a top down approach to business units. As by definition a risk limit does not have to be operated at full capacity, "used" risk capital can be equal or smaller than allocated risk capital. Used risk capital corresponds to the total daily value at risk of a business unit or the whole bank calculated bottom up. Our definition of risk capital is different from that of Merton / Perold (1993)[8], but we find it to be used in a lot of banks, at least in all big German banks.[9]

Banks often interpret value at risk to be a measure of "unexpected" loss that might occur in worst case scenarios. This misinterpretation might be due to the fact that value at risk characterizes a value in the far left of a probability distribution (*p* is commonly fixed by banks within the range of 1 % to 5 %). But value at risk measures potential loss for going concern scenarios and overshooting is expected with probability *p*. Risk managers and the banking supervision should be worried if on average losses overshoot value at risk less frequently. When considering the going-concern-case there are diversification effects within value at risk calculation as correlations are usually smaller than one. In contrast a worst case situation is typically characterized by correlations between all assets converging to one, which means that all asset prices are falling at the same

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proach of asset allocation under shortfall constraint. See Leibowitz / Henriksson (1989) and Leibowitz / Kogelman (1991).

[7] In accordance to the use of internal models to measure market risks the daily risk limit is one third of the available regulatory capital. See Basle Committee on Banking Supervision (1996).

[8] Merton / Perold (1993) define risk capital as the smallest amount that can be invested to insure the value of the firm's net assets against a loss in value relative to a risk-free investment.

[9] See Perold (2001) for evidence for the US market.

time. As a consequence, banks have to install extra worst case simulations beside value at risk calculation.[10]

A couple of papers deal with the capital allocation problem within financial firms. Saita (1999) gives an overview of different possibilities to organize the capital allocation process within the firm. Bühler / Birn (2001) show the effect of forecast-uncertainty when estimating correlations between business units. They demonstrate that unstable negative correlations induce significantly higher capital requirements.[11]

One first approach of modeling interdependent divisions is developed by Froot / Stein (1998). They model a firm that has to raise costly external funds due to uncertain investment payoffs and a potential cash shortfall for which penalties have to be paid. Stoughton / Zechner (1999) extend this approach. Their incentive model focuses on capital budgeting decisions of banks with multiple business units. The main purpose is to derive an optimal capital allocation mechanism in order to achieve overall value maximization from a shareholder's perspective. Perold (2001) models a firm that has to provide guarantees for its performance on customer contracts. Due to these guarantees the firm has to suffer deadweight costs. Denault (2001) models the capital allocation problem as a coalitional game. If portfolios are not separable the Shapley-Value provides a coherent capital allocation. If in contrast portfolios are separable, the Aumann-Shapley-Value gives a coherent capital allocation that assigns the marginal rate of risk capital to each portfolio.

Crucial to all approaches except Denault (2001) is the determination of capital costs and with it the determination of a hurdle rate. However, cost of risk capital is a function of

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[10] Stress tests are also required by the banking supervision. See "the amendment to the capital accord to incorporate market risks", Part B, section B.5.

[11] See Rothschild / Stiglitz (1970).

the covariance of a business unit's profit with firm-wide profits. But this covariance remains unknown ex ante if business units decide independently about the size and direction (long or short) of their exposures. Therefore these models need some simplifications. Froot / Stein (1998) derive the hurdle rate for the limit case when the size of new investments is small. Then the effect of a new investment on risk capital of other business units is small and can be neglected. If investments are not of this small size interdependencies between the investments arise and an optimal decision making can only be reached by a central decision instance. This approach is also adopted by Stoughton / Zechner (1999). However, especially for the trading business in which positions are adjusted almost continuously the centralized decision making does not seem to be a practical approach because costs and delays associated with transmitting new information to headquarters each time an investment is made may be prohibitive. Perold (2001) only considers uncorrelated profits between business units. So risk effects of one investment decision on other business units are ignored in his analysis.

To conclude, the coordination of risk effects on business units' investment decisions is so far only solved theoretically for some very simplifying assumptions, which do not seem to be of practical use at least in trading business.

In the next chapter we develop a value at risk limit system that is of practical use in daily risk management. Thereby our focus is on ex ante risk management and thus on ex ante capital allocation. Actually, a lot of unsolved questions exist about how to design a value at risk limit system, but we do not find solutions for these problems neither in financial industry nor in literature. One major problem is that risk capital is allocated only once or twice a year whereas the time horizon of the trading business is short, sometimes only some seconds. Beeck / Johanning / Rudolph (1999) transform annual limits into daily limits by using the square-root-of-time-rule. They also adjust value at risk limits for realized profits and losses of the business unit. But the most challenging problem is how to account for correlations between the exposures of business units and risk factors. This is necessary as a bank with multiple businesses requires less risk capital than the sum of these businesses operated on a stand-alone basis.[12] Perold (2001) reports for a New York based investment bank with twenty trading businesses that diversified risk capital is only 29.8 % of stand alone risk capital of all units. This large diversification effect is due to very small (even negative) correlations between the major product segments interest rate, equity, foreign exchange and commodity.

As this diversification effect is essential for our analysis, we want to analyze its genesis in greater detail. The problem of unused risk capital emerges when business units decide independently about size and direction (long or short) of their investments. First consider an example with two traders 1 and 2 (trader 1 can be interpreted as the first business unit and trader 2 analogously as the second business unit). Each trader deals only one stock. The stocks *A* and *B* have a given and commonly known correlation ρ_{AB} and covariance σ_{AB}. Both traders always invest independently a maximum exposure V_A and V_B. Then the correlation between the payoffs of traders' exposures $\rho_{T1,T2}$ only depends on the directions (long or short) of the investments; $\rho_{T1,T2}$ equals either ρ_{AB} or $-\rho_{AB}$. We adopt this simple setting for our simulation model in the next chapter. Consider now a small extension of our example. Trader 1 is allowed to deal two stocks *A* and *B* and trader 2 can deal only one stock *C*. The traders decide independently about the size of V_A, V_B and V_C and the directions (long or short) of their exposures. Thus the correlation between the payoffs of traders' exposures is:

$$\rho_{T1,T2} = \frac{V_A \sigma_{AC} + V_B \sigma_{BC}}{(V_A + V_B)\sigma_{T1}\sigma_{T2}}$$ [13].

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[12] See Saita (1999).

[13] See appendix 1.

σ_{T1} (σ_{T2}) is the standard deviation of trader 1's payoff (trader 2's), σ_{AC} (σ_{BC}) is the co-variance between stock *A* and *C* (*B* and *C*). Assume trader 2 first determines V_C and then trader 1 fixes V_A and V_B so that the overall budget restriction $V_A + V_B + V_C = 1$ holds. As a result the correlation between the first and second trader´s payoff $\rho_{T1,T2}$ can vary tremendously (within the range of 0 and 0.65) which is shown in figure 1 for a given vector of standard deviations, a given correlation matrix and $V_C = 0.2; 0.5; 0.8$.[14]

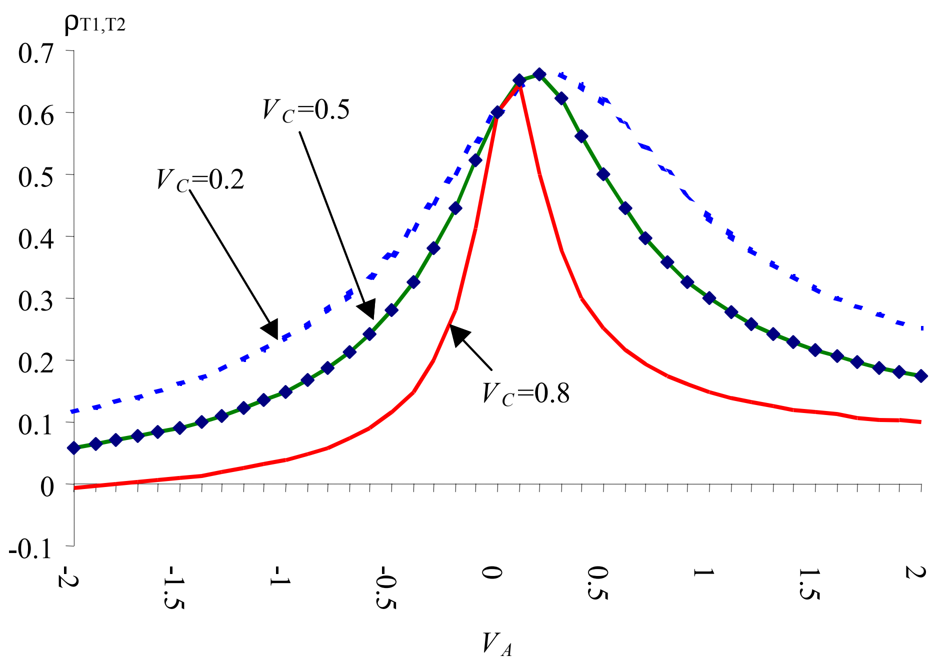

Figure 1: Correlation between the exposures of trader 1 and trader 2

These two examples illustrate the management problem to design a value at risk limit system that accounts for correlation effects if traders decide independently about size and direction (long or short) of their exposures. Next we want to address this problem in greater detail.

[14] For the correlation matrix see appendix 1.

3 VALUE AT RISK LIMITS CONSIDERING CORRELATIONS – A SIMULATION STUDY

3.1 Standard capital allocation in banks – the basic model

The biggest German banks – and this seems to be common practice for almost all investment banks – have organized their risk capital allocation in a top-down process. Risk limits are sometimes assigned without taking correlation effects into account. In this case total limit equals the sum of the limits of sub-portfolios. Thereby a contingency reserve for a worst case is installed. Other banks do account for correlation by allowing the sum of sub-limits to be larger than the total limit. In our basic model we adopt the latter approach.[15]

The bank's management delegates the trading decision to the trading division which consists of thirty traders. Each trader is only allowed to trade one stock.[16] The key proposition can be summarized in the management's objective to maximize the total return of the trading division under the constraint of not breaching the total limit as well as each individual limit. The optimization problem is:[17]

(1) $\max \mu_{PF,t}$ s.t. $VaR_{PF,t} \leq VaR - Limit_{PF,t}$ and $VaR_{i,t} \leq VaR - Limit_{i,t}$ for i = 1 to 30.

$\mu_{PF,t}$ represents the expected profit of the trading portfolio at time t. $VaR_{PF,t}$ is the value at risk and $VaR - Limit_{PF,t}$ the value at risk limit for the total trading portfolio at

[15] See for the setup of the model Dresel / Härtl / Johanning (2002).

[16] We simplify our analysis in permitting each trader to trade only one stock. Stocks are easy to handle, especially in simulation studies. It is more adequate, like it is in real trading situations, to think of risk factors instead of stocks.

[17] We do not need a budget restriction because risk capital not cash is the limiting factor in banks.

time *t*. Accordingly $VaR_{i,t}$ is the value at risk and $VaR - Limit_{i,t}$ the value at risk limit for a single trader *i* at time *t*. The confidence level for the value at risk is 99 % and the holding period is one day.[18] As value at risk fails to measure coherent and increasing risk (see chapter 2), we assume normally distributed stock returns.[19]

The correlation matrix as well as the vectors of returns and standard deviations of the traders' stocks are exogenous and known.[20] Each trader can choose whether to invest long or short whereby it is supposed that the stocks are arbitrarily separable. Central to our study is that like in Stoughton / Zechner (1999) traders' decisions are independent of each other. Further it is assumed that all traders always exploit their entire value at risk limit.[21] All positions are opened in the morning of one day and closed the next morning. There is no intraday-trading. With a chance of 55 % the traders correctly anticipate the direction of the next day's price-movements. Consequently, if the trader predicts a price increase (price decrease), he will decide to invest long (short). On aver-

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[18] The value at risk is calculated by $VaR = \sqrt{L(p)^2 V^2 \sigma^2}$.

[19] To be precise, we suppose the stock prices to follow a geometric Brownian motion, which yields log-normally distributed discrete €-returns. For portfolio aggregation discrete returns have to be normally distributed. But as it is well known, the difference of log-normal and normal distribution for daily returns can be neglected. For an explicit discussion of the properties of discrete and continuous returns see Dorfleitner (2002).

[20] The parameters used here are based on the German stock index DAX, which also consists of thirty stocks. For details see appendix 2.

[21] As a consequence of this assumption and our modelling of traders' forecasting ability (they receive a noisy signal about the direction of their stock' s price movement) traders can be seen to be risk neutral. It follows that they always exploit their risk limit. The structure of the model follows Beeck / Johanning / Rudolph (1999).

age in 55 % of all cases this decision is right, in 45 % it is wrong.[22] The model abstracts from any agency-problems between the traders and top-management in the way that there is no strategic trading, i.e. a trader does not invest strategically against the exposure of other traders but only follows his forecasted price movement. Ex post performance is measured using RORAC (Return On Risk Adjusted Capital).

The total value at risk limit for the trading division shall equal 3 million €. We assign a value at risk sub-limit for each individual trader. These sub-limits are calculated in a way that each trader can invest the same market value V_i given the exogenous standard deviations and correlations. Knowing the value at risk limit a trader can specify his exposure $\overline{V_i}$ by estimating the standard deviation of the stock on basis of the returns of the last 250 trading days. Although we use exogenous correlations and standard deviations for simulating geometric Brownian motions the estimated standard deviation might deviate from the exogenous value. As a consequence the traders' exposures $\overline{V_i}$ in t are not exactly the same. Therefore if fifteen traders have invested long and fifteen have invested short the sum of the exposures does probably not equal 0.[23] As in our special case all correlations between the thirty stocks are positive, the value at risk of the portfolio will obtain its maximum, when all traders invest long at the same time (or short at the same time). Although this constellation rarely happens because of the supposed independency of trading decisions, we have to calculate the sub-limits on basis of this unlikely scenario to make sure that the total value at risk limit is never exceeded.

Table 1 presents the resulting value at risk sub-limits for all thirty traders. The individual value at risk limits range from 108,391 € (trader 19) to 315,735 € (trader 26).

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[22] Within the model the traders' forecasting skills are implemented via equally distributed random numbers ranging from 0 to 1.

[23] The detailed calculation of the traders' limits is documented in appendix 3.

Whereas the nominal sum of the sub-limits equals 5,043,514 €, the aggregated limit accounting for stock correlation is exactly 3 million €.

Trader 1 160,745	Trader 2 151,974	Trader 3 134,048	Trader 4 154,401	Trader 5 162,087	Trader 6 162,302
Trader 7 144,072	Trader 8 167,734	Trader 9 145,099	Trader 10 183,360	Trader 11 182,060	Trader 12 136,092
Trader 13 217,744	Trader 14 127,159	Trader 15 220,535	Trader 16 158,131	Trader 17 138,132	Trader 18 273,877
Trader 19 108,391	Trader 20 131,304	Trader 21 251,019	Trader 22 129,265	Trader 23 147,245	Trader 24 167,952
Trader 25 131,501	Trader 26 315,735	Trader 27 134,115	Trader 28 203,129	Trader 29 148,867	Trader 30 155,437

Table 1: Value at risk limits for the thirty traders in €

We are aware of the fact that we are using some very restrictive simplifications and assumptions within the model, which abstract from real trading situations. We found one bank measuring the trading skills by looking at the forecasting ability of the traders. The assumed forecasting ability of 55 % is comparatively high. But our assumption intends to emphasize the results. Another simplification is that each trader deals only one stock and always exploits his individual value at risk limit entirely. Instead of a single stock real traders deal in a universe of securities. Further, value at risk limits are commonly not allocated to a single trader but to a group of traders. These assumptions do not change our results in general but simplify our analysis tremendously. Altering these assumptions will change the amount of individual trader's profit, but not the main propositions of the model. For the same reason the possibility of intraday-trading is excluded. Further, each trader has only to decide whether to invest long or short. This abstracts from real trading situations in which traders also determine the size of the exposure. This effect could be modeled by incorporating a utility function or a μ,σ-

preference function for each trader accounting for individual risk aversion.[24] But this way of modeling would add other difficulties, especially the mapping of return expectations i.e. of μ and σ. The most crucial assumption might be the independency of trading decisions. Trades of a bank are often motivated by clients, e.g. an investment management firm that wants to unload a block position.[25] Then, it is likely that overall trading decisions are indeed more or less independent. A smaller part of trading decisions (e.g. 5 %) are induced by speculations of traders and it might be more appropriate to assume dependency for these trades. Herding effects have to be discussed within this context.[26] Whenever trading decisions of multiple traders are independent, at least to some degree, the above optimization approach can't be solved analytically. Therefore we run Monte-Carlo-Simulation and simulate 20,000 trading days with the above described data as input parameters.[27] Cholesky-Factorization is applied to incorporate correlations into the vectors of iid standard normal random variables.[28] The traders know their individual value at risk sub-limit. A trader's exposure V_i is derived via the historically estimated (250 days) standard deviations.[29] The results of the simulation are presented in table 2.

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[24] Another way to model this effect would be to change the kind of the noisy signal appropriately.

[25] See Perold (2001), p. 4.

[26] For a study considering herding effects within a trading department see Burghof / Dresel (2002).

[27] The simulation was realised with E-Views 4.0.

[28] The factorisation of the correlation matrix presumes this matrix to be positive semi-definite. This applies to the correlation matrix we used for the simulation. For a detailed description of the Cholesky factorisation see J.P. Morgan / Reuters (1996) and Hull (2000), p. 409.

[29] See appendix 3.

	Mean	Std.Dev.	Median	25%-Quantile	75%-Quantile	Minimum	Maximum
VaR of the trading division in €	941,404	220,564	881,497	785,946	1,038,383	549,279	2,427,325
Utilization of available limit (in %)	31.38	--	29.38	26.20	34.61	18.31	80.91
Total profit of trading division in €	180,317	416,360	173,465	-81,969	434,242	-2,134,473	3,013,116

Table 2: Results for 20.000 simulated trading days – basic model

The value at risk is shown in line 2. On average the total “used” value at risk amounts to 941,404 €, which is only 31.38 % of “allocated” risk capital of 3 million €. Even the maximum value at risk with 2,427,325 € is far below the “allocated” risk capital of 3 million € (80.91 %).[30] These figures show the tremendous diversification effect. It is interesting to note that this result corresponds closely to real trading situations.[31] Line 4 in table 2 gives information about the profit of the trading division, which is on average 180,317 €.

3.2 Dynamic capital allocation – the benchmark model

As on average only 31.38 % of the risk capital of 3 million € is used, the trading profit should obviously be far below its optimum. Looking for the best limit allocation process we develop a benchmark model in a way that the value at risk limit of 3 million € is totally used each trading day. This can be achieved if the correlation structure between the traders’ exposures is known. Therefore each trader has to report his trading decision (whether he invests short or long) to a central authority (the risk controlling division) –

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[30] Within the 20,000 trading days we did not observe the case that all traders invested long or short at the same time.

[31] See chapter 2 and Perold (2001).

similar to the Stoughton / Zechner (1999) assumption. Knowing all thirty trading directions the authority calculates the exposure each trader can invest. The exposures are calculated in a way that each trader can invest the same market value V_i.[32] The simulation results are shown in table 3.[33]

	Mean	Std.Dev.	Median	25%-Quantile	75%-Quantile	Minimum	Maximum
VaR of all traders divided by 30 in €	559,923	110,250	571,312	484,241	641,512	207,152	920,133
Total profit of all traders in €	545,443	1,277,300	578,576	-282,873	1,403,039	-5,525,159	5,589,909

Table 3: Results for 20.000 simulated trading days – benchmark model

By assumption the total value at risk of the trading division is always 3 million €. The average value at risk of a single trader (the sum of individual values at risk - without taking correlation effects into account - divided by 30) is 559,912 € with a standard deviation of 110,250 (line 2). The average profit of the trading division more than triples compared to the basic scenario from 180,714 € to 545,443 € but the range of profits increases as well (see the standard deviation of 1,277,300 € or maximum and minimum values).

Of course, this model does not seem to be a practical application for real trading situations. But it illustrates a benchmark: the traders' forecasting abilities are always exploited entirely and the total value at risk limit is never exceeded. Obviously, this should result in the highest possible profit of the trading division assuming the given forecasting ability. This result documents as well the cost of not accounting for diversi-

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[32] In contrast to the basic model the traders' sub-limits are determined every day using the estimated standard deviations and correlations. Therefore absolute value of the exposures V_i are the same for all traders in t.

[33] We used the same random numbers as in the basic model. So all differences in the results are due to model specification.

fication effects. These costs are equal to the large shortfall in average daily profits of 365,126 € (545,443 € minus 180,317 €).

3.3 A practical risk capital allocation approach – the treasurer model

The next step in our analysis is to find an easy way to apply allocation approach, which always assures full usage of risk capital. The same type of limit system as in the basic model is installed for controlling the thirty traders (see 3.1). In addition a thirty first player is allowed to invest the unused (or residual) part of risk capital. This thirty first player shall be called "treasurer" because his job is in some way similar to the one of a real treasurer, which is liquidity planning and overall financing of a bank.[34] In fact, it might even be better not to have in mind a single person being the treasurer but rather a group of traders i.e. chief traders and risk managers, who often join to the so called risk committee.

We model two types of treasures which respond to treasurer model I and treasurer model II. In both cases the treasurer has no market forecasting skills and can only buy or sell a stock index that is equally weighted by the thirty stocks. The treasurer chooses his position after all thirty traders have invested. He calculates the value at risk of the total position of the thirty traders using the correlations between the traders´ positions and then invests exactly the amount needed to adjust the value at risk of the whole trading department to the desired level of 3 million €. As the treasurer has no forecasting ability he invests long if the net trading position of all traders is positive (i.e. if more traders invest long than short) and vice versa.[35]

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[34] See Newman / Milgate / Eatwell (1992).

[35] In the hypothetical case of a net exposure of exactly 0 € the treasurer makes a long investment.

In the first model (treasurer model I) the sub-limits for the thirty traders correspond exactly to the limits in chapter 3.1. Figure 2 illustrates the investment behavior of the treasurer. The treasurer's exposure is largest, if the net position of the traders is close to zero as then the traders hedge themselves widely.

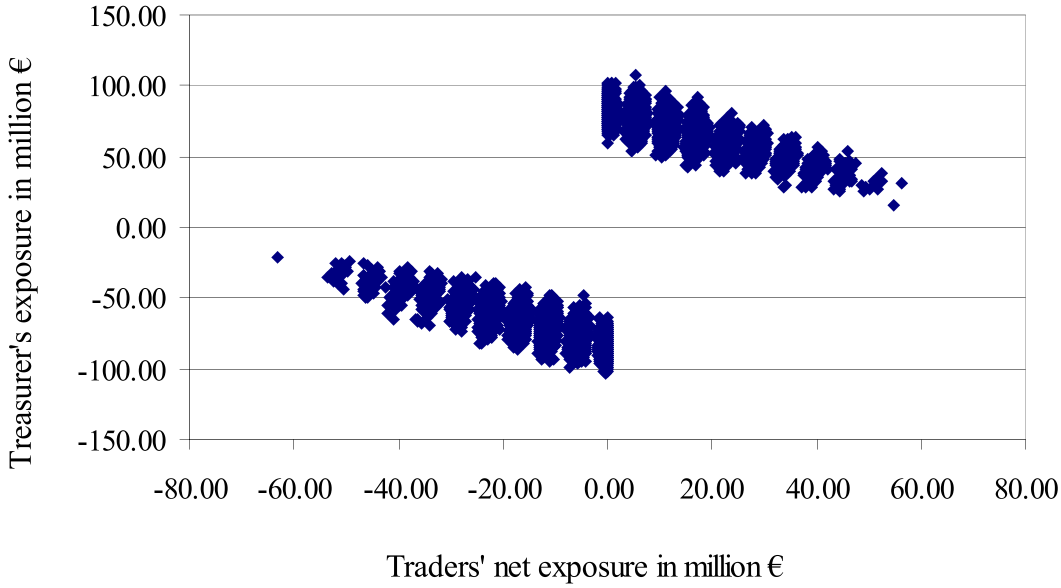

Figure 2: Treasurer's exposure subject to traders´ net exposure

Table 4 shows the results of the simulation for the treasurer model I.

	Mean	Std.Dev.	Median	25%-Quantile	75%-Quantile	Minimum	Maximum
Treasurer's VaR in €	2,456,283	405,625	2,493,990	2,200,017	2,748,969	584,102	3,597,229
Treasurer's profit in €	182,968	1,050,728	216,654	-490,294	869,403	-5,061,119	4,517,660
Total profit of the trading division in €	363,284	1,270,754	397,477	-460,264	1,229,516	-5,645,284	5,369,455

Table 4: Results for 20,000 simulated trading days – treasurer model I

As the limit system for the thirty traders has not changed their results are exactly identical to the ones shown in chapter 3.1. Other than before the value at risk of the whole bank (i.e. the thirty traders and the treasurer) is exactly 3 million € each day. The average utilization of risk capital (used value at risk limit) increases from 31.38 % to 100.00%.

As can be seen in figure 2 the treasurer's exposure is on average approx. 2.5 times larger than the traders' exposure. This can be explained with great diversification effects between the traders' positions and thus big parts of idle risk capital.

As the treasurer invests in the same direction (long or short) as the majority of the traders he makes use of the traders' forecasting ability. As a result the treasurer's index investment is on average profitable. On average he earns even a bit more than the traders altogether (182,968 € vs. 180,714 €) but the standard deviation (1,050,728 €) of his profit is more than 2.5 times larger than that one of the traders' profit. As a result, total profit of the whole trading division (treasurer's profit plus traders' profit) rises but the standard deviation increases to 1,270,754 € which is almost the same as in the benchmark model (1,277,300 €).

As traders' value at risk is larger in comparison to treasurer's value at risk, one could think of increasing the overall traders' exposure (e.g. to triple each individual trader's value at risk limit) as then the traders' forecasting skills could be better exploited and overall profit should rise even more.

But an enhancement of each trader's limit might end up in an overall traders' value at risk larger than 3 million €. For example suppose the individual sub-limits are doubled. Ceteris paribus this leads to a maximum value at risk of all traders of 4,865,354 (compare to table 2 line 2). In cases when the traders' value at risk is larger than 3 million € the treasurer has to scale down the value at risk to the desired level of 3 million €. However reducing the total value at risk to 3 million € is only possible if the value at risk of all traders does not exceed a certain threshold. This is because of the following argumentation: Total value at risk of the bank is calculated by

$$(2)\ VaR_{total} = \sqrt{VaR_{traders}^2 + VaR_{treasurer}^2 + 2 \cdot VaR_{traders} \cdot VaR_{treasurer} \cdot \rho_{traders,treasurer}}\ ,$$

with $VaR_{traders}$ being the value at risk of the thirty traders altogether, $VaR_{treasurer}$ being the value-at-risk of the treasurer and $\rho_{traders,treasurer}$ being the correlation between the payoffs of the treasurer's and traders' exposures. Solving this equation for $VaR_{treasurer}$ yields:

$$(3)\ VaR_{treasurer} = -VaR_{traders} \cdot \rho_{traders,treasurer} \pm \sqrt{VaR_{traders}^2 (\rho_{traders,treasurer}^2 - 1) + VaR_{total}^2}\ .$$

If $VaR_{traders}$ is larger than 3 million €, the term under the square root would not be necessarily positive. In fact this depends on the correlation $\rho_{traders,treasurer}$. Solving the term under the square root for being positive results in:

$$(4)\ VaR_{traders} \leq \sqrt{-\frac{VaR_{total}^{\ 2}}{(\rho_{traders,treasurer}^2 - 1)}}\ .$$

Figure 3 illustrates this relationship. For a correlation of 0.5 (0.8) in absolute terms e.g. the maximum possible traders' value at risk is 3.646.102 € (5.000.000 €). Only if the treasurer deals a security that correlates with the traders' portfolio by 0.5 (0.8) or more in absolute terms total value at risk can be reduced to 3 million €. Otherwise a reduction cannot be achieved. Therefore an uncontrolled enhancement of individual traders' value at risk limits bears risks because the treasurer might not be able to bring down the total value at risk to the desired level of 3 million €.

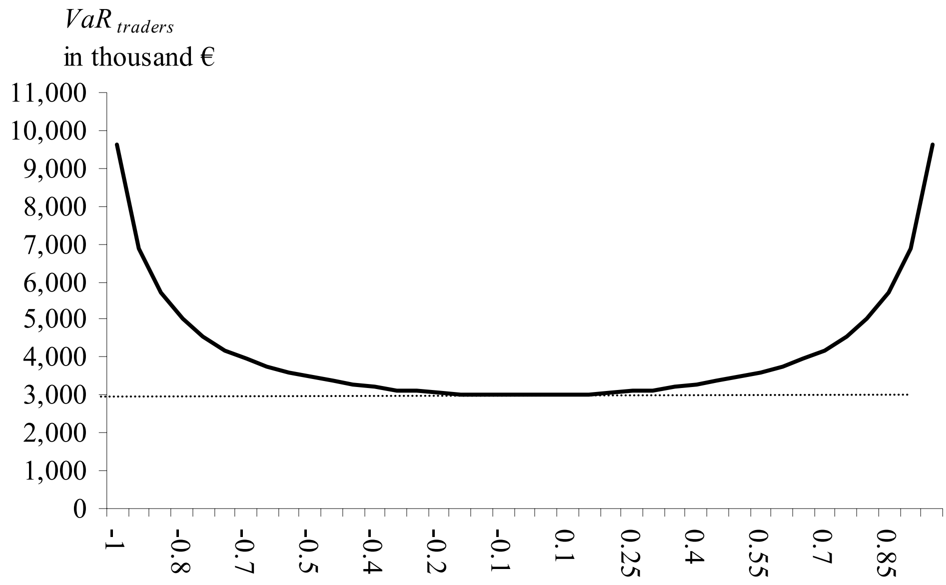

Figure 3: Max. possible traders' value at risk subject to correlation $\rho_{traders,treasurer}$

In our model the absolute correlation between the payoffs of traders' portfolio and the equally weighted stock index, the treasurer is allowed to trade, lies within the range of 0.0000 and 0.9759.[36] This correlation is small if the index is uncorrelated with the traders' portfolio, i.e. the traders hedge themselves widely. If the correlation is small the treasurer can only slightly reduce the overall value at risk. But if the traders widely hedge themselves their value at risk will be relatively small and probably there is no need to reduce overall value at risk. Vice versa if all traders invest more or less into the same direction their value at risk is larger (probably larger than 3 million €) and the correlation between the stock index and traders' exposure will converge to 1. So the treasurer can reduce overall value at risk limit to 3 million €. But the enhancement has an

[36] The treasurer can invert the correlation by taking the opposite position, i.e. if the treasurer is long and his position has a correlation of +0.5 to the position of the traders, he can change the correlation to -0.5 by short-investing.

upper bound which is shown in figure 3. Traders' positions larger than the upper bound can't be reduced to the predetermined level of 3 million €.

The increment of single trader's value at risk limits depend very much on the correlation between the traders' exposure and the financial products the treasurer is allowed to trade. If the treasurer is allowed to trade a universe of products the probability of high correlation (in absolute terms) between traders' portfolio and treasurer's position rises. In these cases the increment of single trader's limits can be large. If the trading universe of the treasurer is limited the probability of high correlations (in absolute terms) is small and with it the possible increment of traders' limits.

In our treasurer model II each trader's limit is increased by the factor 2.5. An increment larger than 2.5 would results in days on which the treasurer could not scale down total value at risk to 3 million and thus risk capital of 3 million € would be exceeded. As in model I the treasurer always fills up or scales down the value at risk to the desired level of 3 million €. If possible the treasurer chooses the direction of his position according to the net investment of the traders (see figure 4). All other simulation parameters are retained unchanged. The results of model II are shown in table 5.

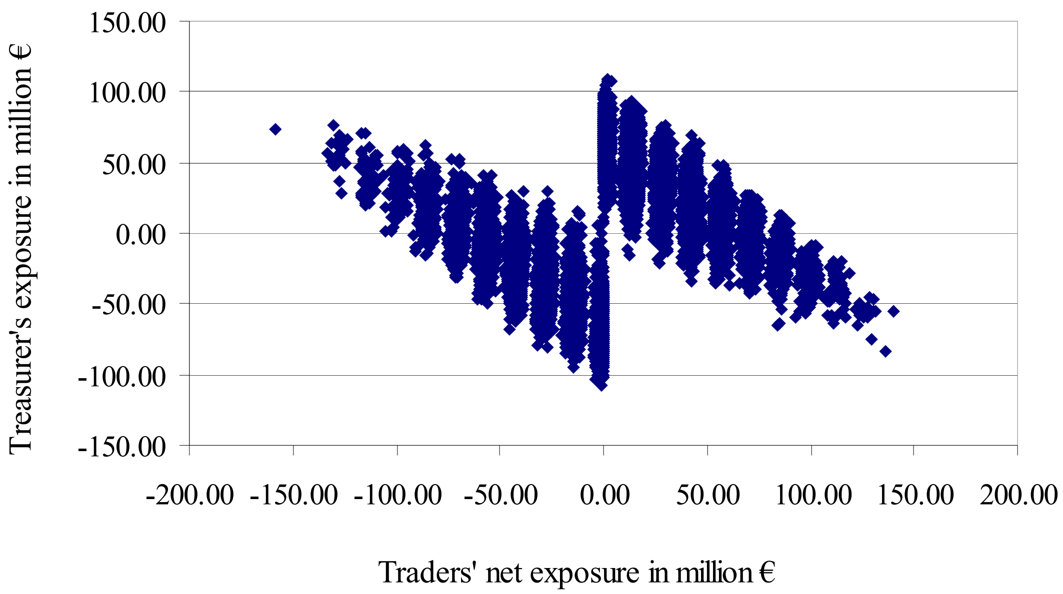

Figure 4: Treasurer's exposure subject to traders' exposure

	Mean	Std.Dev.	Median	25%-Quantile	75%-Quantile	Minimum	Maximum
VaR of the 30 traders in €	2,535,509	551,409	2,203,742	1,964,865	2,595,957	1,373,197	6,068,313
Treasurer's VaR in €	1,356,109	790,036	1,329,742	709,954	1,949,054	151	3,871,594
Profit of the 30 traders in €	450,792	1,040,900	433,663	-204,921	1,085,604	-5,336,181	7,532,790
Treasurer's profit in €	34,308	667,385	21,153	-263,457	346,695	-3,851,301	4,071,178
Total profit of the trading division in €	485,100	1,278,079	531,727	-340,088	1,344,135	-4,909,145	5,322,876

Table 5: Results for 20,000 simulated trading days – treasurer model II

The value at risk of the whole trading division (traders and treasurer) equals exactly 3 million € each day. As the thirty traders are controlled by the same limit system as in chapter 3.1 with the exception of the limit upscale of 2.5, their value at risk and profit is 2.5 times the corresponding values in table 2. Interesting to note is that the traders' value at risk is on average almost twice the amount of treasurer's value at risk. The minimum treasurer's value at risk is only 151 € (see line 3). Therefore, the treasurer's impact on profits is small.

The overall profit of the trading division increases to 485,100 € (compared to 363,284 € for the treasurer model I). But standard deviation of total profits is almost the same as for the treasurer model I (1,278,079 € versus 1,270,754 €). So profits can be increased without enhancing risk.

3.4 <u>Evaluation of results</u>

In order to evaluate our results we first take a look at the cumulative frequency distributions of the profits of the trading division in the basic model, in the benchmark model and in the two treasurer models. The distributions are shown in figure 4.

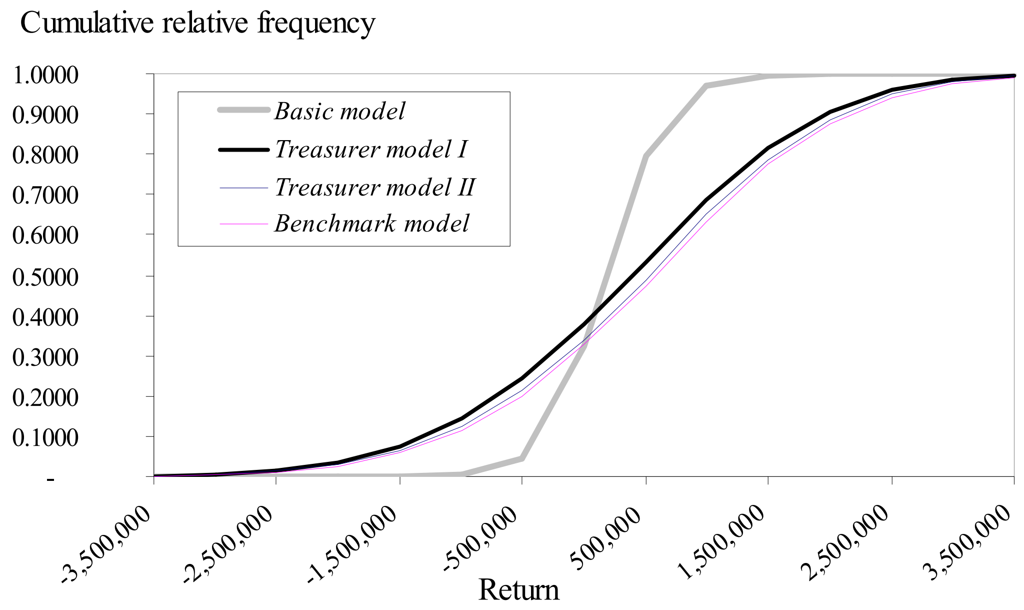

Figure 5: Risk profiles for the four models

As the graphs for the basic and the other three models intersect around zero-profit-level no approach strictly dominates the basic model, at least from the perspective of a risk averse investor. A risk averse bank can either choose the less risky alternative with - of course - smaller expected return (basic model) or a more risky alternative (one of the treasurer models) with higher expected return. Leaving aside the border areas the benchmark model dominates the treasurer model II which itself dominates the treasurer model I.[37] As the standard deviations of all three models are merely the same the differences are caused by different average profits. It is interesting to note that the treasurer model II is only slightly dominated by the benchmark model.

If we assume a risk neutral bank, both treasurer models dominate the basic model because of higher expected profits. As well in the case of a risk neutral bank the benchmark model also dominates all other models.

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[37] However this dominance relation does not hold for the border areas which becomes evident by the comparison of the maximum and minimum values in tables 3, 4 and 5.

It is common practice in banks to use RORAC for ex post performance measurement.[38] We calculate RORAC (Return On Risk Adjusted Capital) for our three alternatives by putting profits first in relation to value at risk ("used" risk capital) and secondly to value at risk limits ("allocated" risk capital):

$$RORAC = \frac{\sum_{t=1}^{N} \mu_t / VaR_t}{N} \quad \text{and} \quad RORACL = \frac{\sum_{t=1}^{N} \mu_t / VaR - Limit_t}{N}$$

with N representing the number of trading days (20,000) and μ_t the average profit of the trading division at day t.

Both approaches are applied in banks. While the first approach measures the traders' performance the latter measures the interest paid for the total risk capital and might be the appropriate measure for management and shareholders. The results are presented in table 6.

	RORAC	RORACL
Basic model	18.24 %	6.02 %
Treasurer model I	8.61 %[1]	12.11 %
Treasurer model II	3.21 %[2]	16.17 %
Benchmark model	18.18 %	18.18 %

[1] 8.61 % is only the treasurer's RORAC, the traders' RORAC is 18.24 %
[2] 3.21 % is only the treasurer's RORAC, the traders' RORAC is 18.24 %

Table 6: RORAC for the basic model, treasurer model and the benchmark model

Due to the low usage of on average 31.38 % of allocated risk capital in the basic model the RORACL is only 6.02 %. But the traders' RORAC is 18.24 % and equals with a small difference the RORAC of the benchmark model (line 5).[39] The treasurer's stand alone performance in the treasurer model I is 8.61 % but the RORACL of the whole

[38] See James (1996), Zaik / Walter / Kelling / James (1996), Matten (2000) for examples.

[39] The deviation between 18.24 % and 18.18 % is caused by small differences in the setup of the basic model and the benchmark model. See chapter 3.1 and footnote 34.

trading division increases to 12.11 % because of full usage of the allocated risk capital each day. Although the treasurer's RAROC is only 3.21%, the RORACL for the treasurer model II is 16.17 % and very close to the benchmark model's RORAC of 18.18 %.

4 SUMMARY AND OUTLOOK

The main purpose of our study was to find a top down risk capital allocation approach for several traders, which might be applied in real trading situations and assures full usage of total risk capital. Therefore, we developed a simulation model and compared the results for four alternatives: the basic model, the benchmark model, the treasurer model I and the treasurer model II.

The main results of our simulation study can be summarized as follows:

- In the basic model which closely corresponds to risk capital allocation systems in banks today, on average only 31.38 % of total risk capital is used. This is due to the independent trading decisions of the traders. Because almost 70 % of risk capital stays idle, the RORACL is only 6.02 %.
- In the benchmark model the value at risk limits system is designed in a way that total value at risk always equals the desired level of 3 million €. RORACL is highest with 18.18 %. However, to setup in a central authority which determines the size of each trader's position is not of practical use for real trading situations.
- A treasurer, who can invest after the traders have chosen their exposures, can always assure full usage of allocated risk capital. Because the treasurer has no forecasting ability, his RORAC is only 8.61 % (treasurer model I) or 3.21 % (treasurer model II) but the RORACL for the whole trading division increases to 12.11 % or 16.17 % respectively.

Especially the treasurer model II seems to be promising. The forecasting skills of the traders are optimally exploited by scaling up their risk limits with the factor 2.5. Because aggregated value at risk of the traders might be larger than the predetermined risk capital the treasurer has to scale down total value at risk to the predetermined level in some cases. The direction (long or short) of the treasurer's investment should be in line with the traders' net investment direction because only then the treasurer makes use of the traders' forecasting skills. It should be possible to implement these tasks in a real risk management system. The risk management system should calculate daily or with higher frequency on basis of historical correlations the size and direction of a treasurer investment. The bank's treasury might check the proposed investments. It has to be kept in mind that a value at risk limit system like in the treasurer model II assures no overshooting in going concern situations. For worst case situations other preventive capital measures have to be installed.

There are a lot of unsolved questions concerning risk capital allocation. First, further important empirical work has to be done. We assumed stationary correlations between stocks (risk factors). It has to be analyzed whether correlations are stationary and if they are not, how to handle these instationarities within the risk capital allocation process.[40]

For future work concerning our model we first want to alter some critical assumptions, like the independence of trading decisions, the traders' forecasting ability, the normal distribution of stock returns and the fact that traders always totally use their limit. Like discussed in 3.1 utility functions and - with them - incentive compensation systems can be introduced.

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[40] For an examination of the effects of the uncertainty of correlations on required risk capital see Bühler / Birn (2001).

However the simulation technique seems to be a very promising tool to evaluate and develop risk capital allocation systems. Our results are the first steps in utilizing correlations between business units, which we consider a very important aspect not only in risk management but also in asset management and even in general investment decisions of non-financial firms.

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Appendix 1:

Assume correlations ρ_{AB}, ρ_{AC}, ρ_{BC} and covariances σ_{AB}, σ_{AC} and σ_{BC} between the stocks are stationary and known. The portfolio variance is given by:

$$\sigma_p^2 = x_A^2\sigma_A^2 + X_B^2\sigma_B^2 + X_C^2\sigma_C^2 + 2X_A X_B \sigma_{AB} + 2X_A X_C \sigma_{AC} + 2X_B X_C \sigma_{BC}.$$

It follows the subportfolio variance of trader 1 σ^2_{T1} and trader 2 σ^2_{T2} to be:

$$\sigma_{T1}^2 = \frac{x_A^2\sigma_A^2 + X_B^2\sigma_B^2 + 2X_A X_B \sigma_{AB}}{(X_A + X_B)^2} \text{ and } \sigma_{T2}^2 = \sigma_C^2.$$

The covariance between the subportfolios of trader 1 and trader 2 is

$$\sigma_{T1,T2} = \frac{X_A\sigma_{AC} + X_B\sigma_{BC}}{(X_A + X_B)}$$

and the correlation between the subportfolios is

$$\rho_{T1,T2} = \frac{X_A\sigma_{AC} + X_B\sigma_{BC}}{(X_A + X_B)\sigma_{T1}\sigma_{T2}}.$$

For the above example the following correlations and standard deviations are assumed:

Correlation matrix

	A	**B**	**C**
A	1	0.4	0.5
B	0.4	1	0.6
C	0.5	0.6	1

Vector of standard deviations

	A	**B**	**C**
A	0.2	0	0
B	0	0.15	0
C	0	0	0.25

Appendix 2: Correlation matrix used in the simulation model

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1.0000	0.1921	0.4327	0.3231	0.3798	0.4081	0.3243	0.3458	0.2950	0.3472	0.4360	0.1757	0.1300	0.2125	0.3081
2	0.1921	1.0000	0.4627	0.3683	0.5334	0.3895	0.5297	0.4802	0.3486	0.5878	0.4882	0.2591	0.4006	0.4716	0.3123
3	0.4327	0.4627	1.0000	0.6936	0.4898	0.5060	0.5168	0.5201	0.5075	0.5568	0.5641	0.1525	0.3416	0.3352	0.3960
4	0.3231	0.3683	0.6936	1.0000	0.3566	0.4159	0.3744	0.3621	0.4386	0.4057	0.3641	0.1156	0.3602	0.2934	0.3475
5	0.3798	0.5334	0.4898	0.3566	1.0000	0.4884	0.5899	0.5248	0.2326	0.6248	0.3756	0.2863	0.3977	0.3031	0.3325
6	0.4081	0.3895	0.5060	0.4159	0.4884	1.0000	0.4141	0.5618	0.4855	0.4870	0.4787	0.2262	0.3147	0.2723	0.3073
7	0.3243	0.5297	0.5168	0.3744	0.5899	0.4141	1.0000	0.4867	0.4045	0.6860	0.5130	0.1559	0.2974	0.3026	0.4056
8	0.3458	0.4802	0.5201	0.3621	0.5248	0.5618	0.4867	1.0000	0.3618	0.5904	0.5294	0.1537	0.4544	0.2942	0.4300
9	0.2950	0.3486	0.5075	0.4386	0.2326	0.4855	0.4045	0.3618	1.0000	0.4174	0.3837	0.0084	0.2163	0.2656	0.3382
10	0.3472	0.5878	0.5568	0.4057	0.6248	0.4870	0.6860	0.5904	0.4174	1.0000	0.5064	0.2336	0.5539	0.3330	0.5396
11	0.4360	0.4882	0.5641	0.3641	0.3756	0.4787	0.5130	0.5294	0.3837	0.5064	1.0000	0.1556	0.2677	0.2741	0.4091
12	0.1757	0.2591	0.1525	0.1156	0.2863	0.2262	0.1559	0.1537	0.0084	0.2336	0.1556	1.0000	0.0948	0.2970	0.1376
13	0.1300	0.4006	0.3416	0.3602	0.3977	0.3147	0.2974	0.4544	0.2163	0.5539	0.2677	0.0948	1.0000	0.2669	0.5611
14	0.2125	0.4716	0.3352	0.2934	0.3031	0.2723	0.3026	0.2942	0.2656	0.3330	0.2741	0.2970	0.2669	1.0000	0.1396
15	0.3081	0.3123	0.3960	0.3475	0.3325	0.3073	0.4056	0.4300	0.3382	0.5396	0.4091	0.1376	0.5611	0.1396	1.0000
16	0.1121	0.2021	0.1815	0.1890	0.0567	0.2141	0.1552	0.1711	0.2199	0.1174	0.1680	0.0400	0.0250	0.2138	0.2013
17	0.1570	0.1251	0.1498	0.1008	0.2042	0.1339	0.2266	0.1267	0.2052	0.2130	0.1285	0.1163	0.1244	0.1814	0.1542
18	0.2906	0.3143	0.2747	0.2666	0.4184	0.3363	0.3609	0.4424	0.1800	0.5249	0.3338	0.1172	0.5057	0.1104	0.5559
19	0.4196	0.3852	0.4605	0.3978	0.3402	0.4196	0.4794	0.3719	0.4438	0.4355	0.4484	0.1294	0.2130	0.3408	0.2872
20	0.1987	0.2939	0.2886	0.2777	0.2432	0.2486	0.2991	0.3352	0.2716	0.3275	0.2310	0.0433	0.2402	0.2364	0.2736
21	0.2000	0.2703	0.2606	0.1673	0.3058	0.2422	0.3263	0.3072	0.1547	0.4667	0.2830	0.1452	0.3501	0.1498	0.3965
22	0.3465	0.4221	0.4399	0.3656	0.2630	0.3553	0.3634	0.3948	0.4836	0.3959	0.4080	0.1029	0.2164	0.4221	0.2671
23	0.2199	0.7783	0.4410	0.3357	0.5097	0.3939	0.5316	0.4666	0.3103	0.5896	0.5106	0.2716	0.3615	0.4695	0.2649
24	0.4236	0.4215	0.4333	0.2977	0.4096	0.4490	0.5347	0.4062	0.3960	0.4749	0.4694	0.2320	0.1925	0.2368	0.3041
25	0.1386	0.4619	0.3327	0.2725	0.2300	0.1901	0.2962	0.3018	0.2812	0.3276	0.2451	0.2251	0.1983	0.6316	0.1740
26	0.2130	0.3723	0.3269	0.2627	0.3132	0.2546	0.3191	0.4222	0.2206	0.5450	0.2613	0.1441	0.4529	0.1755	0.4491
27	0.0698	0.3463	0.2804	0.2523	0.2631	0.2266	0.2835	0.2194	0.2594	0.3247	0.1573	0.1454	0.2310	0.3380	0.1502
28	0.3234	0.4871	0.4751	0.4109	0.4953	0.4440	0.4611	0.6081	0.3808	0.6808	0.4932	0.1739	0.6365	0.3451	0.7100
29	0.4641	0.3011	0.4624	0.3540	0.3451	0.4549	0.3939	0.4181	0.3484	0.4663	0.3424	0.2979	0.2574	0.2876	0.3167
30	0.4222	0.3985	0.5585	0.3880	0.5688	0.6666	0.4889	0.6106	0.4132	0.5616	0.5111	0.2092	0.3353	0.1798	0.4241

	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	0.1121	0.1570	0.2906	0.4196	0.1987	0.2000	0.3465	0.2199	0.4236	0.1386	0.2130	0.0698	0.3234	0.4641	0.4222
2	0.2021	0.1251	0.3143	0.3852	0.2939	0.2703	0.4221	0.7783	0.4215	0.4619	0.3723	0.3463	0.4871	0.3011	0.3985
3	0.1815	0.1498	0.2747	0.4605	0.2886	0.2606	0.4399	0.4410	0.4333	0.3327	0.3269	0.2804	0.4751	0.4624	0.5585
4	0.1890	0.1008	0.2666	0.3978	0.2777	0.1673	0.3656	0.3357	0.2977	0.2725	0.2627	0.2523	0.4109	0.3540	0.3880
5	0.0567	0.2042	0.4184	0.3402	0.2432	0.3058	0.2630	0.5097	0.4096	0.2300	0.3132	0.2631	0.4953	0.3451	0.5688
6	0.2141	0.1339	0.3363	0.4196	0.2486	0.2422	0.3553	0.3939	0.4490	0.1901	0.2546	0.2266	0.4440	0.4549	0.6666
7	0.1552	0.2266	0.3609	0.4794	0.2991	0.3263	0.3634	0.5316	0.5347	0.2962	0.3191	0.2835	0.4611	0.3939	0.4889
8	0.1711	0.1267	0.4424	0.3719	0.3352	0.3072	0.3948	0.4666	0.4062	0.3018	0.4222	0.2194	0.6081	0.4181	0.6106
9	0.2199	0.2052	0.1800	0.4438	0.2716	0.1547	0.4836	0.3103	0.3960	0.2812	0.2206	0.2594	0.3808	0.3484	0.4132
10	0.1174	0.2130	0.5249	0.4355	0.3275	0.4667	0.3959	0.5896	0.4749	0.3276	0.5450	0.3247	0.6808	0.4663	0.5616
11	0.1680	0.1285	0.3338	0.4484	0.2310	0.2830	0.4080	0.5106	0.4694	0.2451	0.2613	0.1573	0.4932	0.3424	0.5111
12	0.0400	0.1163	0.1172	0.1294	0.0433	0.1452	0.1029	0.2716	0.2320	0.2251	0.1441	0.1454	0.1739	0.2979	0.2092
13	0.0250	0.1244	0.5057	0.2130	0.2402	0.3501	0.2164	0.3615	0.1925	0.1983	0.4529	0.2310	0.6365	0.2574	0.3353
14	0.2138	0.1814	0.1104	0.3408	0.2364	0.1498	0.4221	0.4695	0.2368	0.6316	0.1755	0.3380	0.3451	0.2876	0.1798
15	0.2013	0.1542	0.5559	0.2872	0.2736	0.3965	0.2671	0.2649	0.3041	0.1740	0.4491	0.1502	0.7100	0.3167	0.4241
16	1.0000	0.0716	0.0326	0.1328	0.1768	0.1217	0.2845	0.1915	0.1942	0.1572	0.0648	0.1764	0.1529	0.1349	0.0994
17	0.0716	1.0000	0.1124	0.1889	0.0520	0.0513	0.2113	0.1607	0.1767	0.1323	0.0863	0.2592	0.1712	0.1331	0.1932
18	0.0326	0.1124	1.0000	0.2047	0.2824	0.2650	0.1658	0.2658	0.3426	0.1390	0.5068	0.2485	0.6552	0.2832	0.3938
19	0.1328	0.1889	0.2047	1.0000	0.3712	0.2191	0.4988	0.4045	0.5288	0.3108	0.1379	0.2218	0.3883	0.4873	0.4997
20	0.1768	0.0520	0.2824	0.3712	1.0000	0.0895	0.3294	0.2230	0.3506	0.3161	0.2798	0.1606	0.3929	0.3578	0.2268
21	0.1217	0.0513	0.2650	0.2191	0.0895	1.0000	0.2222	0.2872	0.2930	0.1410	0.3335	0.2145	0.4218	0.2757	0.2937
22	0.2845	0.2113	0.1658	0.4988	0.3294	0.2222	1.0000	0.4362	0.3826	0.4417	0.2505	0.3340	0.3473	0.3629	0.3176
23	0.1915	0.1607	0.2658	0.4045	0.2230	0.2872	0.4362	1.0000	0.3922	0.4140	0.3281	0.3603	0.4241	0.2726	0.4163
24	0.1942	0.1767	0.3426	0.5288	0.3506	0.2930	0.3826	0.3922	1.0000	0.2147	0.2240	0.2475	0.4238	0.5175	0.4675
25	0.1572	0.1323	0.1390	0.3108	0.3161	0.1410	0.4417	0.4140	0.2147	1.0000	0.2406	0.3985	0.3549	0.1993	0.2094
26	0.0648	0.0863	0.5068	0.1379	0.2798	0.3335	0.2505	0.3281	0.2240	0.2406	1.0000	0.2503	0.5734	0.2860	0.3703
27	0.1764	0.2592	0.2485	0.2218	0.1606	0.2145	0.3340	0.3603	0.2475	0.3985	0.2503	1.0000	0.2812	0.1631	0.1974
28	0.1529	0.1712	0.6552	0.3883	0.3929	0.4218	0.3473	0.4241	0.4238	0.3549	0.5734	0.2812	1.0000	0.4593	0.5509
29	0.1349	0.1331	0.2832	0.4873	0.3578	0.2757	0.3629	0.2726	0.5175	0.1993	0.2860	0.1631	0.4593	1.0000	0.4418
30	0.0994	0.1932	0.3938	0.4997	0.2268	0.2937	0.3176	0.4163	0.4675	0.2094	0.3703	0.1974	0.5509	0.4418	1.0000

Annualized expected returns of the thirty stocks (in percent)

Stock 1	Stock 2	Stock 3	Stock 4	Stock 5	Stock 6	Stock 7	Stock 8	Stock 9	Stock 10
13.50	-31.34	-4.74	-3.07	-46.31	-2.15	-52.97	-7.96	-13.67	-32.09
Stock 11	Stock 12	Stock 13	Stock 14	Stock 15	Stock 16	Stock 17	Stock 18	Stock 19	Stock 20
-57.37	-23.63	-59.41	-2.68	-40.94	-22.71	3.38	-68.15	-9.96	-42.29
Stock 21	Stock 22	Stock 23	Stock 24	Stock 25	Stock 26	Stock 27	Stock 28	Stock 29	Stock 30
-49.55	-20.42	-12.31	-20.45	-0.43	0.48	-6.60	-31.41	-23.39	-22.22

Annualized standard deviations of the thirty stock returns (in percent)

Stock 1	Stock 2	Stock 3	Stock 4	Stock 5	Stock 6	Stock 7	Stock 8	Stock 9	Stock 10
38.81	36.70	32.37	37.28	39.14	39.19	34.79	40.50	35.04	44.28
Stock 11	Stock 12	Stock 13	Stock 14	Stock 15	Stock 16	Stock 17	Stock 18	Stock 19	Stock 20
43.96	32.86	52.58	30.71	53.25	38.18	33.36	66.13	26.17	31.71
Stock 21	Stock 22	Stock 23	Stock 24	Stock 25	Stock 26	Stock 27	Stock 28	Stock 29	Stock 30
60.61	31.21	35.56	40.56	31.75	76.24	32.39	49.05	35.95	37.53

Appendix 3: Calculation of traders' individual value at risk limits

The value at risk of a single position is calculated by:

$$(1)\ VaR_i = \sqrt{L(p)^2 \cdot V_i^2 \cdot \sigma_i^2}$$

with *L(p)* being the adequate quantile of the standard normal distribution, V_i being the nominal exposure of trader i and σ_i being the standard deviation of stock i.

The portfolio-value at risk is calculated according to

$$(2)\ VaR_{PF} = \sqrt{\mathbf{VaR}^{\mathbf{T}} \cdot \mathbf{R} \cdot \mathbf{VaR}}$$

with **VaR** being the vector of all individual value at risk measures of the traders and **R** being the correlation matrix of all stocks.

Therefrom results:

$$(3)\ VaR_{PF}^2 = \mathbf{VaR}^{\mathbf{T}} \cdot \mathbf{R} \cdot \mathbf{VaR}\,.$$

Assuming identical positions for all traders leads to:

$$(4)\ VaR_{PF}^2 = V_i^2 \cdot L(p)^2 \cdot \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_i \cdot \sigma_j \cdot \rho_{ij}\,.$$

Here from the maximum position each trader can build, can be derived by:

$$(5)\ V_i = \sqrt{\frac{VaR_{PF}^2}{L(p)^2 \cdot \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_i \cdot \sigma_j \cdot \rho_{ij}}}$$

This position is by assumption identical for all traders. Thus the individual value at risk limit for trader *i* can be derived from:

$$(6)\ VaR - Limit_i = V_i \cdot L(p) \cdot \sigma_i$$

Given this limit the traders derive their exposure with the estimated (250 days) historical standard deviation $\hat{\sigma}_i$:

$$\overline{V}_i = \frac{VaR-Limit_i}{L(p)\cdot\hat{\sigma}_i} \tag{7}$$