The Minimum-Variance-Hedge and the Bankruptcy Risk of the Firm

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Abstract:

In this paper, we analyze the influence of hedging with forward contracts on the firm´s probability of bankruptcy. The minimization of this probability can serve as a substitute for the maximization of shareholders wealth. It is shown that the popular minimum-variance-hedge is generally neither necessary nor sufficient for the minimization of the firm´s probability of bankruptcy. Moreover, our model suggests a correction of the widespread view that a reduction in the variance of the future value of the firm is inevitably accompanied by a reduction in its default risk. We derive an analytical solution for the variance-minimizing hedge ratio of a firm exposed to both input and output price uncertainty that takes into account the issue of correlation. Based on this solution we provide a graphical analysis to prove our claim that there is a *fundamental* difference between hedging policies focused on bankruptcy risk and those following conventional wisdom even if positive correlation constitutes a “natural” hedge.

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I. Introduction

The reduction of the costs of bankruptcy and financial distress is one of the most prominent motives for corporate hedging. In their seminal article *Smith and Stulz* (1985) formally analyse the impact of a firm´s hedging policy on the market value of bankruptcy costs within a simple state-preference model. Although not explicitly dealing with the firm´s probability of bankruptcy, they claim the existence of a direct link between total risk in terms of variability or variance and bankruptcy risk: „By reducing the variability of the future value of the firm, hedging lowers the probability of incurring bankruptcy costs.”[1] Using a graphical argument *Rawls and Smithson* (1990) come to a similar conclusion: „...by reducing cash flow volatility, risk management reduces the probability of the firm getting into financial difficulty and bearing the consequent costs.“ [2] This view is also shared by *Fite and Pfleiderer* (1995) who, in their survey article summarize the indirect effect of hedging on shareholders´ wealth as follows: „Hedging generally lowers the probability of financial distress and bankruptcy. By lowering the probability, hedging lowers the expected costs of distress and increases the expected cashflows available for shareholders.”[3]

In this paper we present a new model to analyze directly the effect of hedging with forward contracts on the firm´s probability of bankruptcy (p. o. b.). The minimization of which can serve as a substitute for the maximization of shareholders wealth. By marginally restricting the probability density function of future prices, we formalize the idea developed by *Stulz* (1996) that when designing their hedging policies, value maximizing firms should focus on bankruptcy risk rather than on measures of total risk. Despite the early contributions of *Roy* (1952) and *Telser* (1955), the bulk of hedging models in the literature deal with the minimiza-

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1 *Smith & Stulz* (1985), p. 396.

2 *Rawls & Smithson* (1990), p. 10 f.

3 *Fite & Pfleiderer* (1995), p. 154.

tion of total rather than default risk.[4] *Stulz* (1996), p. 8 even refers to the variance-minimization-paradigma as the "prevailing academic theory of risk management" and *Froot, Scharfstein and Stein* (1993), p. 1630 argue that "much of the previous work has the extreme implication that firms should hedge fully – completely insulating their market values from hedgeable risk."

Firstly, we show that – in contrast to the statements cited above – a reduction in the variance of the future value of the firm is not inevitably accompanied by a reduction in its p. o. b. Secondly, we prove that full-cover hedging is generally neither necessary nor sufficient for the minimization of the firm´s p. o. b. The policy which leads to a minimal p. o. b. therefore differs *fundamentally* from the variance-minimizing hedge.

II. The Model

1. Assumptions

We follow *Holthausen* (1979) and consider a single-product manufacturing firm in a one period model (t = 0, t = 1). The firm was founded before t = 0 and the only assets in t = 0 are cash holdings. During the manufacturing period, the cash is transformed completely into input factors (e. g. raw materials, wages) which are utilized in the process of production, so that in t = 1, the output quantity is the firm´s only asset. The quantity of homogeous output in t = 1 $Y \in\ ]0; \infty[$ is certain and taken as given, because the production process is deterministic and we do not want the production decision to enter our model.

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4 *Cooper and Mello* (1999) model explicitly the impact of corporate hedging on firm value. They therefore have to make rather strong assumptions about taxation and bankruptcy costs. Their Proposition 6 states that with zero tax savings the optimal level of hedging is always less than the minimum variance amount. See *Cooper & Mello* (1999), p. 210 f.

The firm is a widely held corporation, whose shares are traded on a stock market. It has the legal form of a joint-stock company, so that shareholders´ liability in t = 1 is limited. Management is assumed to act in t = 0 in the best interest of the diversified shareholders, who want the firm´s p. o. b. to be minimized.

The goods produced are traded on spot as well as forward markets. In t = 0 management can choose between selling its output on the forward market or waiting until t = 1 in order to sell the output on the spot market. In both markets, the firm acts as price-taker. In t = 0, the current spot price s_0 is known, whereas the future spot price $\tilde{s}_1$ is uncertain. The future spot price is represented by a continuous random variable. Unlike the previous literature, we assume that all strictly positive price realizations are possible. For the probability density function of the future spot price w follows:

$$w(s_1) > 0 \, , \;\; s_1 > 0 \quad ; \quad 0, \;\; \text{otherwise}. \tag{1}$$

To avoid a possible discontinuity in the probability density function at $s_1 = 0$ we further assume that:

$$\lim_{s_1 \to 0} w(s_1) = 0. \tag{2}$$

The function w can be subjective and need not be specified further. Although some theoretical distributions – e. g. the normal and the uniform distribution – are excluded through (1), the underlying assumption has intuitive appeal.[5] Moreover, it is not a severe restriction. On the one hand, a multitude of distributions satisfy (1) and (2). The lognormal density, which would

5 See *Sandmo* (1971), p. 66 to justify the exclusion of negative prices.

result if s_0 were supposed to follow a Geometric Brownian Motion, constitutes only the most conspicuous example. On the other hand, our results hold for those continuous distributions that do not obey (1) over the complete domain, at least in those intervals where the densities are stricty positive.

For the corresponding cumulative probability density function W follows from (1):

$$W(s_1) = P\{\tilde{s}_1 \le s_1\} = \int_0^{s_1} w(u)\, du > 0\,,\ \ s_1 > 0 \quad ; \quad 0,\ \text{otherwise.} \tag{3}$$

In t = 0, the current forward price f is also known. Management can sell $N \in [0; Y]$ units of output forward at this price f. To exclude from the analysis speculative policies that result from advantages in information about future prices, the hedge ratio management can choose in t = 0 is restricted to $H = N / Y \in [0; 1]$. The price f is formed in the forward market in such a manner that the contracts do not have a direct effect on the initial wealth of the parties involved in the transaction.[6]

In t = 1 the forward contract is settled: The firm receives $N \cdot f$ dollars and the contracted N units of output are transferred to the forward partner. The remaining quantity Y – N is sold at the random spot price $\tilde{s}_1$. Note that as the volume of the forward contract is restricted to $N \in [0; Y]$, the claim of the forward partner cannot be subject to default if the firm was all-equity financed. Therefore, the forward price f does not include a premium for default risk.

6 See e. g. *Hull* (2000), p. 59: „The value of a forward contract at the time it is first entered into is zero.“ Of course, this statement refers only to the market value of the firm prior to bankruptcy costs.

To introduce the possibility of bankruptcy, we assume – like *Smith and Stulz* (1985) – that the firm issues a zerobond in t = 0, which embodies the promise to pay in t = 1 D_1 dollars to the creditors. The nominal amount of debt is given exogeniously. Although not explicitly considered here, other kinds of fixed claims against the firm could be included in the same way. To make sure that the claim of the forward partner is still not threatened by default we follow *Bessembinder* (1991) and assume, that it is senior to the claim of the bondholders. Otherwise, the correct forward price f would depend on the level of outstanding debt and could not be treated as a constant. Bankruptcy is triggered in t = 1, if the market value of the firm´s assets $\tilde{V}_1$ is not sufficient to satisfy the claims of the creditors D_1.[7] That is, if

$$\tilde{V}_1 = N \cdot f + (Y - N) \cdot \tilde{s}_1 < D_1 \ . \tag{4}$$

2. Results

If management intended to minimize the variance in the future value of the firm, the optimization problem would simply be:

$$\mathrm{Var}[\tilde{V}_1] = (Y - N)^2 \cdot \mathrm{Var}[\tilde{s}_1] \rightarrow \min_N \tag{5}$$

with the variance of the future spot price defined by:

$$\mathrm{Var}[\tilde{s}_1] = \int_0^\infty \left(s_1 - \int_0^\infty s_1 \cdot w(s_1)\, ds_1 \right)^2 \cdot w(s_1)\, ds_1 > 0 \, . \tag{6}$$

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[7] See e. g. *Merton* (1974).

The first partial derivative of (5) with respect to N, assumes negative values for $0 \le N < Y$, so that the variance of the future value of the firm is a strictly monotically decreasing function of the forward position:

$$\frac{\partial Var[\tilde{V}_1]}{\partial N} = -2 \cdot (Y - N) \cdot Var[\tilde{s}_1] < 0\,,\, 0 \le N < Y \quad ; \quad 0\,,\, N = Y. \tag{7}$$

For a hedge ratio of one, the variance is reduced to zero, so that both the necessary and sufficient condition for a minimum are met. Thus, the full-cover hedge is the only solution to (5): $N^* = Y$ resp. $H^* = 1$.

If, as assumed, management attempts to minimize the firm´s p. o. b. ζ instead, the optimization problem is:

$$\zeta = P\{\tilde{V}_1 < D_1\} \to \min_N \tag{8}$$

with the p. o. b. defined by:

$$\zeta = P\{N \cdot f + (Y - N) \cdot \tilde{s}_1 < D_1\} = \begin{cases} P\{\tilde{s}_1 < \dfrac{D_1 - N \cdot f}{Y - N}\} = W\left(\dfrac{D_1 - N \cdot f}{Y - N}\right), 0 \le N < Y; \\ \left.\begin{matrix} 0\,,\; Y \cdot f \ge D_1\,; \\ 1\,,\; Y \cdot f < D_1 \end{matrix}\right\} \qquad , \; N = Y\,. \end{cases} \tag{9}$$

The first partial derivative of (9) with respect to N which is defined for $0 \le N < Y$, describes the change in the firm´s p. o. b. if management marginally increases the volume of the forward contract:

$$\frac{\partial \zeta}{\partial N} = w\left(\frac{D_1 - N \cdot f}{Y - N}\right) \cdot \frac{D_1 - Y \cdot f}{(Y - N)^2} \quad . \tag{10}$$

The comparative statics and the resulting optimal forward positions respectively hedge ratios are given in Table I. *[Referee, please see the appended page not to be published.]* The relationship between output quantity, outstanding debt and forward price (situations A, B, C) plays a central role for the influence of the forward contract on the firm´s p. o. b. and the optimal hedge ratio. The two shaded cells in column three are not filled, since these situations would either imply N = Y, so that (10) was not defined, or N > Y, which, by assumption, was excluded from our analysis.

	$\partial\zeta / \partial N$		N*	H*
	$0 < N < D_1 / f$	$D_1 / f \leq N < Y$		
$D_1 < Y \cdot f$ (situation A)	< 0	$= 0$	$[D_1 / f; Y]$	$[D_1 / (Y \cdot f); 1]$
$D_1 = Y \cdot f$ (situation B)	$= 0$	$(N \geq Y)$	Y	1
$D_1 > Y \cdot f$ (situation C)	> 0	$(N > Y)$	0	0

Table I: Comparative statics, optimal forward positions N^* and hedge ratios H^*.

If the firm is in situation A, so that the fixed credit obligation is lower than the maximum revenue which the firm can rely on earning ($D_1 < Y \cdot f$), and if the certain payment from the forward sale does not yet cover the creditors´ claim ($0 < N < D_1 / f$), then the p. o. b. is reduced by a marginal increase in the hedge ratio. If, in converse, the certain payment from the forward sale already covers the credit obligation ($N \geq D_1 / f$), then a marginal increase in the hedge ratio has no further impact. The p. o. b. remains zero. Thus, in situation A, both a complete and a partial protection of the price risk with $N \geq D_1 / f$ lead to the minimum p. o. b.: $N^* \in [D_1 / f ; Y]$ resp. $H^* \in [D_1 / (Y \cdot f); 1]$. There is no unique solution in this situation, because (8) is satisfied by the entire interval of hedge ratios. The full-cover hedge is sufficient, but not necessary for the minimization of the firm´s p. o. b. Taking into account that an increased volume of the forward contract typically leads to additional transaction costs, the recommendation is to reduce total risk only to the degree that default can be completely ruled out: $H^* = D_1 / (Y \cdot f)$. The more detailed explanation for the result that complete protection against market price risk is not necessary in situation A, lies in the exclusion of negative future spot prices for the production good according to (1). If, in contrast to (1), e. g. by assuming a normal distribution, negative prices were not ruled out, a corresponding statement as to the minimal hedge ratio that solves (8) could not be made.

If the firm is in situation B, where the credit obligation exactly equals the maximum revenue which the firm can earn with certainty ($D_1 = Y \cdot f$), a marginal increase in the hedge ratio has no effect on the firm´s p. o. b. As can be seen from (9), the p. o. b. equals the probability of making a profit on the forward contract – $P\{\tilde{s}_1 < f\} = W(f)$ – for every $0 < N < Y$. Indeed, the forward contract earns a profit whenever the firm goes bankrupt, but with only partial hedging, this profit is not sufficient to enable the firm to meet its obligations. Only through complete protection against price risk, the can p. o. b. be reduced to zero: $N^* = Y$ resp. $H^* = 1$. In this case only, the minimization of variance of the future value of the firm and the minimization of its p. o. b., lead to the same optimal hedging policy.

If the firm is in situation C, in which the credit obligation exceeds the maximum revenue which the firm can be sure of earning ($D_1 > Y \cdot f$), then a marginal increase of the hedge ratio even lifts the firm´s p. o. b. With a full-cover hedge the company would certainly go bankrupt. Without hedging, the firm has still the chance to avoid bankruptcy if the spot price of the production good rises sharply. The optimal hedge ratio in this case is $H^* = 0$.

3. Extension: Input factor price uncertainty and the "natural" hedge

In this section we discuss the robustness of our main result – the *fundamental* difference between the variance-minimizing and the bankruptcy risk minimizing hedge. In order to clarify that it is not the absence of other kinds of uncertainty that causes this difference, we now explicitly include uncertainty about the price of the firm´s (single) input factor. In the interest of simplicity, we assume a one-to-one correspondence between units of output produced and units of input used, i. e. a production coefficient of one. With the random input factor price $\tilde{b}_1$, payable in time one, the firm´s future total asset value can now be written as

$$\tilde{V}_1 = N \cdot f + (Y - N) \cdot \tilde{s}_1 - Y \cdot \tilde{b}_1 \ . \tag{11}$$

Clearly, (11) indicates that we do not consider a second hedging instrument to manage the risk of the procurement cost. Thus, the minimization of the variance of future firm value is given by

$$\mathrm{Var}[\tilde{V}_1] = (Y - N)^2 \cdot \mathrm{Var}[\tilde{s}_1] + Y^2 \cdot \mathrm{Var}[\tilde{b}_1] - 2 \cdot (Y - N) \cdot Y \cdot \mathrm{Cov}[\tilde{s}_1;\ \tilde{b}_1] \rightarrow \min_{N} . \tag{12}$$

Assuming that the input price is a continuous random variable with a probability density function v, that has the same properties as the density function of the output price w (1) and (2) the variance of the input price is given by

$$\mathrm{Var}[\tilde{b}_1] = \int_0^\infty \left(b_1 - \int_0^\infty b_1 \cdot v(b_1)\, db_1 \right)^2 \cdot v(b_1)\, db_1 > 0 \ . \tag{13}$$

While the two variances in (12) refer only to the two corresponding univariate distributions, defining the covariance requires knowledge of the full bivariate distribution, not only its two marginals. To extend our basic model to the case of a bivariate distribution, it seems most straightforward to assume that the *joint* probability density function j has the property

$$j(s_1; b_1) > 0\,,\ \ s_1 > 0 \quad \wedge \quad b_1 > 0; \quad 0,\ \text{otherwise}. \tag{14}$$

This means that any combination of positive input and output prices is possible. Note that (14) does not immediately follow from the fact that both the marginal density functions w and v have property (1), but that it is a stronger assumption.[8]

With the joint probability density function j according to (14) the covariance in (12) is defined as

$$\mathrm{Cov}[\tilde{s}_1; \tilde{b}_1] = \int_0^\infty \int_0^\infty \left(s_1 - \int_0^\infty s_1 \cdot w(s_1)\, ds_1 \right) \cdot \left(b_1 - \int_0^\infty b_1 \cdot v(b_1)\, db_1 \right) \cdot j(s_1; b_1)\, ds_1\, db_1\ . \tag{15}$$

The first-order condition of (12) yields

$$\frac{\partial \mathrm{Var}[\tilde{V}_1]}{\partial N} = -2 \cdot (Y - N) \cdot \mathrm{Var}[\tilde{s}_1] + 2 \cdot Y \cdot \mathrm{Cov}[\tilde{s}_1; \tilde{b}_1] \overset{!}{=} 0\,, \tag{16}$$

so that the optimal number of the forward contracts respectively the optimal hedge ratio emerges as a function of the correlation coefficient k:

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8 In general, the construction of a multivariate distribution from its marginals can be achieved by specifying a so-called *copula function* – an approach which is becoming more and more wide-spread in default risk measurement. See e. g. *Li* (2000).

$$N^{**} = Y \cdot \left(1 - \frac{\text{Cov}[\tilde{s}_1; \tilde{b}_1]}{\text{Var}[\tilde{s}_1]}\right) \quad \text{resp.} \quad H^{**} = \frac{N^{**}}{Y} = 1 - k \cdot \sqrt{\frac{\text{Var}[\tilde{b}_1]}{\text{Var}[\tilde{s}_1]}} \,. \tag{17}$$

The sufficient condition for a minimum is always met. Hence, solely with uncorrelated input and output prices ($k = 0$), the variance-minimizing hedge ratio H^{**} is again 100% . Positive correlation ($k > 0$) constitutes a so-called “natural” hedge that results in an optimal hedge ratio of less than 100%.[9] We leave further interpretation of (17) to the reader and now turn to the optimization problem (8) again in order to explain why and how its solution does in general differ from (17).

Firstly, we extend the bankruptcy condition (4) to allow for the uncertain factor costs included in (11):

$$V_1 = N \cdot f + (Y - N) \cdot s_1 - Y \cdot b_1 < D_1 \Leftrightarrow b_1 > \frac{N \cdot f - D_1}{Y} + \frac{Y - N}{Y} \cdot s_1 = \bar{b}_1(s_1; N; f; D_1; Y) \,. \tag{18}$$

Hence, the firm will be bankrupt if the realization of the input price b_1 exceeds a critical level that depends principally on the realization of the output price s_1, as well as on the number of forward contracts N into which the firm enters. The critical input factor price that just enables the firm to avoid bankruptcy is denoted by $\bar{b}_1 = \bar{b}_1(s_1; N; f; D_1; Y)$. The dependence of the critical price $\bar{b}_1$ on different values of s_1, is illustrated in Figure 1 for three different hedging levels, supposing that the firm is in situation A ($D_1 < Y \cdot f$). With given parameter values for f, D_1, Y and N, the function $\bar{b}_1(s_1)$ is represented by a straight line whose gradient displays the hedge ratio chosen: $N_1 = 0$ ($H_1 = 0$), $N_2 = 0.5 \cdot Y$ ($H_2 = 0.5$), $N_3 = Y$ ($H_3 = 1$). Only for an

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[9] See *Froot, Scharfstein and Stein* (1994), p. 97 ff. who give an illustrative example of a “naturally” hedged oil company.

output price realization s_1, that just equals the forward price f, the critical input price $\bar{b}_1$ does not depend on the hedge ratio (see the intersection of the three straight lines).

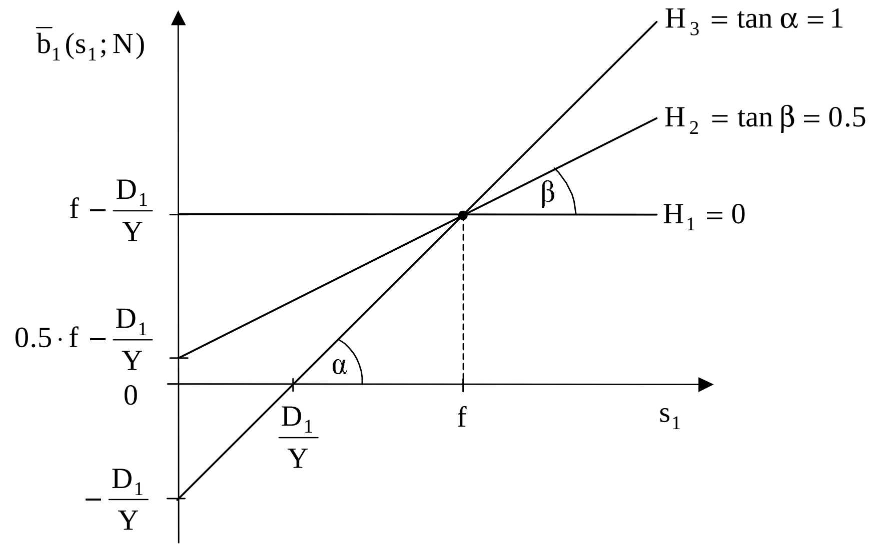

Figure 1: Critical input factor price $\bar{b}_1$ as a function of the realized output price s_1 for differing hedge ratios: $H_1 = 0$, $H_2 = 0.5$ and $H_3 = 1$.

Secondly, we use (18) to show in what way the firm's p. o. b. depends on the chosen hedge ratio. The p. o. b. can be defined by means of an indicator function $I = I(b_1; \bar{b}_1)$, which refers to the realization of the input factor price in relation to its critical value. This indicator function takes on the value one, if the firm is bankrupt, and zero, if not:

$$\zeta = P\{\tilde{V}_1 < D_1\} = P\{\tilde{b}_1 > \bar{b}_1\} = \int_0^\infty \int_0^\infty I(b_1; \bar{b}_1) \cdot j(s_1; b_1)\, ds_1\, db_1 \tag{19}$$

with

$$I(b_1; \bar{b}_1) = 1,\ b_1 > \bar{b}_1; \quad 0,\ \text{otherwise.} \tag{20}$$

Figure 2 illustrates this mathematical representation of the firm's p. o. b. for a hedge ratio of 50%. The bivariate density function that is assumed to be strictly positive for s_1, $b_1 > 0$ obviously displays a positive correlation. The area that contains those combinations of s_1 and b_1 that lead to bankruptcy for the given hedge ratio, $b_1 > \overline{b}_1(s_1; H = 0.5)$, lies in the upper left corner of the base and is hatched. The p. o. b. is the volume of the solid space between the hatched part of the (s_1, b_1)-plane and the density function j. The indicator function I in (19) ensures that only that portion of the total probability mass of one that lies above the hatched part of the plane is taken into account.

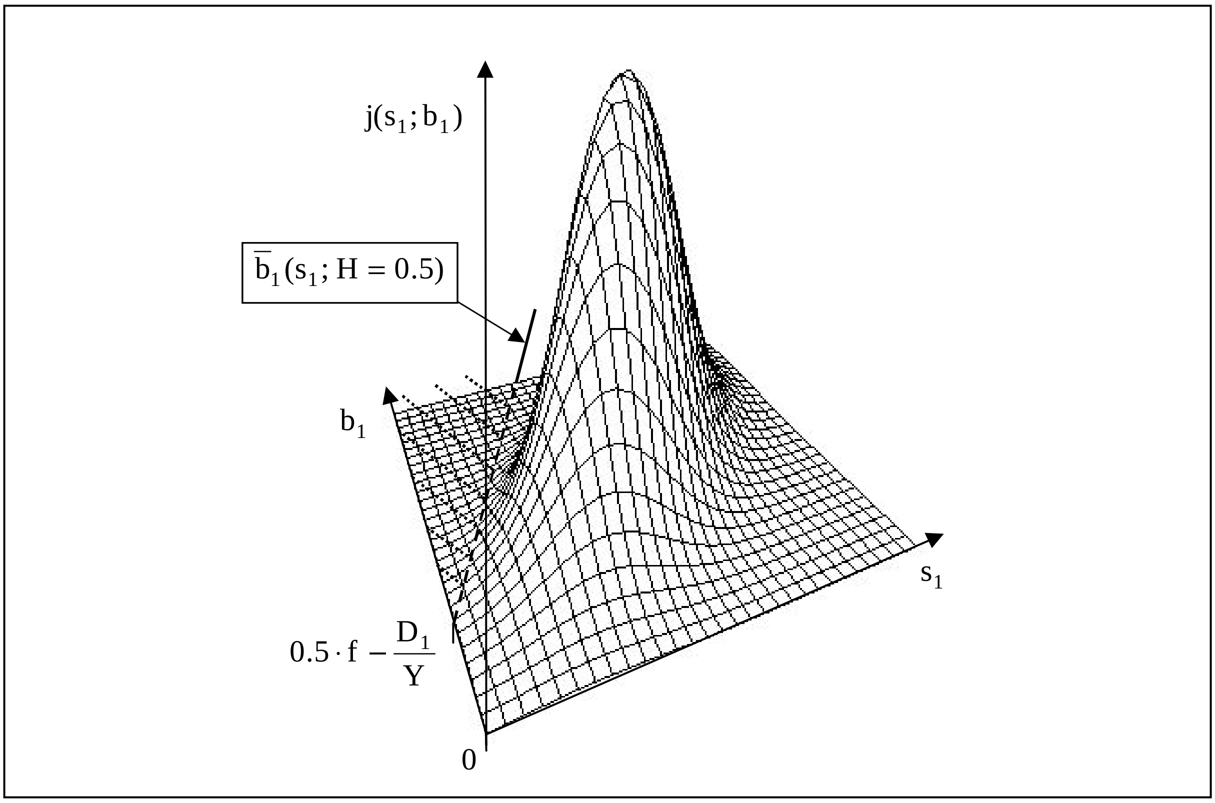

Figure 2: Joint probability density function of positively correlated output and input prices $j(s_1; b_1)$, critical input factor prices for a hedge ratio of 50% $\overline{b}_1(s_1; H = 0.5)$ and the firm's p. o. b.

Thirdly, we combine the relationships made transparent in Figures 1 and 2 in order to show that the variance-minimizing solution in (17) does not provide a general solution for the minimization of (19). Our argument is illustrated in Figure 3. Only for simplicity, it is assumed that $\mathrm{Var}[\tilde{s}_1] = \mathrm{Var}[\tilde{b}_1]$ and k = 0.5, so that H** = 0.5. Thus, the variance-minimizing hedge ratio is 50%, so that the appropriate function $\bar{b}_1(s_1; H = 0.5)$ can be taken from Figure 1 without revision. Furthermore, let us suppose that the iso-density ellipses of the bivariate density function j which are given in Figure 3, are fully compatible with this calibration of the three parameters.

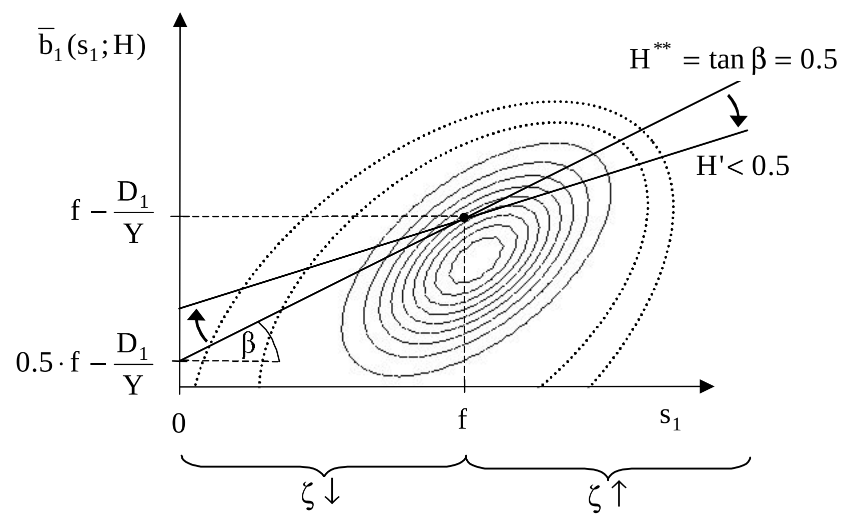

Figure 3: Ambiguous influence of a decrease in the hedge ratio below the variance-minimizing-level on the firm’s p. o. b.

Figure 3 illustrates the effect of a decrease in the forward position below the variance-minimizing level H** on the firm‘s p. o. b. The shift in the hedge ratio (from H** to H’ < 0.5) is indicated by the two arrows. Its effect is twofold: On the one hand there is an *increase* in the p. o. b. by the probability mass that lies above the area between the two straight lines to the *right* of the fulcrum (f; f - D_1 / Y). But on the other hand, there is also a *decrease* in the

p. o. b. by the probability mass above the area between the two straight lines to the *left* of the fulcrum. Hence, the total effect of a deviation from the variance-minimizing solution on the firm's bankruptcy risk is ambiguous.

Of course, according to (19), the p. o. b. can be calculated for any pair of hedge ratios and any parameter set by numerical integration, if the functional form of the bivariate distribution is given. But to get to know the sign and amount of variation in the p. o. b., it is generally not sufficient to refer to the (first and) second moments of the two marginal distributions and to a single correlation parameter.[10] The three parameters that enter the solution (17), therefore provide only a very limited picture of the full bivariate distribution, especially with respect to it's lower tail behavior. The importance of the lower tails is illustrated in Figure 3 by the two dashed iso-density ellipses that obviously play a decisive role for the net effect of a change in the forward position on the p. o. b. Furthermore, the variation in the p. o. b. caused by a variation in the firm's hedge ratio does, in the case of the bivariate as well as in the case of the univariate distribution of asset value, in general depend on the forward price f and on the debt level D_1. Neither of these parameters enters in the derivation of (17) and they therefore cannot influence the variance-minimizing solution.

It should have become clear from Figure 3 and from the discussion above, that the hedge ratio that minimizes the company's p. o. b. is not generally identical with that given by (17). Having graphically clarified the robustness of our proposition for the case of two correlated output and input prices that establish a "natural" hedge, we leave possible analytical derivations of closed-form solutions to the minimization of (19) for future research.

10 Although the bivariate *normal* distribution is completely characterized by the first and second moments of the marginal distributions together with a single correlation parameter, this cannot be generalized to other distributions.

III. Conclusions

The commonly held view that a reduction in a firm´s total risk generally results in a reduction in its bankruptcy risk, served as the starting point for our analysis. Our result that hedging with forward contracts may at the same time reduce risk in terms of variance and may not reduce or even raise default risk, refutes such statements. Hedging policies focused on bankruptcy risk therefore differ *fundamentally* from the recommendations following conventional wisdom. This result holds for a firm exposed to input and output price uncertainty even if positive correlation gives rise to a “natural” hedge.

As long as neither the production nor the capital structure decision are explained endogenously, there is no reason to exclude one of the three possible situations A, B and C from the theoretical considerations. Nonetheless the question remains as to what these cases represent in reality. It seems quite obvious to associate the only moderately levered firm of case A with the standard situation. An intuitive interpretation of our situation C is given by *Stulz* (1996) who alludes to the speculative positions taken by already financially distressed banks during the S&L-crisis.[11] In this case, management should not protect the firm against market price fluctuations at all, but “gamble for resurrection”.

Our optimal hedge ratio of $0 < H^* = D_1 / (Y \cdot f) < 1$ for financially sound firms that are exposed to output price uncertainty, is presumably much more in line with actual firm behavior than the 100 %-hedge.[12] However, it is necessary to find out through empirical tests, whether this implication can be supported by real data. It would be particularly interesting to test our prediction that firms´ hedge ratios decrease with the coverage of total fixed charges (rather than increase with leverage alone) on a cross-sectional basis.

11 See *Stulz* (1996), p. 17.

12 See *Haushalter* (2000), p. 117, who reports hedge ratios of about 15 % for the oil and gas industry.

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Attn. Referee:

$D_1 < Y \cdot f$ and $0 < N < D_1 / f$:

$$\frac{\partial\zeta}{\partial N} = \underbrace{w\underbrace{\left(\frac{D_1 - N\cdot f}{Y - N}\right)}_{>0}}_{>0} \cdot \underbrace{\frac{D_1 - Y\cdot f}{(Y-N)^2}}_{<0} < 0 \ .$$

$D_1 < Y \cdot f$ and $D_1 / f \leq N < Y$:

$$\frac{\partial\zeta}{\partial N} = \underbrace{w\underbrace{\left(\frac{D_1 - N\cdot f}{Y - N}\right)}_{\leq 0}}_{=0} \cdot \underbrace{\frac{D_1 - Y\cdot f}{(Y-N)^2}}_{<0} = 0 \ .$$

$D_1 = Y \cdot f$ and $0 < N < D_1 / f$:

$$\frac{\partial\zeta}{\partial N} = \underbrace{w\underbrace{\left(\frac{D_1 - N\cdot f}{Y - N}\right)}_{>0}}_{>0} \cdot \underbrace{\frac{D_1 - Y\cdot f}{(Y-N)^2}}_{=0} = 0 \ .$$

$D_1 > Y \cdot f$ and $0 < N < D_1 / f$:

$$\frac{\partial\zeta}{\partial N} = \underbrace{w\underbrace{\left(\frac{D_1 - N\cdot f}{Y - N}\right)}_{>0}}_{>0} \cdot \underbrace{\frac{D_1 - Y\cdot f}{(Y-N)^2}}_{>0} > 0 \ .$$