Occupational Choice and the Private Equity Premium Puzzle

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Abstract

This paper suggests a solution to what has become known as the "private equity premium puzzle" (Moskowitz and Vissing-Jorgensen (2002)). We interpret occupational choice as a dynamic portfolio choice problem of a life-cycle investor facing a liquidity constraint and imperfect information about the profitability of potential businesses. In this setting, becoming an entrepreneur is equivalent to investing in non-traded private equity capital subject to transaction costs. We model the return on private equity as the sum of two components, the individual ability of the entrepreneur and idiosyncratic business risk. Information is imperfect, because only entrepreneurs observe their own business risk realizations. Using numerical techniques we find that the model generates the observed return structure for private equity using standard CRRA-preferences and fully rational expectations.

Keywords: Portfolio choice, Life-cycle models, Private equity

JEL classification: G11, D91

1 Introduction

Recently, Moskowitz and Vissing-Jorgensen (2002) have documented in a highly interesting paper the risk-return structure and the portfolio allocation of private equity in the US. Private equity is defined as the equity capital of firms that are not listed in the stock market and which is therefore not traded publicly. They find that average private equity ex-post returns are comparable to average ex-post returns on public equity, but also that the distribution of individual returns is very wide and skewed to the left. At the same time, investment in private equity is extremely concentrated with most entrepreneurial households investing the majority of their wealth in a single firm. In addition, the majority of the equity capital of a single firm is regularly held by a single private investor who also has a management interest in the firm. The authors conclude that, given the concentration in individual portfolios and the large idiosyncratic risk associated with private equity, investment in private firms seems to be dominated by investment in publicly traded equity for risk-averse individuals and term this finding the "private equity premium puzzle".[1]

[1] In the previously circulated version of their paper Moskowitz and Vissing-Jorgensen (2002) claimed the existence of the puzzle. The published version qualifies this claim and calls for more theoretical work taking into account the correlation of returns to financial and human capital and optimal dynamic consumption choice. Our model is designed in

Studying the empirical results in more detail, there are two different dimensions of this puzzle. The "corporate finance"-dimension is why most private firms are controlled by a single shareholder. As shown by Bitler, Moskowitz and Vissing-Jorgensen 2002, a simple moral hazard model goes a long way toward explaining this finding. The "portfolio choice"-dimension seems to be more difficult to explain. The issue is, why entrepreneurs are willing to hold the equity of a single firm given the unfavorable variance-return characteristics of the asset. As correctly observed by Moskowitz and Vissing-Jorgensen (2002), this behavior violates the participation constraint in the moral hazard game. In the following, we propose a resolution to the "portfolio-dimension" of the puzzle by showing that the observed structure of ex-post private equity returns and the concentrated portfolio allocations of private equity emerge from a simple model of occupational choice over the life-cycle. We find that the realized rate of return on an index of non-traded private equity is only slightly higher than the return on a riskless, frictionlessly traded asset. We also confirm the conjecture that the index return overestimates the average cross-sectional return on private equity and find that this average basically equals the return on the frictionlessly traded asset. At the same time, cross-sectional returns to private equity are very volatile and for agents holding private equity, these holdings represent a large share

order to address precisely these points.

of their total wealth.

In addition to these findings, our model also replicates other facts about entrepreneurship and private businesses. The aggregate value of private equity roughly equals 1/2 of the aggregate value of traded assets, as reported by Moskowitz and Vissing-Jorgensen (2002). Firm survival rates are low for young firms and the average age of private firms is 12.6 years, roughly in line with the 10.7 years reported by Moskowitz and Vissing-Jorgensen (2002). The distribution of total wealth is skewed to the left and much more unequal than the distribution of income. Entrepreneurs are on average 3.6 times wealthier than agents not holding private equity and 38% of total wealth is owned by entrepreneurs, who represent a share of only 13.6% of the population, which are statistics that roughly match the estimates provided by Gentry and Hubbard (2000).

Following Moskowitz and Vissing-Jorgensen (2002) we take the decision to become an entrepreneur as being equivalent to the decision of investing in private equity. While in principle, business ownership does not necessarily imply that the business owner also chooses to be an entrepreneur and spends his time running the firm, Moskowitz and Vissing-Jorgensen (2002) find that 70% of business owners also have an active management interest in the single firm they are investing in. Hence, they find that the "single proprietor

model" of entrepreneurship accounts for most of the data and we agree with their conclusions. When giving this interpretation to the choice of becoming an entrepreneur however, it is also important to consider the special kind of investment environment in which this choice is embedded. First, the choice of becoming an entrepreneur has strong implications for the life-cycle consumption profile. We argue that the appropriate setting for studying occupational choice is a life-cycle model with liquidity-constrained, finitely-lived agents who in each period can decide to be entrepreneurs or workers. The decision to accumulate assets is endogenous in this framework and motivated by the fact that post-retirement income is considerably lower than wage income. We allow for stochastic lifetimes, but take wage and retirement income as exogenous and stochastic over the lifecycle.

Second, the financial market in this setting is inherently incomplete and it is necessary to take into account both, the risk component of starting and running a business, and the transaction costs associated with it. Becoming an entrepreneur is equivalent to forgoing the wage earned in the labor market and starting a new business. This implies creating a new type of financial asset by assembling various capital goods into a legal entity called a firm and issuing control rights in the form of private equity. This newly created asset is initially not traded in a public asset market[2]. It is likely that full

[2] The procedure to convert non-traded private equity into traded public equity is called

information about the return characteristics of the asset becomes available only upon creation of the asset and that the transaction costs associated with entrepreneurship are quite high. In our model, all agents are informed about the conditional distribution of the returns to entrepreneurship, but only entrepreneurs observe their business risk realization. It is a central assumption of the model that the rate of return on private equity depends on the skills of the entrepreneur. Due to the incompleteness of financial markets, agents cannot sell their own private equity or hold other agent's private equity. It is this short-selling constraint, the existence of transaction costs and the serial correlation in returns to entrepreneurship, which makes the optimal portfolio choice inherently dynamic and requires numerical solution methods.

Two factors mainly drive the result. First, agents with standard utility functions and standard levels of risk aversion do not require a large excess return of 10% to hold an asset with empirically observed levels of volatility. This is a well-established result first reported by Mehra and Prescott (1985).

Initial Public Offering. We believe that our model is a good starting point for modeling IPO's, but limit our analysis to firms that do not ultimately go public. This is not likely to affect our results much, since Moskowitz and Vissing-Jorgensen (2002) show that accounting for IPO's increases the relevant rate of return on private equity by only 0.5% annually.

Second and more importantly, average rates of return on private equity are not independent of the age of the entrepreneur or the age of the business. Clearly, the existence of start-up costs which will be amortized over time implies that young businesses have lower average rates of return than mature businesses. In addition, very young businesses have a high failure rate, because of entrepreneurs learning about their business risk realization.

A little less obvious, young agents require lower average rates of return to start a business than older agents. The persistence in private equity returns and the imperfect information about returns make it attractive for wealthy young agents to start a business even if the average expected return is low. The reason for this is the possibility of exit, if the business return realization is bad. Effectively, this means that the present value of the loss associated with low realizations of business returns is bounded. On the other hand, if the business return realization is good, the present value of the gain associated with it can be fully reaped by the young agents. Since agents cannot borrow against future income, some highly skilled agents are deterred from starting a business by insufficient wealth. Since the expected rate of return on private equity depends positively on the skills of an agent, especially wealthy individuals with high wage income tend to start private businesses early in life.

Further, young agents face increasing income profiles and their current

consumption is low for exogenous reasons (the liquidity constraint and an increasing income profile). Hence, negative return shocks do not have a strong impact on lifetime utility, since their income is guaranteed to increase in the future. Later in life, income is projected to decrease and agents therefore seek to avoid holding a risky asset, even if its returns are relatively high. Together with the fact that the weight of young agents in the US population is relatively large, these effects lead to relatively low average individual returns in the cross-section. The index return is not affected that much however, because young businesses tend to be small and young agents own young businesses.

The next section provides a brief overview of the literature on occupational choice and optimal portfolios in life-cycle models. Section 3 presents the model in detail and discusses calibration issues. In section 4 we present the results for our baseline calibration and analyze the mechanism. Section 5 provides results from alternative calibrations and section 6 concludes and points to interesting research topics in the future. The appendix contains a description of our computational procedure.

2 Related literature

The theoretical literature on entrepreneurship is rather scarce. Following Schumpeter's treatise on economic development and entrepreneurship (Schumpeter (1934)), little academic attention has been devoted to studying this phenomenon. Laffont and Kihlstrom (1979) consider risk aversion as the main determinant of becoming an entrepreneur in a static general equilibrium setting, but fail to provide convincing empirical evidence for their hypothesis. In an interesting and challenging paper, Banerjee and Newman (1993) analyze the effect of liquidity constraints on occupational choice in a dynamic growth model. They show that the equilibrium wage level and the occupational structure of the economy depend on the current distribution of wealth if financial markets are imperfect and investment projects are indivisible. Since the current distribution of wealth is itself endogenous in a dynamic model, they show that various paths of economic development are possible for economies with the same technological characteristics and that ultimate outcomes crucially depend on the initial distribution of wealth. Their model is able to rationalize radical redistribution policies in order to avoid negative long-run outcomes of aggregate dynamics. Other than Moskowitz and Vissing-Jorgensen (2002), there also exists little empirical work on the return distribution of private equity. Empirical studies of entrepreneurship

have focused instead on finding determinants of the decision to become an entrepreneur or be self-employed. Evans and Jovanovic (1989) were among the first to study the determinants of entrepreneurship, focusing in particular on the effect of receiving a large gift or bequest on the probability of becoming an entrepreneur. Subsequent studies such as Holtz-Eakin, Joulfaian and Rosen (1994), Hurst and Lusardi (2002) and Hamilton (2000) enlarge the set of determinants considered, but find similar results concerning the positive effects of large positive income shocks.

The paper that is probably most closely related to ours is Cagetti and DeNardi (2002), who study the decision of becoming an entrepreneur in an overlapping generations setting. However, they focus on the wealth distribution rather than the returns to entrepreneurship and use a much more stylized model, in which entrepreneurs operate in a secluded sector without hiring labor, allowing them to jointly determine the rates of return and the amount of assets accumulated through an entrepreneurial production function. Our model instead takes the distribution of rates of return to entrepreneurship as exogenous to the savings decision, but endogenously determines the share of wealth invested in private equity at each point in time.

A large literature exists on optimal saving decisions over the life-cycle, if liquidity constraints are binding and one asset is available. Auerbach and

Kotlikoff (1987) and Hubbard and Judd (1987) were among the first to study this issue. More recent studies including a portfolio-choice component, participation costs and stochastic labor income are Campbell, Cocco, Gomes and Maenhout (1999), Haliassos and Michaelides (2002) and Laibson, Repetto and Tobacman (1998). Campbell, Cocco, Gomes and Maenhout (1999) study the implications of investing retirement assets in the stock market rather than government bonds. Laibson, Repetto and Tobacman (1998) build a detailed model of the US economy taking into account heterogeneity in wealth and education levels, but focus on the optimal saving behavior of consumers with time-inconsistent preferences and its implications for public pension schemes.

3 Modeling occupational choice

Like Campbell, Cocco, Gomes and Maenhout (1999) and Laibson, Repetto and Tobacman (1998) our life-cycle problem is set in a partial equilibrium context, taking the stochastic processes of asset returns and wages as given. For computational reasons and well-known problems of dynamic general equilibrium models to generate empirically plausible asset returns, this is a standard approach in the dynamic portfolio choice literature. We also abstract from aggregate uncertainty and assume that the aggregate economy grows at a deterministic rate. Therefore, we are not able to capture business cycle

effects and focus on the relative return structure of private to public equity instead. Individuals have finite, stochastic lifetimes and we set a fixed retirement date. There is no bequest motive.

Household $i \in \{1, 2, ..., I\}$ maximizes utility over a finite horizon of T periods:

$$\max_{c_{it}, B_{i,t+1}, S_{i,t+1}} E_0 \left[\sum_{t=1}^{T} \delta^t \left(\prod_{j=1}^{t} p_j \right) u(c_{it}) \right], \tag{1}$$

where δ is a discount factor and p_t denotes the probability of being alive at date t, conditional on being alive at date $t-1$, subject to the following budget constraint

$$c_{it} = w_{it} + (1+r)B_{it} - B_{i,t+1} - \theta w_{it} + (1+v_{it})S_{it} - S_{i,t+1} - ... \tag{2}$$

$$... - \mathbf{1}(S_{it} = 0, S_{i,t+1} > 0)\Phi - \mu \max(0, S_{it} - S_{i,t+1})$$

and short-sale constraints on the two available assets

$$B_{it} \geq 0, S_{it} \geq 0, \forall t \tag{3}$$

Exogenous income has an age-dependent deterministic component

$$\bar{w}_t = g(t), \text{ for } 1 \leq t \leq P \tag{4}$$

and

$$\bar{w}_t = \bar{b}, \text{ for } P < t \leq T \tag{5}$$

where P denotes the fixed retirement age. In addition, there is uninsurable idiosyncratic income risk ε_{it} such that

$$\log w_{it} = \log \bar{w}_t + \varepsilon_{it} \tag{6}$$

The stochastic income component is described by

$$\varepsilon_{it} = \phi \varepsilon_{i,t-1} + \upsilon_{it}, \tag{7}$$

where υ_{it} is an i.i.d. shock distributed as

$$\upsilon_{it} \sim N\left(0, \sigma_\upsilon^2\right).$$

There are two types of assets available in the economy, a financial asset traded in public markets and denoted by B_{it}, which yields a certain rate of return r. The other type of asset is non-traded private equity capital invested in a firm which is managed by the same entrepreneurial household. This asset yields an idiosyncratic return

$$v_{it} = \bar{v} + \rho \varepsilon_{it} + \zeta_{it}, \tag{8}$$

which consists of an average guaranteed return, $\bar{v}$, a skill component, perfectly correlated to the uninsurable idiosyncratic income risk, $\rho \varepsilon_{it}$, and a business

risk component, ζ_{it}. The business risk component is orthogonal to the skills component and evolves according to

$$\zeta_{it} = \psi\zeta_{i,t-1} + \xi_{it}, \tag{9}$$

where ξ_{it} is an i.i.d. shock distributed as

$$\xi_{it} \sim N\left(0, \sigma_{\xi}^{2}\right).$$

and every newly created business receives an initial draw from the steady state distribution of ζ.

A crucial feature of the model are the assumptions relating to the information on the uncertainty realizations available to the agents. We assume that each individual can observe the realization of idiosyncratic income risk at each point in time. However, it is assumed that business risk is unobserved prior to starting a business. This implies that only entrepreneurs, defined as having a portfolio with a strictly positive level of private equity, can use conditional distributions of returns in their optimal decisions by applying transition probabilities over the states of business risk. Non-entrepreneurs are facing the unconditional distribution of business risk.

Investment in private equity is subject to transaction costs, which represent start-up costs of becoming an entrepreneur, $\mathbf{1}(S_{it} = 0, S_{it+1} > 0)\Phi$, and costly investment irreversibility, $\mu \max(0, S_{it} - S_{it+1})$. As in Abel and

Eberly (1994) or Dixit and Pindyck (1998) μ represents the wedge between the purchase and sale price of private equity[3].

The structure of this problem is close to a standard dynamic portfolio choice problem for a finite horizon investor introduced by Samuelson (1969) and Merton (1969). The distinguishing features of our model are the existence of transaction costs, the fact that returns to private equity capital are correlated with exogenous income, and the informational assumption that returns to the entrepreneurial asset are observed only when its share in the portfolio is strictly positive. These assumptions together with the short sale constraints require the use of numerical techniques to find the solution of the problem.

4 Calibration

We set the maximum lifetime of agents in our model to 89 years and assume that agents enter the model at age 20. In order to economize on computational resources, we choose a period-length of 3 years and hence arrive at a total lifetime of 23 periods. The mandatory retirement date is the end

[3] The idea here is that selling private equity essentially means selling some of the capital goods used in the business. The wedge therefore implicitly exists between the purchase and sale price of capital goods.

of period 16, which is equivalent to an age of 68 years. Death probabilities are taken from the lifetables of the US National Center for Health Statistics, which report conditional survival probabilities[4].

Agents are assumed to maximize a standard instantaneous utility function of the CRRA-class,

$$u(c_{it}) = \frac{c_{it}^{1-\sigma} - 1}{1-\sigma} \tag{10}$$

setting the coefficient of relative risk aversion $\frac{1}{\sigma}$equal to 0.5 and the discount rate δ to 0.97. These values are common in the literature (see Conesa and Krueger (1999), Campbell, Cocco, Gomes and Maenhout (1999) and Cagetti and DeNardi (2002)) and consistent with estimates by Gourinchas and Parker (1999) in their analysis of lifetime consumption profiles.

Fixed Parameters	Value
σ	2
δ	0.97
r	0.05
θ	0.185
ϕ	0.688
σ_v^2	0.052

[4]U.S. National Center for Health Statistics (1994)

We take the calibration of the wage income process from the detailed study by Laibson, Repetto and Tobacman (1998). These authors estimate age-income profiles for three educational groups from PSID data by regressing log income on powers of age and some control variables. We select the median educational group as being most representative for the average household.

$$g(t) = \exp(8.835 + 0.058t - 0.017t^2/100 - 0.055t^3/10000)$$

We convert their results obtained for annual data to a 3 year-frequency by time-aggregation. The autocorrelation coefficient ϕ of the stochastic wage process is equal to 0.688 for annual data, which implies a coefficient of 0.326 for our calibration. The variance of the innovation to log income σ_v^2 is 0.052 for annual data, resulting in a value of 0.0883 for our modeling frequency. We approximate the tax system by a proportional tax on exogenous income, choosing a tax rate θ of 18.5% and calibrate the deterministic component of retirement income to match the average replacement ratio[5] of 45% reported in Laibson, Repetto and Tobacman (1998) and Engen, Gale and Scholz (1994). Draws of initial wealth are taken from a lognormal distribution, using the empirical mean of the wealth distribution for the youngest cohort (equal to $\$23,183$) and the corresponding coefficient of variation of 6.53 given in Budria, Diaz-Gimenez, Quadrini and Rios-Rull (2001). In the aggregation we

[5] Defined as the ratio of pension benefits to wage income in last working period.

used population weights from the 1998 issue of the CPS (Current Population Survey), truncated below age 20, assuming a long-run real income growth rate of 1%.

Calibrated Parameters	Value
ψ	0.867
σ_ξ^2	0.004
$\bar{v}$	0.06
Φ	1.835
μ	0.1
ρ	0.4

We now discuss the calibration of the asset returns and transaction costs. For obvious computational reasons, we have restricted the type of assets in our model to two. The traded asset is supposed to capture the investment possibilities in the public equity and bond market. The return on this composite asset is assumed to be fixed to further reduce computational costs. We want to stress that this assumption does not bias the results in our favor, because in our model there is no motive for holding private equity for portfolio diversification reasons at all. Such a motive might lower the required rate of return on private equity, if the correlation between public and private equity returns was less than perfect. The assumption is justified by the fact that

in the data the index returns to public and private equity are very highly correlated and therefore portfolio diversification is unlikely to constitute a major motive for holding private equity. The annual real rate of return on the fixed asset is assumed to be 5%, in line with estimates of long-run average returns to public equity and bonds reported in the literature (see for example Mehra and Prescott (1985) or Heaton and Lucas (1997)). The deterministic component of the non-traded asset return $\bar{v}$, equal to the unconditional mean of the return distribution, is chosen to imply an empirically plausible share of entrepreneurs in the population. Gentry and Hubbard (2000) consider various definitions of entrepreneurship and report shares of 8.7% to 11.5% in the population with 8.7% being their preferred interpretation.

The total return on non-traded private equity also includes a skill and a business risk component. As pointed out above, the skill component is equal to the stochastic component of the income process. We choose the autocorrelation coefficient ψ, the weight of skills in the return distribution and the variance of the innovation to business risk σ_ξ^2 in order to achieve a relatively wide distribution of possible asset returns. Our baseline calibration sets ψ equal to 0.867, ρ equal to 0.4 and σ_ξ^2 equal to 0.11. These values translate into a span of -56.31% to 94.52% for the unconditional distribution of private equity returns in the calculation using a grid that is 6 standard

deviations wide. Transaction costs are chosen to be of a plausible magnitude compared to wages. We set the start-up cost parameter Φ equal to 1.835, equivalent to about one annual income for a young household. This value is subject to some uncertainty, but our choice seems to be realistic and relatively prudent as Caballero and Hammour (1994) used a value equal to half a year of production costs for manufacturing in their calibration. Costly reversibility is captured by the parameter μ, which we take to be relatively low with a value of 0.1. Also for this parameter estimates in the literature vary widely. Dixit and Pindyck (1998) for example assumed a wedge of 40% in their illustration of the importance of costly reversibility for the behavior of firm investment. Obviously, the calibration of the parameters pertaining to business risk and transaction costs is the most difficult, because very little empirical evidence is available. For this reason, we will provide some sensitivity analyses for the values of these parameters in the following section.

4.1 Pure life-cycle results

Eliminating transaction costs and persistence in business risk allows us to determine the features of our calibrated model implicit in the canonical life-cycle setup. Without persistence in business risk, information about the current realization of business risk does not help in forecasting future business

risk realizations and therefore also the informational assumptions are not relevant. In such a stripped down model, the only reason for observing results different from standard static portfolio choice models is the existence of the liquidity constraint and the fact that wage and private equity returns are highly correlated. This model probably comes quite close to the setup that Moskowitz and Vissing-Jorgensen (2002) had in mind when interpreting their empirical results.

The rate of return difference between the traded and the non-traded asset in this setup is 5.99% over a 3-year period. This is very far from the 10% annually hypothesized by Moskowitz and Vissing-Jorgensen (2002), but in line with other results in the literature. As expected, the index (22.29%) and cross-sectional average (21.75%) return on private equity are basically equal. Since there are no transaction costs and there is no persistence in returns, the only source of divergence of these two measures is the different weighting of individually realized returns implied by different sizes of the asset stocks held by agents. Conditional on holding the non-traded asset, non-traded equity is very concentrated in entrepreneur's portfolios. The average portfolio share is very high at 94%. Portfolio choice is essentially a 0-1 choice due to the observable skill component in returns, which makes returns conditional on a good skill realization very attractive in comparison with the riskless asset and induces agents to shift their wealth holdings into the non-traded asset. An

important implication of this is however, that portfolio shares are extremely volatile. The average number of start-ups per agent (defined as the event that an agent not holding the non-traded asset holds some of it in the next period) is measured at 3.1 start-ups per agent. Average rates of return are independent of the age of the business and due to the liquidity constraint are slightly higher for young agents, than for older agents. In sum, the canonical version of the life-cycle model with a portfolio choice component does not suffice for resolving the portfolio choice dimension of the private equity premium puzzle. The required excess returns are substantial and the portfolio shares are much too volatile.

5 Private equity as a non-tradable asset

Taking into account the importance of transaction costs and incomplete information about private equity returns, the model performs much better. It roughly reproduces the relative size of the private equity market to the public equity and bond market, which Moskowitz and Vissing-Jorgensen (2002) find to be about 1 : 2. Also concerning the wealth distribution our model fits the empirical data quite well. The wealth distributions are generally skewed to the left with median wealth lower than average wealth for any age group and both entrepreneurs and non-entrepreneurs. The share of to-

tal wealth held by entrepreneurs is 38.1%, which corresponds closely to the 37.7% reported by Gentry and Hubbard (2000). As in US data, entrepreneurs are also significantly wealthier on average than non-entrepreneurs. In our baseline calibration, entrepreneurs hold on average 3.9 times the average wealth of non-entrepreneurs, while Gentry and Hubbard (2000) report an average wealth ratio of 6.8 : 1. In general, the consumption profile of entrepreneurs is different from the consumption profile of the general population, with entrepreneurs having increasing rather than hump-shaped consumption profiles. This behavior is consistent with different savings incentives for entrepreneurs and workers, a hypothesis formulated by Quadrini (1999) in his study of wealth concentration, social mobility and entrepreneurship.

We calculate private equity returns by drawing random samples from the population of entrepreneurs and compute them as they are defined in Moskowitz and Vissing-Jorgensen (2002) p. 30

$$ret_{it} = v_{it} + \left(\frac{\text{Value of business at time } t}{\text{Value of original investment}} \right)^{1/(\text{Years since founded})} - 1 \quad (11)$$

where "Value of business at time t" takes into account costly reversibility and "Value of original investment" includes start-up costs.

The cross-sectional average return equals 5.03% annually (16.0% for a 3 year horizon) and hence is almost equivalent to the rate of return on the

riskless asset. The return on an index of all private equity held is somewhat higher and calculated as 7.13% annually (22.9% for a 3 year horizon). At the same time, the cross-sectional ex post-return distribution is very wide, ranging from -64.06% to $+85.2\%$ for a 3 year horizon, with a standard deviation of 0.18. Hence, although the average entrepreneur does NOT earn a rate of return above the public market rate, and holding private equity is associated with substantial idiosyncratic risk, agents are willing to take the risk of entrepreneurship.

Although average returns are low and the volatility of returns is high, entrepreneurs hold large shares of their wealth in private equity of a single firm. The model implies an average of 88.7%, which is quite close to the 82% reported by Moskowitz and Vissing-Jorgensen (2002). Figure 1 shows that, while most agents do not hold private equity at all, those that do have large holdings relative to their own wealth.

A central aspect of the model is the fact that average rates of return on private equity vary systematically with the age of entrepreneurs and the age of the firm. Young agents with high skills require low average rates of return to start a business, since by starting a business they can use their skills more productively. These agents do not know their business risk realization, and they are inclined to take the risk of paying the start-up cost to learn it, since

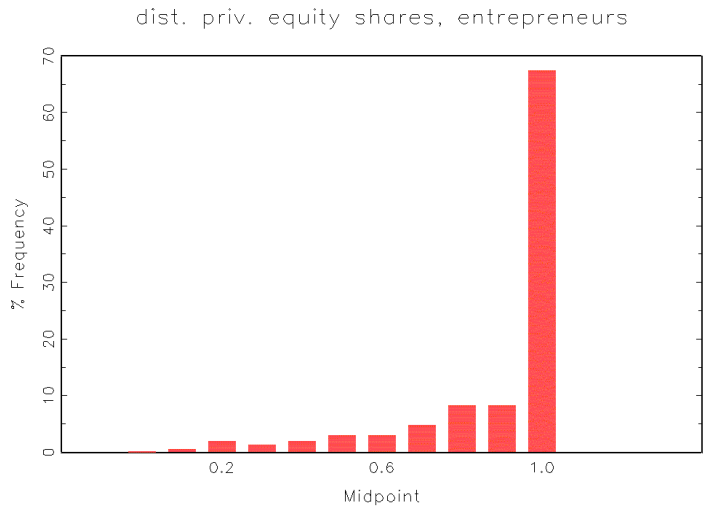

Figure 1: Portfolio shares of private equity

they can reap the benefits of running a profitable business for a long time. Later in life, agents are wealthier on average, but they require larger rates of return to start a business because they are able to exploit high rates of return from a shorter period of time and as retirement nears, they become effectively more risk-averse. The second fact also affects the required return for agents holding private equity already. The reason for this is that when the certain component of lifetime income shrinks and the slope of the earnings profile decreases, agents lower their demand for risky assets (see Campbell, Chan and Viceira 2003). As a result, the average rate of return on private equity increases with the age of the entrepreneur.

The model also implies that young firms have relatively low rates of return

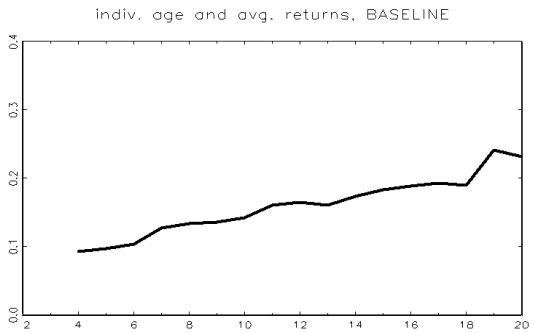

Figure 2: Average returns by agent age

and relatively large failure rates. Average firm age in our baseline calibration is 12.6 years and only slightly higher than the 10.7 years reported by Moskowitz and Vissing-Jorgensen (2002). Median firm age is less than average firm age, indicating that most firms have relatively short lives. In fact, the survival rate of firms above 12 years is only 36.1%, approximately equal to the 34% survival rate for a 10 year horizon quoted Moskowitz and Vissing-Jorgensen (2002).

The following section is dedicated to an analysis of the determinants of becoming an entrepreneur and sensitivity of these results to changes in the calibration of the model and an exploration of the mechanism generating the results presented above.

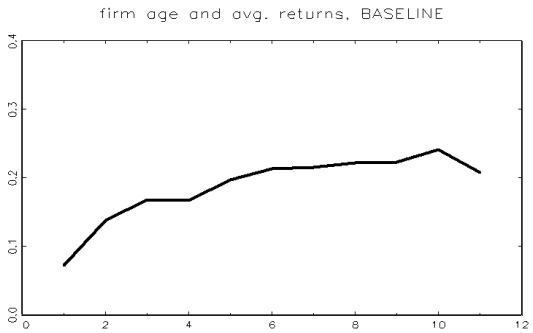

Figure 3: Average returns by firm age

5.1 Determinants of entrepreneurship

The characteristics of entrepreneurs implied by the model are also quite appealing. Entrepreneurs are more skilled and wealthier than the population average. The empirical literature (Evans and Jovanovic (1989), Holtz-Eakin, Joulfaian and Rosen (1994), Hurst and Lusardi (2002) and Hamilton (2000)) has established that large positive income shocks have a positive effect on the probability of becoming an entrepreneur in the future. Our model generates a similar result. Agents that are relatively wealthy are more likely to become entrepreneurs. This effect is particularly strong early in life and young agents might enter entrepreneurship even if their skill level is very low. Figures 4 - 6 show the combinations of skill and wealth that lead to business start-ups for three different ages. At age 2, wealth and skills are positively correlated with trying entrepreneurship. Wealthy agents become entrepreneurs, even

Figure 4: Wealth and skills at start-up at age 2

Figure 5: Wealth and skills at start-up at age 8

when the expected return to private equity is quite low. On the other hand, skilled agents that have high expected returns do not become entrepreneurs, if their wealth is low. While the effect of the liquidity constraint is present for all ages, the former effect diminishes as the agent's age increases. At age 15 (the retirement date), only agents with high skill realizations enter entrepreneurship, in order to exploit their high expected return on non-traded equity.

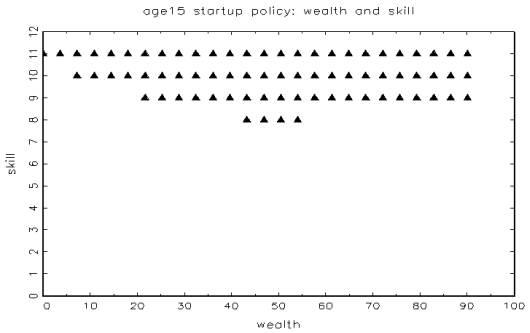

Figure 6: Wealth and skills at start-up at age 15

The reason for the diminishing incentive for wealthy individuals to enter entrepreneurship even if expected returns are low, is that the value of the informational advantage they gain from becoming entrepreneurs diminishes with age. Older agents have less incentive to acquire information on their business risk realization by becoming entrepreneurs, because they cannot profit from the potential wealth increases implied by good business risk realizations as long as young agents - simply because of their shorter horizon. The value of information about the current business risk realization is also sharply reduced, if the persistence of the stochastic process governing it is low. The effect of reducing the persistence of business risk is shown in Figures 7 - 9. The incentive to become an entrepreneur early in life is less wealth-dependent than in the previous case. The effect of wealth also dimishes with age however. As before, the liquidity constraint deters agents

with high skills and low wealth from entering entrepreneurship.

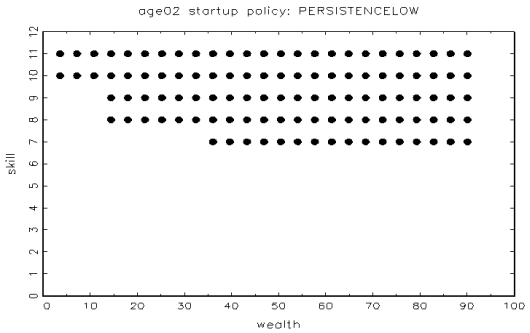

Figure 7: Wealth and skills at start-up at age 2

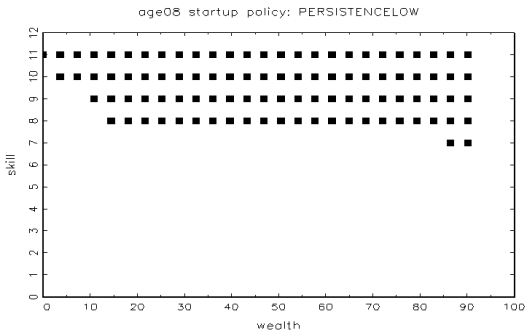

Figure 8: Wealth and skills at start-up at age 8

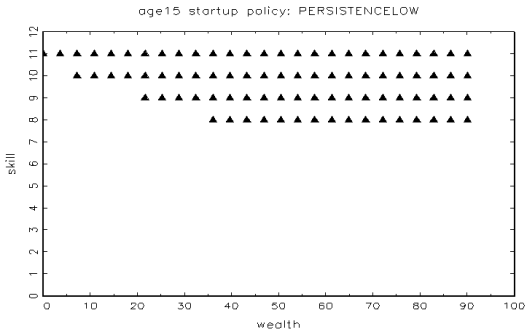

Figure 9: Wealth and skills at start-up at age 15

Exiting entrepreneurship not surprisingly is related more to the business risk realization than to skills or wealth. The decision to exit is also quite independent of age prior to retirement. Only after retirement, more and more agents exit entrepreneurship to invest their wealth into the safe asset. This feature is a consequence of the motive for "strategic risk allocation" outlined in Campbell, Chan and Viceira 2003. The following section will outline the results derived from alternative calibrations of transaction costs and the persistence of business risk.

6 Some sensitivity analysis

In the following, we take the set of fixed parameters and the deterministic component of private equity returns $\bar{v}$ as given and concentrate on the remaining calibrated parameters. Our results are summarized in Table 1,

which gives summary statistics for the average, median and standard deviation of cross-sectional returns to private equity, the return on an index of private equity and the average share of total wealth held in private equity conditional on holding private equity at all.

Table 1:

	Cross-section of private equity returns			Mean	index return on
	Excess mean return	Median return	Stand. Dev.	portfolio share	private equity
BASELINE	0.23%	16.32%	0.182	0.88	23.0%
FIXEDCOSTHIGH	0.27%	17.07%	0.198	0.83	23.8%
DISINVESTHIGH	−2.37%	13.51%	0.186	0.76	23.0%
PERSISTENCELOW	−1.14%	15.65%	0.181	0.80	19.7%

The first column contains the name of the parameter set used. The actual parameter values used for each calibration can be read from Table 2 below.

Table 2:

	ψ	σ_ξ^2	Φ	μ	ρ
BASELINE	0.867	0.004	1.835	0.1	0.4
FIXEDCOSTHIGH	0.867	0.004	3.670	0	0.4
DISINVESTHIGH	0.867	0.004	0	0.25	0.4
PERSISTENCELOW	0.546	0.011	1.835	0.1	0.4

6.1 Alternative transaction cost structures

Varying the structure of transaction costs does not have a large effect on our main results. Neither the presence of fixed start-up costs, nor the presence of costly reversibility are necessary to reproduce the main findings. With respect to other interesting statistics, disregarding either of these costs worsens the performance of our model however. Neglecting start-up costs has the effect of changing the age structure of entrepreneurs and the size distribution of firms. Most entrepreneurs start their business early in life and some keep very small businesses. This is also related to the assumption of non-observable and persistent business risk, meaning that it is worth running a very small, but not profitable business, in order to be informed about business risk realizations. Figure 10 shows that the share of entrepreneurs rises very quickly and is too high for this alternative set of parameters.

Neglecting costly reversibility and setting start-up costs twice as high as before ($\$36,900$) leads to start-ups later in life. The average age of entrepreneurs rises to 60.8 years, from an already relatively high number of 56.6 years in the baseline run. Moskowitz and Vissing-Jorgensen (2002) report an average age of entrepreneurs of 46.5 years, closer to the 50.6 years resulting from the case without start-up costs, but high disinvestment costs. Also, the increased start-up costs lead to a bimodal distribution of portfolio shares

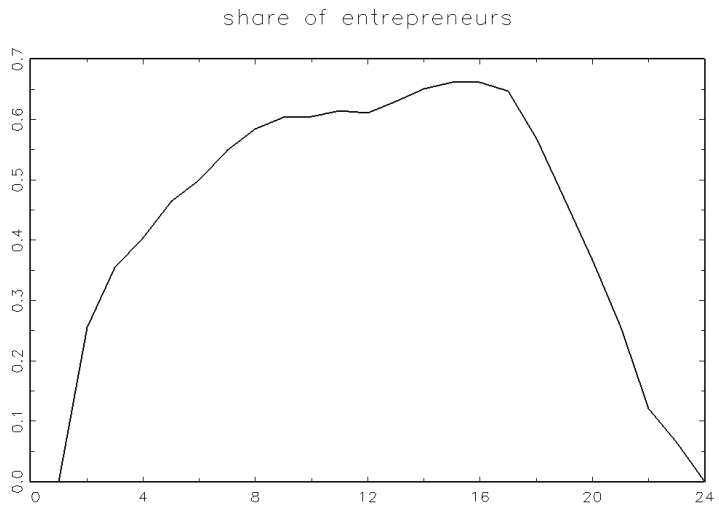

Figure 10: The share of entrepreneurs

(Figure 11), with some agents avoiding to exit the business completely, but rather allocating only a small share of their wealth to private equity in some periods.

6.2 Persistence in business risk

The parameters defining the distribution of private equity returns are hard to calibrate, but at the same time fundamental for the question we would like to answer. Both, the persistence of business risk realizations and the volatility of the private equity return play an important role in the model. Concerning the persistence of business risk realizations, there is no information to be gained from the SCF, since it is not a panel study and few alternative information

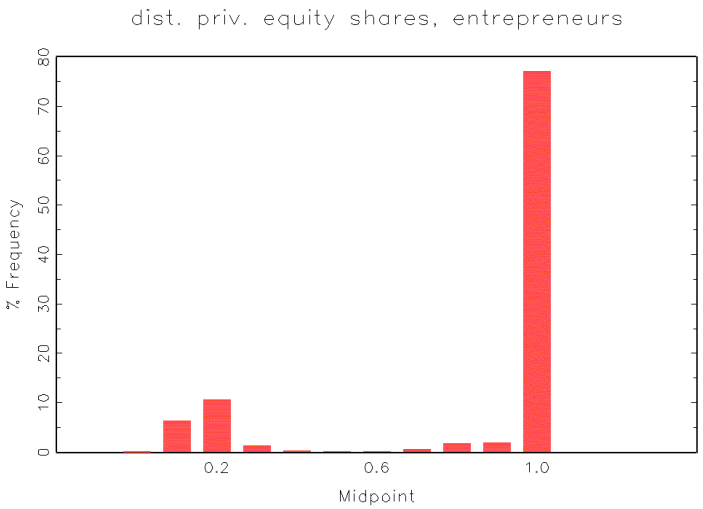

Figure 11: Portfolio shares of private equity

sources exist. Studies (e.g. Quadrini (1999)) on the wealth distribution and on the savings choices of entrepreneurs suggest that the persistence in total returns to private equity is relatively high. In order to check the robustness of results with respect to the persistence in business risk, we have also computed a run with little persistence in business risk, such that the main source of serial correlation in private equity returns is the skill component. Results are robust in the sense that our main findings do not change even if business risk is less persistent. In fact, average private equity returns are even lower than in the baseline scenario because entrepreneurs do not exit as quickly after observing a bad business risk realization. However, big changes occur in a different dimension. The average lifetime of a private firm increases

drastically if business risk is less persistent and the shape of the survival rate distribution changes. Figure 12 shows the histogram of firms' lifetimes for the case of low persistence in business risk, labeled PERSISTENCELOW. Very few firms live for less than twelve years, and the average is 10.16 periods, corresponding to approximately 30 years in real time. This is much shorter than estimates in the literature reported above.

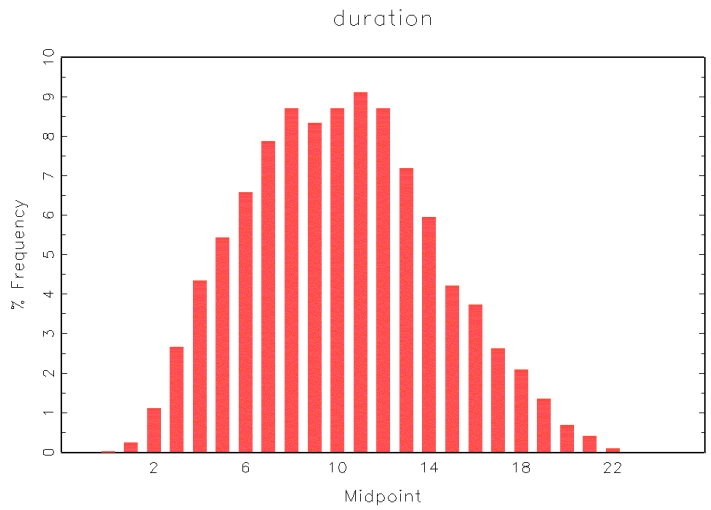

Figure 12: Histogram of firm duration

7 Conclusions

We have shown that the characteristics of private equity returns and portfolio allocations documented by Moskowitz and Vissing-Jorgensen (2002) emerge

from a simple model of occupational choice over the life cycle. More specifically, the average return on private equity is not higher than the return on the riskless asset, but much more variable and agents holding private equity allocate a large share of their total wealth to it. In our model, average private equity returns are increasing in both, the age of the firm owner and the age of the business. Transaction costs associated with investing into and disinvesting from a private business lower the average cross-sectional return on private equity in comparison to the return on an index of private equity and make these returns roughly equal to the return on a riskless asset. Since returns to private equity are not directly observable by non-entrepreneurs, average firm life is relatively short and entrepreneurs with bad business risk realizations exit quickly. The implied survival rates of firms are in line with estimates in the literature.

Our baseline calibration also yields attractive results with respect to the wealth distribution. Entrepreneurs are on average 3.9 times wealthier than non-entrepreneurs. The sensitivity analysis shows that results are quite robust to different specifications of transaction costs. However, the existence of transaction costs and high persistence in business risk realizations are necessary ingredients to generate these results. The main determinants for becoming an entrepreneur are the agent's age, wealth and skills. Young agents

with high skills or high wealth hold private equity, despite the low average return realization. Older agents are less likely to start-up a business, they do so only when the expected rate of return is quite high. We have largely neglected tax-issues so far, but intend to study the impact of tax systems on the propensity of becoming an entrepreneur in the future. We also think that this framework would be a good basis for studying the decision to convert private equity to public equity through an initial public offering (IPO).

8 Appendix

8.1 Numerical procedure

The numerical method we use is finite state, finite horizon dynamic programming. We discretize the state space, defined over asset stocks and uncertainty states. Fineness of the grid basically determines computation time, which is linear in lifetime and quadratic for each asset in the number of gridpoints considered.

Given some discretization of the state space, for each point in time $t = 1, 2, ..., T$, optimal policy rules are computed, describing the optimal adjustments in the levels of the two assets in the portfolio between t and $t + 1$. These rules are made given the level of each asset when taking a de-

cision and conditional on the states of uncertainty over idiosyncratic income risk and business risk observed prior to the decision. Optimal decisions are recursively computed, starting by setting the level of assets in $T+1$ to zero, and then updating the continuation value at each point in time, according to the optimal decisions between the final period and the current period. While those optimal policies are found by going backwards from the end of life to the beginning of life, the actual solution paths for each individual $i \in \{1, 2, ..., I\}$ are found by applying the optimal policy rules going forward from the beginning of life to the end of life. At the very beginning of life, individuals differ only according to the amount of initial wealth they are endowed with. During lifetime they differ according to the their individual histories of idiosyncratic income shocks and business risk realizations. This explains the distribution of solution paths over assets and consumption in the population of I individuals.

Having obtained the solution paths for the entire population according to individual draws of initial wealth and shock histories, it is straightforward to aggregate and to calculate statistics for the population. The basic problem is to translate a set of longitudinal observations for a population of individuals into a representative cross-section at a given point in time. First, we make sure that the age-structure in the observed population (as taken from the CPS) corresponds to the age-structure of the simulated economy. Therefore,

starting from the longitudinal simulated paths for the I individuals, we split the sample into T age groups, such that for each age the its share in the population corresponds to its share in the CPS. Second, the simulated values for each age group are translated into a common unit of account, by adjusting for the aggregate real income growth rate.

Aggregation and the number of individuals do not increase computation time by much, once the optimal decision rules are computed, the marginal computational cost of further numerical analysis is low.

The application of dynamic programming techniques in this case requires a very special structure, due to the informational assumptions specific to returns on private equity. In the computation of optimal decisions we must distinguish two groups of agents. The first group are the entrepreneurs, who can observe business risk, for whom we compute optimal decisions conditional on the observed realizations of uncertainty in both the dimension of business risk and idiosyncratic income risk. The second group are the non-entrepreneurs, defined as holding zero entrepreneurial assets, for whom policy rules are computed conditional on the observed realizations of idiosyncratic income shocks only, while using the unconditional distribution over business risk. When simulating the optimal paths for the individuals those two sets of rules must be applied appropriately, depending on whether at the point of decision the individual does hold the entrepreneurial asset or not.

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