Are minimum return guarantees really as expensive?

Regulation in the German Pension Reform of 2001

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Abstract:

This paper analyzes the costs and benefits of minimum return guarantees for capital invested in pension plans as introduced by the German pension reform of 2001. Data on stock and bond returns between 1954 and 2001 are used to compare different investment strategies that insure the portfolio against losses with a pure equity investment strategy. Using historical and simulated data, final wealth levels for individuals with different investment horizons are compared. Certainty equivalent wealth levels for different degrees of risk aversion are calculated. The capital guarantee reduces mean returns by about 0.5 percentage points, while reducing the portfolio risk. Investors with reasonable degrees of risk aversion prefer the guarantee.

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1 Introduction

In order to cope with the problems of population aging, many governments in the OECD countries seek to reform their pension systems through a reduction of the fraction of pension benefits that are financed by the Pay-As-You-Go system.[1] The most common reform measures are lowering replacement rates and raising the official retirement age. Lower replacement rates of the PAYG pension system can be stocked up through voluntary or compulsory contributions to funded pension plans that are usually tax-advantaged. The tax preferred pension plans are usually subject to regulations, which differ across OECD countries. Whereas Anglo-Saxon countries prefer "prudent person rules", many countries in continental Europe resort to extensive regulatory frameworks using quantitative rules.

Germany is an example of a country that faces a dramatic population aging and has a very large PAYG pension system. In the year 2003, public pensions amount to 13 percent of GDP. The PAYG pension benefits make up 80 to 85 percent of the retirement income of the average German household (see Börsch-Supan, Reil-Held, Schnabel 2001). The German pension Reform of 2001 will gradually reduce the net replacement rate of the public pension system from around 70 to 64 percent[2]. Further reductions are inevitable given the severity of population aging. Parallel to the reduction of replacement rates, the reform of 2001 introduces a tax-preferred[3] treatment of contributions to private pension plans. The German pension reform requires (among many other rules) a nominal capital guarantee for pension contributions, which is equivalent to a nominal zero percent return rule. The guarantee has to be met at the end of the accumulation phase. There are at least two motivations for this regulation. First, it is argued that uninformed consumers should be protected by a minimum return rule. Second, the down side risk of investment strategies is limited due to the welfare system which guarantees a minimum income at old age. This may encourage inefficiently risky investment strategies; a capital guarantee limits this moral hazard problem. In this paper we assess the costs of this capital guarantee in terms of final wealth levels and certainty equivalents for different degrees of risk aversion.

The costs of quantitative investment regulations have been the subject of much controversy in the literature (see Davis 2001a, 2001b). The requirement of a minimum return guarantee

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[1] Whether this is a means of alleviating the demographic burden is debated. This paper does not intend to contribute to this discussion.
[2] This net replacement rate is calculated as the net pension based on 45 years of average earnings divided by the average earnings in a given year.
[3] The term "preferred taxation" refers to different tax deductions and advantages associated with occupational or private pension schemes. This may include deferred taxation (the German case) as well as special tax rates.

is one of the most widely discussed forms of regulating pension plans, especially in the light of recent reforms in Latin American countries which have switched to funded pension schemes (see Vittas 1998). The pension literature is divided regarding the benefit or harm associated with minimum return guarantees. For one, the issue of appropriate pricing of guarantees in terms of option pricing theory is controversial (see Fischer 1998). Moreover, if there are minimum guarantees and the portfolio choice is not restricted, the moral hazard problem might become very costly. Smetters (2002) suggests guaranteeing a „standardized portfolio“ and letting individuals carry the risk of choosing a non-standard portfolio. Furthermore, he proposes to tax portfolio returns in good states of the world and subsidize returns in bad states of the world. Jensen and Sorensen (2000) consider minimum return guarantees for pension plans in terms of an embedded put-option which the individual is forced to buy, thus, indirectly restricting the portfolio choice. They find that moderately risk averse investors suffer utility loss if forced to purchase such a guarantee.

Lachance and Mitchell (2002) have considered the costs of quantitative portfolio rules. They analyze standardized portfolios, which invest 100 % in equity, 50% in bonds and 50 % in equity, and 100 % bonds. However, these portfolios are not designed to guarantee the invested capital. In contrast to Lachance and Mitchell (2002), we consider an explicit guarantee at expiry of the contract. Maurer and Schlag (2002) consider the effect of money back guarantees in the context of the German pension reform. They use shortfall risk as well as regulatory capital that is needed for solvency regulation as the main indicators of risk. They do not take individual attitudes towards risk into account. Maurer and Schlag (2002) consider several investment strategies and find that the likelihood of regulatory capital charges diminishes with the length of the investment horizon. Therefore hedging costs are relatively low. Moreover, shortfall risk is almost negligible in the presence of hedging strategies.

Our paper is concerned with the effects of the capital guarantee introduced by the pension reform of 2001. We estimate final wealth levels and compare individual utilities based on certainty equivalents for different degrees of risk aversion and different time horizons. We first discuss several investment strategies that guarantee a zero percent return and select two simple rules for further consideration which entail very low transaction costs. As a benchmark we use a pure equity strategy.[4] For each investment strategy we calculate the rates of return and the final wealth for different dates of retirement. The dates of retirement can be thought of as defining a cohort. We also consider different investment horizons

[4] Note that this pure equity strategy is very extreme, since all stocks are sold off at the date of retirement. Thus there is no diversification across exit periods. Very simple rule of thumb strategies can reduce the risk greatly (e.g. selling off the stocks over, say, 12 months instead of in one single month).

ranging from a minimum of 10 years to a maximum of 35 years. The empirical analysis starts with the historical data on equity and bond returns. We find that a pure equity investment strategy has not always dominated the guarantee strategies for all investment horizons. For shorter periods, strategies that guarantee the invested capital have outperformed the pure equity strategy for a few cohorts – notably cohorts retiring during a bearish equity market. In a further step, the equity returns are modeled as a generalized autoregressive conditional heteroscedasticity process (GARCH (1,1)). Using the econometric estimates, a Monte Carlo simulation is performed to evaluate the performance of the different investment strategies. The simulation shows that the probability of bad draws is not negligible: with a 20-year investment plan the probability of a negative return is estimated to be 2.4 percent.

Finally, a constant relative risk aversion utility function is considered to take account of the individuals' attitude towards risk. The analysis shows that for reasonable degrees of risk aversion, individuals prefer insured investment strategies. In the absence of moral hazard and information problems, those individuals would buy products with insurance. For risk neutral investors with an investment horizon of 35 years the costs amount on average to 0.5 percentage points of the expected annual return. Using certainty equivalents for final wealth, the costs of the capital guarantee for risk averse individuals are calculated. We conclude that the capital guarantee implies relatively low costs for individuals with a low coefficient of relative risk aversion and are presumably not harmful for the largest part of the population.

The paper is organized as follows: in the next section we describe the basics of the pension reform and the regulation of the funded pension plans. In Section 3 we discuss the different investment strategies. Section 4 presents the performance of the investment strategies based on historical data. Section 4.2 is concerned with the analysis of simulated data and section 4.3 evaluates the costs associated with a capital guarantee using the concept of certainty equivalent final wealth. Section 5 concludes.

2 Tax Preferred Pension Plans and Regulation

The German pension system consists of three pillars. The first pillar is the PAYG pension system, which is compulsory for about 90 percent of the work force.[5] It is currently the predominant source of retirement income. The second and the third pillar are voluntary. The second pillar comprises occupational pension schemes, which are usually defined benefit plans. It is mostly the large firms that provide occupational pension schemes (see Bundesverband Deutscher Banken 2000). The third pillar used to comprise voluntary, individual contributions – mostly to life-insurance plans. Contributions to life-insurance plans are usually made out of taxed income with tax-free accumulation of interest and capital gains.

2.1 Taxation and Subsidies

The pension reform of 2001 will reduce the net replacement rate of the PAYG pension system. At the same time, a *preferred tax treatment* of private and occupational pension plans is introduced. For private pension plans maximum contribution of 4 % of the social security ceiling (53.376 € in 2001 in West Germany) and no more than 2100 € will be deductible from income taxation. Actually, the maximum rate in 2002 is only 1 % and will increase in steps of one percentage point every two years until the full level of 4 % is reached in 2008. Interest and capital gains are also exempt from income taxation. Withdrawals during retirement (as an annuity or as lump-sum payment[6]) will be fully taxed. However, the personal income tax allowance is such that average and marginal tax rates are very low for a median pensioner. Thus, it may be justified to talk of a tax subsidy.

In order to encourage the participation of low-income groups, a minimum subsidy is granted, which depends on marital status and number of dependent children only. If the income tax savings exceed the flat subsidy, a tax rebate is granted. Eligibility will require a minimum contribution of 4 % of labor earnings (of up to 54.000 €). In the year 2008, the subsidy will reach 154 € per person and year and 185 € per dependent child. In 2002, the subsidies are only one quarter of the final amount and will be raised in two-year steps until 2008. Also, during the phasing-in period, the tax advantages and the minimum contributions to be eligible for subsidy and tax deduction are lower (e.g. only 1 % of earnings in 2002).

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[5] There are basically two different forms. The largest one is financed by a pay-roll tax with a federal subsidy of about 30 percent of the budget. The smaller one is only for civil servants and is completely tax financed.

6 Lump-sum withdrawals of up to 20 % are allowed at the time of retirement for some pension instruments (such as investment funds).

Tax preferred pension plans can belong to the second pillar (occupational pension) or to the third pillar (individual pension plan). Occupational pension schemes are favored by the pension reform, since contributions to these plans up to 4 % can be made from pre-tax income in 2002. Furthermore there are three ways in which occupational pension schemes are tax-preferred. Apart from the tax-free level of up to 4 % of the social security limit income, a further 1752 € can be contributed with a flat tax rate of 20 % and grants as well as income tax rebates can be benefited from if contributions are paid from net salary.

2.2 Certification of Pension Plans

Together with the pension reform of 2001, a regulatory framework was introduced in a separate law (Altersvorsorge-Zertifizierungsgesetz). Only contributions to *certified* pension plans are subject to deferred taxation. In August 2001, a new federal institution was created which is part of the (old) institution that regulates insurance products and companies. This institution certifies pension plans. Among other rules, in order to be certified, pension plans have to provide a guarantee that at the time of retirement at least the accumulated gross contributions are paid out. This is equal to a nominal minimum return guarantee of zero percent.

This guarantee has been criticized as being too costly compared to “Prudent Person”-rules (see for instance Davis/Steil 2001). The European Union also opposes minimum return guarantees (see European Union 1999). In June 2002 the European Union has agreed on a pension fund directive that explicitly states “prudent person rules” as the standard for investment practice for occupational pension schemes (see EFRP 2002). However, the minimum return rules may serve important goals of economic policy. They may reduce moral hazard (especially in conjunction with the annuity requirement) and they may protect uninformed consumers. Thus, regarding the German pension reform of 2001 the question arises how costly these kinds of restrictions really are.

3 Investment strategies for minimum return guarantees

There are various ways in which minimum return guarantees can be implemented into the investment strategy of asset managers. These different strategies can be either of a static or a dynamic nature. In the following, we briefly survey the main strategies that are applied in practice.

3.1 Investment strategies

Static strategies are generally simple to implement and associated with low transaction costs. Dynamic strategies are more elaborate and more difficult to implement at the price of higher transaction costs. This is due to the fact that dynamic strategies are associated with a higher portfolio turnover than static strategies. Dynamic investment strategies imply that the portfolio structure has to be adjusted to movements in asset prices as soon as these movements occur. In the static strategies the transactions are limited to the monthly investment of fresh contributions. Since the goal is to guarantee a zero percent return at the expiry date of the contract, investment strategies must use this level as a floor. The following static investment strategies are suitable to provide the capital guarantee.

Stop-loss

The stop-loss strategy starts with a pure equity portfolio. The designated minimum final wealth value is the floor of the portfolio. If the market value of the equity portfolio drops to a level (the stop-loss point) such that the minimum final wealth value is put at risk, all equities are sold and substituted by bonds that are kept until the end of the investment period. This insures that the minimum value is reached upon expiry. This strategy is very extreme in that the portfolio structure is either 100 % stocks or 100 % bonds. In the latter case only the minimum final wealth level is reached. If the stop-loss value is reached, transaction costs are incurred for the restructuring of the portfolio (for a detailed discussion see Bird/Dennis/Tippett 1988).

Protective put

The protective put strategy insures the portfolio through the purchase of a put-option on the equity in the portfolio (usually index certificates). This strategy is inhibited by the fact that often puts with the appropriate duration are not available or too expensive.[7] For this reason, investment companies tend to use the synthetic put strategy, which is described below.

Bond-equity

The bond-equity strategy uses bonds to guarantee the desired minimum value of the portfolio upon expiry. The remaining capital is invested in equity. In its extreme variant the share of bonds is determined such that the guarantee can be fulfilled even if the stocks become worthless. Since it is not necessary to restructure the portfolio to match the guarantee, man-

[7] Over the last decade markets for put options that are suitable for pension plans have grown in the US and are likely to also develop in Europe in the future once partially funded pension systems have grown.

agement costs are very low.[8] Moreover, transaction costs are incurred only when the monthly contributions are invested.

Bond-calls

This strategy differs from the bond-equity strategy in that calls are bought instead of stocks. This provides a higher leverage, since calls are cheaper to purchase than the underlying equity. The downside risk is limited. However, in the case of a long-run bear market the calls become virtually worthless and the strategy only yields the guaranteed level.

Bond-futures

The strategy is similar to the two previous strategies with the difference that futures are bought with the capital that can be exposed to the risky asset. Since contributions are paid over a long period of time, a small investment in bonds suffices to provide the capital guarantee. Thus, a relatively large fraction of contributions can be invested in equity. The longer the investment horizon, the smaller has to be the fraction of bond investment.

The simple stop-loss strategy is not considered, since it generates either the same results as the pure equity portfolio benchmark ("good times") or the minimum guarantee ("bad times"). The protective put strategy is not considered due to the lack of appropriate and inexpensive put options in the market. Bond-call and bond-futures strategies are not considered, as the available time series data is too short to draw any valid conclusions. Therefore, bond-equity strategies will be used to evaluate the performance of pension products with a capital guarantee. A pure equity portfolio is used as a benchmark in all scenarios.

Dynamic investment strategies respond to a changing market environment by reallocating the portfolio of equity and bonds. There are several strategies that could be used to insulate a portfolio against possible losses. One strategy that is of particular interest is the synthetic put strategy.

[8] There may be additional management and transaction costs in case the stock portfolio itself is managed actively. However, for large and broad based *index* funds, the total expense ratios are less than 25 basis points in the U.S. (Poterba and Shoven 2002).

Synthetic put

This strategy synthesizes a bond-put strategy by using the formula introduced by Black and Scholes (1973). In order to follow the synthetic put strategy, the portfolio has to be adapted to changes in the parameters especially equity prices, this entails high transaction costs (see also Aschinger 1993). As the transaction costs that are incurred by this strategy are substantially higher than the transaction costs that are associated with a pure equity strategy, this strategy is not considered in this paper.

In our analysis the simple bond-equity strategy is used to insulate the portfolio against nominal losses. The reason for this is twofold. For one, dynamic strategies are associated with too high transaction costs and are complicated to implement. Secondly, the more elaborate strategies entail financial instruments such as long-term put options that are either expensive or unavailable in the German capital market.

4 Analysis of Historical Returns, Simulation and Certainty Equivalents

The empirical analysis is based on monthly data on German stocks and bonds over the period 1954 to 2001[9]. As a measure of stock market performance we use the DAX 30 performance index of the largest listed German firms (measured by market capitalization). This index is chosen, as the DAX 30 performance index comprises about 90 % of market capitalization of all German stock indices.[10] A large market capitalization is necessary, as institutional investors cannot trade in small markets, as they would move prices too much. The last trading day of a month is used. As a measure of bond market performance we use the Bundesbank index of government bonds with at least 3 years to maturity and durations

[9] For the time period from 1987-2001 data of the DAX 30 performance index is used, for the period 0f 1954-1987 data of estimates of DAX returns which have been kindly supplied by the chair of Prof. Stehle (Humboldt University of Berlin) is used. For a detailed analysis of this data see also Stehle 1996. Note that the 2002 version of these data was used.

[10] In December 2002, the market capitalization of all German performance indices (DAX, MDAX, SDAX, NEMAX) was 331.971 million €. The market capitalization of the DAX 30 performance index was 294.604 million €. The second largest index the MDAX which comprises the 70 stocks which are listed in the DAX 100 but not in the DAX 30. The market capitalization of the MDAX in December 2002 was 28.030 million €. Therefore the DAX 30 comprised 88,7 % of total market capitalization.

of more than 4 years.[11] The geometric average of bond returns over the entire period under consideration has been 6.8 percent in nominal terms.

The distribution of monthly stock returns is depicted in figure 1. The excess kurtosis is 1.47 (compared to the kurtosis of the standard normal of 3). The null-hypothesis of a (unconditional) normal distribution has to be rejected based on the Jarque-Bera Test. The fat tails of the empirical distribution of stock returns are typical for financial data (see Lo, McKinlay, Campbell, 1997, 16). This is important for the choice of an econometric model, which has to be able to deal with this feature of the data.

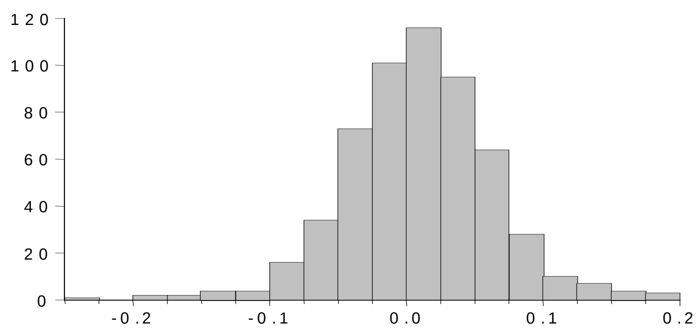

Figure 1: Distribution of historical DAX returns.
Note that the return is calculated as: $r_t = (DAX_t - DAX_{t-1}) / DAX_{t-1}$.

We consider an investor who is saving for retirement. The annual contribution to the pension plan is fixed at 2.000 € and is paid in monthly rates. This is approximately the maximum tax-exempt amount. We consider the following three investment strategies:

- Pure equity investment into the DAX performance index (“strategy 1”).
- Investment of a fixed share in bonds at a risk free return of 6.8 %. The share of bonds depends on the investment horizon and is calculated such that the zero-percent return guarantee of the portfolio is fulfilled at the time of retirement (“strategy 2”).
- Investment of a variable, rising share in bonds at a risk free return of 6.8 %. The initial share of bonds also depends on the investment horizon and is calculated such that the zero-percent return guarantee of the portfolio is fulfilled at the time of retirement. The share of equity investment is highest at the beginning and is declining to zero percent at the end (“strategy 3”).

[11] This index is calculated by the Bundesbank on a monthly basis and provides data starting in 1960. We have used the geometric average of nominal returns of German govt. bonds over the last 40 years.

The first strategy has the highest expected return but is also associated with the highest risk, although the "cost-average-effect" reduces the risk considerably compared to a single investment. The cost-average effect ensures that a large number of stocks is bought in times of low stock prices and a small number of stocks is bought in times of high stock prices. However, the pure equity strategy is still very vulnerable to low stock market valuations at the time of retirement, since we assume that the entire capital is withdrawn as a lump-sum upon retirement (and then transformed into an annuity).

The fraction x of the contribution that is invested in bonds in the case of the second strategy is calculated as follows:

$$(1) \qquad \sum c = \sum_{t=1}^{n} x(1+r)^t ,$$

where c is the contribution per period, n is the time of retirement, r is the interest rate, and t is the respective period.

In the third strategy, the fraction of bonds is calculated such that each monthly investment in bonds will equal 166,67 € (the monthly contribution) at the expiry date. This will satisfy the capital guarantee. Thus the fraction that has to be invested in the safe asset is calculated as follows for period t:

$$(2) \qquad x_t = \frac{c}{(1+r)^t} ,$$

where x_t denotes the part of the contribution that is invested in bonds. The fraction of the contribution that is invested in bonds rises over time and reaches 100 % in the last month. Therefore, a relatively large fraction of total contributions is invested in equity at an early stage of the pension plan. This has the effect that the risk of low returns on equity is minimized with longer duration. Note, that this third strategy can only guarantee the invested capital if the individual stays with the particular pension plan until the time of retirement.

For strategies 2 and 3, figure 2 shows how the percentage invested in equity varies with the duration of the pension plan. The left bar displays the constant fraction of bond investment (strategy 2). The right bar displays the initial fraction of bond investment which is gradually reduced to zero by the time of retirement (strategy 3). Note that the share of bonds depends heavily on the rate of return of government bonds. The higher the interest rate, the less money has to be invested in bonds in order to satisfy the capital guarantee, and the higher the share of equity investment.

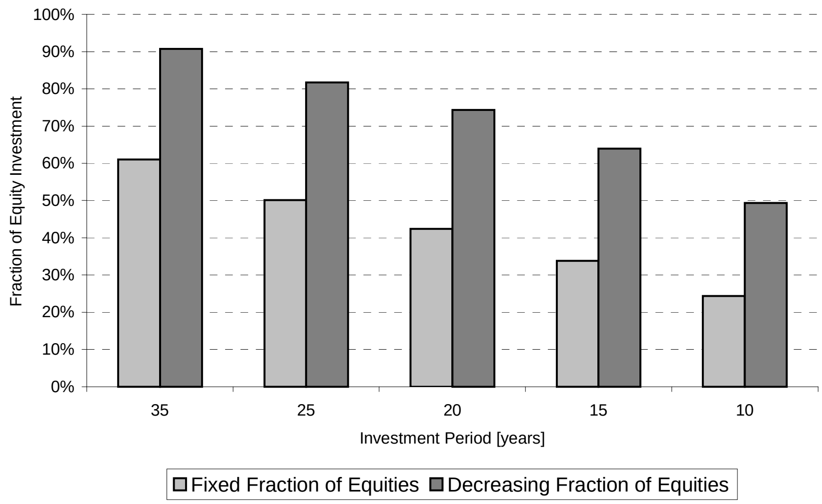

Figure 2: Percentage of monthly contributions invested in equity. Source: own calculations.

4.1 Individual Analysis

Assume that the age at the time of introduction of the pension plan, and therefore the length of individual contributions to the partially funded element of the pension system up to the time of retirement, varies. This is introduced to mimic the fact that the partially funded pension system in Germany is introduced at a specific point in time from which on all working individuals are expected to invest in the partially funded pension system and their PAYG pension benefits are gradually lowered. Thus some individuals who are young have the chance to contribute to the partially funded pension plan over their whole working life, while others who are older only have the chance to contribute to the pension plan for part of their working life. This fact does of course influence the returns that can be generated by investing in the funded element of the pension system as investments in equity are less risky the longer the investment horizon of the individual. On the other hand, as pointed out by Bodie (2002), longer investment horizons do not automatically provide protection from low returns on stock market investments. Rather, while the likelihood that the return of stock investment outperforms investments in the risk free asset rises, there still remains a positive likelihood that stocks under-perform. This underperformance can be very severe with long investment horizons putting savings for old-age security at risk.

The individual enters the respective pension contract on the 1st of January of a specific year and payout of benefits starts on the first of February either 10, 15, 20, 25 or 35 years later. Consider an investment period of 35 years. An individual enters the plan in 1955, the returns of the respective pension plans are compared to an individual with the same characteristics as the first individual the only difference being that he enters into the pension plan in later periods and stays with the plan until the time of retirement 35 years later.

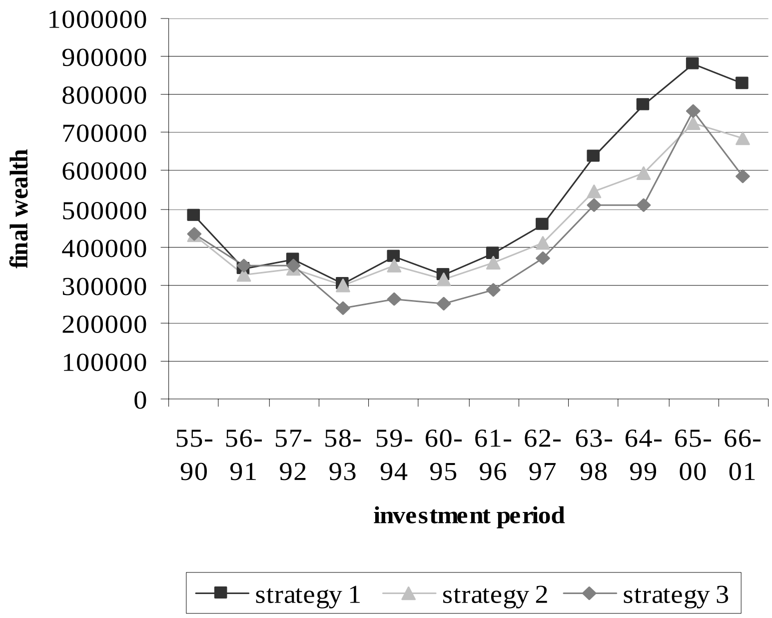

Figure 3: Final wealth after 35 years; Source: own calculations.

An individual who entered the plan in 1955 would have generated the largest wealth level upon retirement if he had chosen a plan that solely invests in equity, while plans that use strategy 2 or 3 would have generated roughly the same final wealth values. Had the individual entered into the plan one year later he would have fared best if he had chosen strategy 3 and invested his contributions in bonds and equity and would have fared worst if he had chosen a pension plan which uses strategy 2.

For an individual who paid into a pension plan using either strategy 1, 2 or 3 and started contributing between 1957 and 1966, the pure equity strategy would have been the best choice[12]. However, the difference in return between the equity strategy and strategy 2 and 3 is sometimes as low as 0.15% higher average annual return of pure equity investment compared to strategy 3 (for instance the investment period 1966-2001).

Now consider the case in which individuals have an investment horizon of 25 years, each individual enters the respective type of plan either in 1955 or any of the subsequent years

[12] This can be explained by the well-known equity premium first discussed by Mehra and Prsecott (1985)

between 1955 and 1976. This generates 23 different sample observations for each type of pension plan.

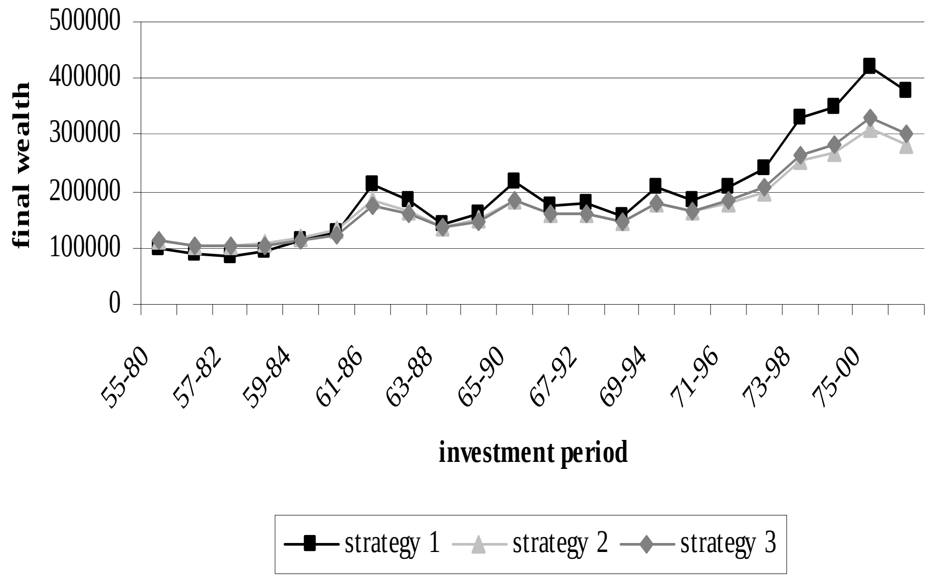

Figure 4: Final wealth after 25 years; Source: own calculations

It can be discerned that for individuals with an investment horizon of 25 years, the final wealth levels that can be generated by the respective investment strategy are quite close. For pension plans with 25-year duration that have been established before 1961, nominal return strategies have been superior to pure equity investment. Moreover, a pension plan of type 3 seems to be superior to a protected plan of type 2.

Figure 5 below depicts final wealth values for an investment period of 20 years. If the capital had been invested for only 20 years, a pension plan with a guaranteed return of 0 % would have beaten pure equity investment on several occasions. These were associated with the stock market crash of 1987 as well as economic slump and oil crisis in the seventies. Although stock markets were booming over the 1990s the stock market decline that started in 2000 and is still continuing due to stagnation of economic growth and the fear of recession, shows that it is quite reasonable to assume that stock market turbulence can occur any time. Also note that again a guarantee with a rising proportion of contributions invested in bonds fares better for most cohorts than the constant proportion plan in a friendly stock market environment.

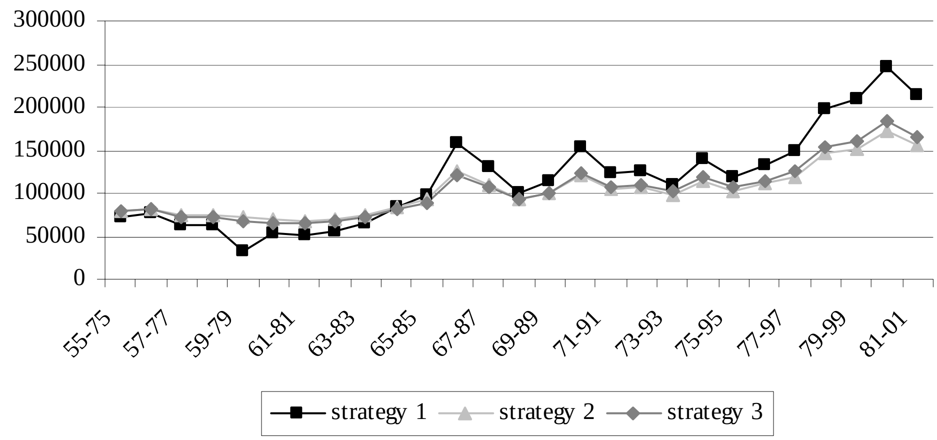

Figure 5: Final wealth after 20 years; Source: own calculations

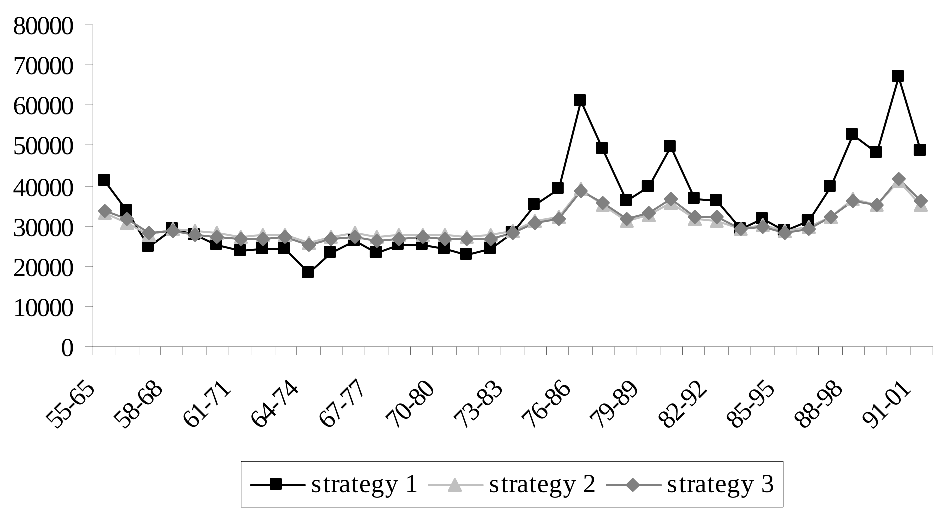

Figure 6: Final wealth after 10 years; Source: own calculations

Finally we look at individuals who have an investment horizon of only ten years. With this relatively short investment horizon, there are even more instances in which the pension plans with a capital guarantee have generated better returns than pure equity investment.

In general, guaranteed pension plans offer returns that are less volatile than pure equity investments. They also appear to protect effectively from economic slump and stock-market turbulence. A pure equity investment strategy seems to be most appropriate for long investment horizons with flexible time of retirement.

4.2 Estimation and Simulation

The question arises of whether the strategies considered would have generated similar results had they been subject to different market conditions. In order to draw more general conclusions, we turn to Monte-Carlo experiments in which we simulate paths for stock market returns and evaluate the alternative investment strategies. We first estimate an econometric model for the stock market returns based on the DAX 30 returns from 1954 to 2001. We use the estimated parameters of the model to simulate returns.

In order to model the fact that volatility in stock market data is generally not constant over time, but is rather distributed in clusters, a GARCH (1,1) (general autoregressive conditional heteroskedasticity) process is estimated. Furthermore a GARCH representation has the advantage that the fat tails that are evident in the distribution of returns can be depicted more accurately.

The model used determines the mean equation as:

(3) $$y_t = c + \varepsilon_t$$

The function is written as a function of a constant c with an error term ε_t.

The variance equation is determined as:

(4) $$\sigma_t^2 = \varpi + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

The conditional variance σ_t^2 is a function of the mean ϖ, the ARCH term ε_{t-1}^2 that is determined by news about volatility in the previous period, and the GARCH term σ_{t-1}^2 that is lasts periods forecast variance (see Bollerslev, Chou, Kroner 1992).

Estimation of the GARCH model of returns of the DAX performance index generate the following model specification:

(5) $$y_t = 0.00743 + \varepsilon_t$$

$$\sigma_t^2 = 0.000105 + (.10421)\varepsilon_{t-1}^2 + (.866991)\sigma_{t-1}^2$$

Based on these results; the simulation proceeds as follows. The GARCH specification is used to generate simulated returns for the respective investment periods. Innovations are normally distributed. 1.000 replications are generated for sample sizes of 420, 300, 240, 180 and 120 months. A pure DAX investment strategy (strategy 1) with an investment horizon of 10, 15, 20, 25 and 35 years is compared to an investment strategy that invests a constant fraction of contributions in government bonds to guarantee the capital that is paid in over the investment period and the remainder in equity (strategy 2). Again, this is done by comparing final wealth levels that can be reached by following the respective investment strategy. The final wealth level that is reached at the end of an investment period of 35 years is classified and displayed in the graphic representation below:

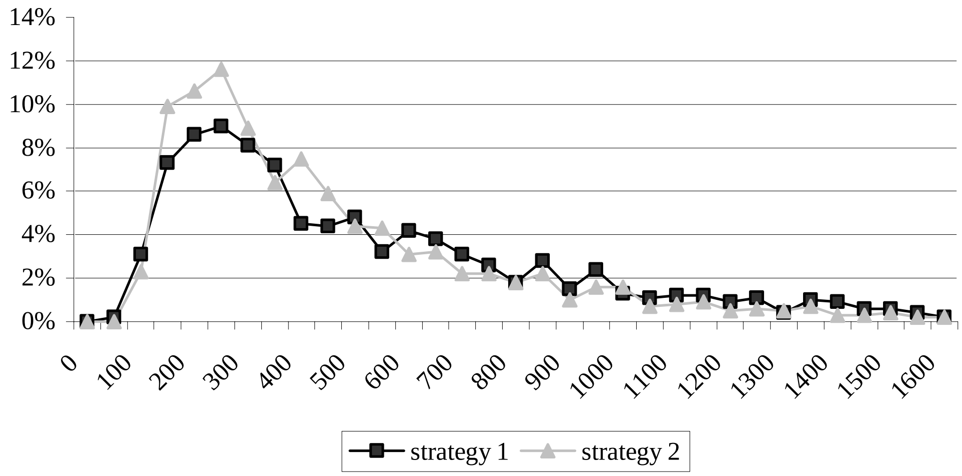

Figure 7: Frequency of final wealth values after 35 years in 1000 €. Source: own calculations.

The black line represents the investment strategy which only invests in the DAX 30 performance index (strategy 1) and the light gray line indicates the investment strategy which provides a capital guarantee (strategy 2).[13] The investment strategy that guarantees 0 % return on capital has a minimum final wealth level of between 150.000 and 200.000 € with a probability of 2 %, whereas final wealth values without guarantee lie in this class with a probability of 3 %. The mode for the guaranteed investment is in the class between 250.000 and 300.000 € with a probability of 11.8 %, the mode of the pure equity investment strategy is in the same class with a probability of 9 %. The probability of ending up with final wealth of more than 500.000 € is 57 % with strategy 1 but 68 % with the insured portfolio of strategy 2.

As can be expected, the chance of getting high final wealth levels is greater in the pure equity investment strategy. This is due to the high volatility inherent in the distribution function. On the other hand, the guaranteed investment strategy offers effective loss protection

[13] Strategy 3 is not used in the simulation as final wealth values differ only slightly from those generated with strategy 2. Therefore, consideration of only one of the guaranteed strategies shall suffice to emphasize the main arguments deducted.

and returns are more stable. This effect can be seen even more clearly when considering an investment horizon of 15 years.

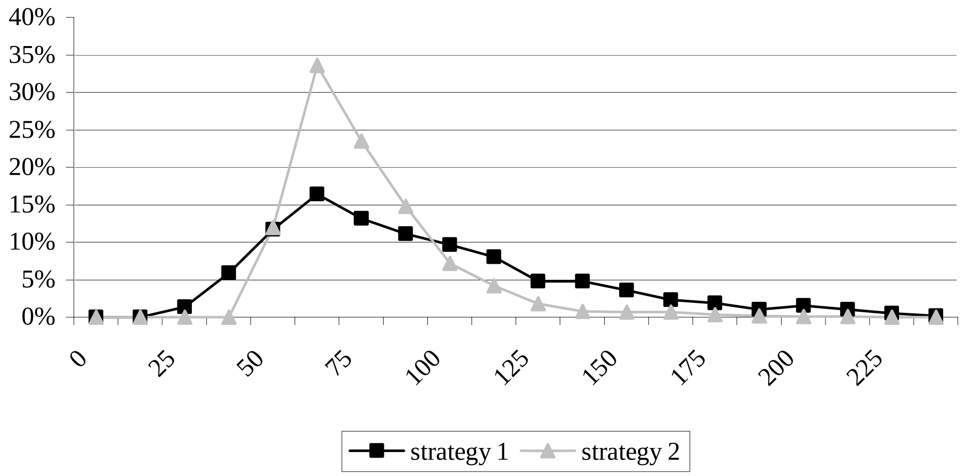

Figure 8: Frequency of final wealth values after 15 years. Source: own calculations.

The gray line again represents the investment strategy with a capital guarantee and the black line represents the 100 % equity investment strategy. In this situation 33.6 % of final wealth values of the guaranteed investment strategy lie between 62.500 and 75.000 €, whereas only 16.4 % of final wealth values lie in that interval if 100 % of capital is invested in equity. Again one can see that final wealth levels are much more stable with an investment strategy that provides a capital guarantee.

Consider the average annual return depending on the investment horizon for each strategy and for simulated and historical data:

	Historical Performance			**Simulated Performance**	
Investment Horizon	Pure equity	Guarantee constant fraction	Guarantee rising fraction	Pure equity	Guarantee constant fraction
10 years	9,12%	7,39%	7,40%	8.4%	7.0%
15 years	8,60%	7,63%	7,43%	8.0%	6.9%
20 years	8,55%	7,98%	8,02%	8.1%	6.8%
25 years	8,71%	8,21%	8,23%	8.3%	7.6%
35 years	9,12%	8,58%	8,11%	8.5%	7.9%

Table 1: Average annual returns of different investment strategies

It can be seen that the historical average annual return for the 100 % equity investment strategy has been higher than the average annual return for the guaranteed investment strategy for every investment horizon. While this is also true for the simulated strategies, they behave contrary to the historic data regarding the extend of returns in relation to the investment horizon. It has been true in the past that longer investment horizons generate higher average annual returns for most investment periods. This is due to the specific development of the underlying DAX performance index which had low returns for many cohorts who started their pension plan in the 1950s, 1960s, 1970s and early 1980s and retired between 1974 and 1991.

Both simulated and historical data are affected by the design of the pension plan regarding contributions. Rather than investing a lump-sum at the beginning of the investment horizon, the representative employee invests a certain amount every month over a long period of time. Therefore, returns that occur towards the end of the investment horizon affect the final wealth value the individual can achieve more strongly than returns that occur at the beginning of the investment horizon as a smaller fraction of total contributions is affected (see Poterba, Shoven, Sialm 2000).

As a consequence, high returns on equity in the early phase of the pension plan have a less favorable effect on the overall performance than high returns on equity in the late phase of the pension plan. Note that this is aggravated for pension plans with a capital guarantee as a smaller fraction of the contribution is invested in equity. The third investment strategy tries to correct this by investing more in equity in the early phase of the pension plan and less in equity in later periods (this is the "base effect" that has been mentioned earlier).

4.3 Evaluation of Individual Costs of a Capital Guarantee

In order to be able to draw more general conclusions, a utility function of the representative individual is introduced. This is done as the individuals' attitude towards risk is relevant for the decision which strategy is superior from the individuals' point of view. A von Neumann-Morgenstern utility function with constant relative risk aversion is assumed. The individual expected utility function takes the form:

$$(6) \qquad E(U) = n^{-1} \sum_{i=1}^{n} \frac{W_i^{1-\alpha}}{1-\alpha}$$

where E(U) is the expected utility, α is the coefficient of relative risk aversion, n is the number of simulations, and W_i is the final wealth level of the respective simulation. Other sources of income are not taken into consideration.

Cost measured by certainty equivalent final wealth levels

In order to evaluate the utility the individual receives from the different outcomes, certainty equivalents of final wealth levels are calculated. This is the level of wealth under certainty that makes the individual indifferent to the outcome of the uncertain prospect of the pension investment.

The advantage of this type of function is that the resulting certainty equivalents show the utility the individual derives from the respective final wealth level measured as the Euro amount of money the individual would be willing to pay today for the resulting expected amount of final wealth generated by investing in the respective pension plan (see also Poterba, Shoven, and Sialm 2000; Shoven and Sialm 2002).

The certainty equivalent is calculated as the inverse function of the expected utility function:

$$(7) \quad CE = U^{-1}(EU) = (EU(1-\alpha))^{\frac{1}{1-\alpha}}.$$

Certainty equivalents are calculated for different degrees of risk aversion based on the simulated final wealth values for investment horizons of 10 and 35 years (figures 9 and 10). We consider coefficients of relative risk aversion between 0 and 10. The range is chosen as there is consensus in the literature that plausible coefficients of risk aversion are below 10 (see Mankiw and Zeldes 1991, Meyer and Meyer 1999). A coefficient of 3 indicates a reasonable degree of risk aversion. In order to compare the results, certainty equivalents of historical final wealth values are presented for the investment period of 10 years also (see figure 11).

Figure 9 compares the certainty equivalents of investment in strategy 1 (pure equity investment) with strategy 2 (portfolio with nominal capital guarantee and constant fraction of contributions paid into bonds). Here, the investment horizon of the individual is set to 10 years, as this is the minimum tenure for pension plans if a guarantee is provided. The x-axis shows the different coefficients of relative risk aversion; the y-axis shows the certainty equivalent wealth levels in Euro.

Individuals with coefficients of relative risk aversion below 1 would prefer the pure equity investment strategy to the investment strategy with capital guarantee. Individuals with a coefficient of relative risk aversion higher than 1 will prefer the capital guarantee. Therefore, if we assume that most individuals are reasonably risk averse regarding their investment for old-age security, only a fraction of individuals would derive less utility if a 0 %

guarantee is required than if they derived without any restrictions regarding return. The loss for persons with a risk aversion of zero (risk neutral) would be around 10 percent of equivalent wealth.

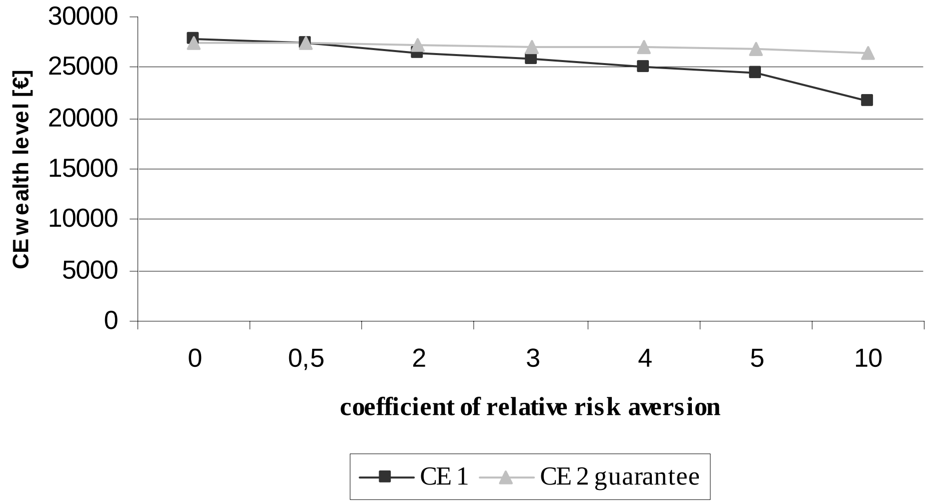

Figure 9: Certainty Equivalents for different degrees of risk aversion for an investment horizon of 10 years using simulated data. Source: own calculations**.**

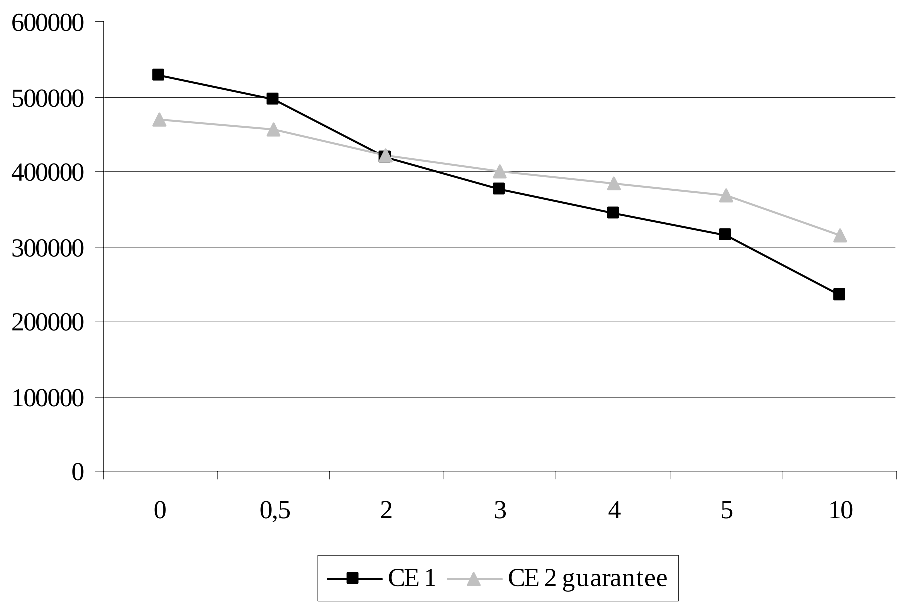

Figure 10: Certainty equivalents for different degrees of risk aversion and an investment horizon of 35 years using simulated data. Source: own calculations**.**

Now consider the certainty equivalent final wealth levels that can be achieved by investing in a pension plan with 35 years duration:

One can discern that the longer duration of the pension plan implies that pure equity investment strategies are preferred by individuals whose utility is characterized by a coefficient of relative risk aversion of up to 2. Again individuals with higher degrees of risk-aversion will prefer the guaranteed pension plan. When looking at the certainty equivalent wealth levels in relation to the coefficient of relative risk aversion one can see that between risk neutral individuals and individuals with a coefficient of relative risk aversion of 2, the certainty equivalent wealth levels drop. Whereas a risk neutral individual would be willing to pay 528.000 € to get any resulting expected final wealth level of the 1.000 simulations, a person with a coefficient of relative risk aversion of 2 would only be willing to pay 417.000 € for the same chance. The same individual would be willing to pay 420.000 € for the guaranteed investment plan.

Certainty equivalents for different degrees of risk aversion for historical data are depicted in the graph below:

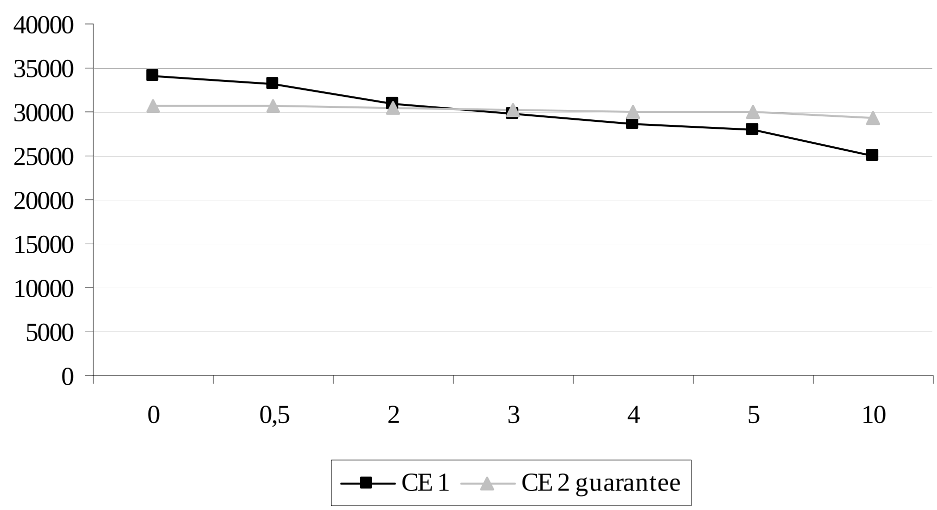

Figure 11: Certainty equivalents for different degrees of risk aversion and an investment horizon of 10 years using historical returns; Source: own calculations

Interestingly, when looking at a ten year investment horizon using historic data, individuals with a coefficient of relative risk aversion of less than 3 prefer the investment strategy that provides a minimum return guarantee of 0 %. Despite the upward trend of the DAX in the nineties, this is only slightly higher than the coefficient of relative risk aversion of 1 generated in the simulated data. This is due to the relative poor performance of the actual DAX realization for ten-year investment horizons.

Costs Measured by Certainty Equivalent Rates of Return

Now that certainty equivalents for different degrees of risk aversion have been calculated, the costs of a strategy that uses portfolio insurance as opposed to a pure equity strategy are analyzed.

As a benchmark, the individual costs depending on the investment horizon of risk neutral individuals are calculated as percentage points of expected average annual returns. Historically the costs for a risk neutral individual with an investment horizon of 10 years who would have invested in an insured pension plan were 0.29 percentage points of average annual return. If instead the investor displays risk aversion with a coefficient of relative risk aversion of 2, these costs turn into a benefit worth 0.24 percentage points of average annual returns. This indicates that individuals with a coefficient of relative risk aversion of slightly below 2 would choose the insured investment plan over the uninsured plan. For an investment horizon of 35 years these costs were 0.5 percentage points of average annual return for risk neutral individuals.

In a further step, costs of simulated realizations are calculated. Individuals who are risk neutral and are forced to pay into the insured pension plan for 10 years up to their time of retirement lose 0.8 % of expected average annual return. Risk averse individuals with a coefficient of relative risk aversion of 2 lose 0.2 percentage points of average annual return in the insured plan. For an investment horizon of 35 years the advantage of a pension plan that only invests in equity over an insured pension plan is 0.6 percentage points of expected average annual return for risk neutral individuals. However, when assuming a coefficient of relative risk aversion of 2, individuals would only lose 0.26 percentage points of expected average annual return if they chose the pension plan that provides a minimum return guarantee of zero percent.

Sensitivity Analysis

As a further step, the sensitivity of results to changes in parameters can be analysed. Simulations of annual returns for a pension plan with of 35 years duration are made in order to analyse the effects of changes in variance and mean compared to the benchmark. One can distinguish that lower variances generate lower average returns whereas higher variances generate higher average returns. This is due to the fact that in the 1.000 runs of the model, higher variances entail the chance of exceptionally high (as well as low) returns. However high returns dominate as the mean is positive and therefore there is a bias for positive values. Note that for changes of the variance the pure equity strategy will consistently generate higher returns than the insured strategy. This can be put down to the long pension plan duration of 35 years.

	Simulated performance 35 year duration			
	variance 20	variance 25	variance 30	variance 36
return pure equity	9,86%	10,70%	11,24%	11,75%
return guarantee	9,49%	10,02%	10,49%	10,88%
	mean 3,5	mean 4	mean 5	mean 6
return pure equity	8,38%	9,08%	10,41%	11,34%
return guarantee	8,32%	8,49%	9,53%	10,52%

Table 2: average annual returns for various mean and variance values, Source: own calculations

Leaving the variance unchanged and analysing different mean values shows that a higher mean will raise average returns for both strategies. However, lower values for the mean result in the superiority of the insured investment strategy as long as returns on the safe asset remain unchanged.

The analysis shows that even for relatively long time horizons, insured investment strategies are frequently superior to pure equity strategies. This is especially true in the light of the recent rapid decline in stock market returns.

5 Conclusion

The analysis has shown that introducing the requirement of a capital guarantee for tax preferred treatment of pension plans is not as costly as feared. The analysis of historical data shows that there are indeed costs associated with capital guarantees: the *expected* rates of return are reduced by some 50 basis points. The potential benefit of a capital guarantee is that it insulates the portfolio against negative returns that may be generated by pure equity investment strategies. As our analysis has demonstrated, those negative returns have indeed been realized in the past with investment horizons below 25 years. The superiority of a pure equity strategy is put into perspective when taking into account that the favorable realizations of equity returns in the past may have been a temporary phenomenon.

Monte Carlo experiments that simulate paths of returns as a GARCH (1,1) specification based on the empirical DAX returns between 1954 and 2001 have been used. The simulation demonstrates that portfolio insurance strategies are superior for market situations that are not quite as favorable as the stock market boom of the 1990s. Moreover, with moderate risk aversion, the more stable but slightly lower returns of investment strategies that are provided by a capital guarantee are favored over the higher expected returns of uninsured investment strategies. This is especially true for shorter investment horizons, which are relevant for cohorts in the process of transition induced by the pension reform of 2001. The German pension reform of 2001 is designed to lower replacement rates over the next decades until a replacement rate of 64 % is reached. In the light of the fact that many cohorts with investment horizons of well below 25 years are affected by the pension reform, the regulation requiring a capital guarantee may be justified. Thus, the capital guarantee is probably non-harming for the large part of the population that is risk averse. The beneficial effect of this regulation may be the protection of uninformed individuals and the elimination of moral hazard in the investment behavior.

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