Does Anonymity Matter in Electronic Limit Order Markets?¹

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Abstract

We analyze the effect of concealing limit order traders’ identities on market liquidity. First we develop a model in which limit order traders have asymmetric information on the true cost of limit order trading (which is determined by the exposure to informed trading). Uninformed bidders draw inferences on this cost from the state of the book. A thin book can be due to untapped profit opportunities or a high cost of limit order trading. The last possibility reduces uninformed bidders’ inclination to add depth when the book is thin. Informed bidders exploit this effect by bidding less aggressively than when bidders have symmetric information. However they bid more aggressively when their identities are concealed than when they are disclosed. For this reason, concealing limit order traders’ IDs affects market liquidity in our model. We test this prediction using a natural experiment. On April 23, 2001, the limit order book for stocks listed on Euronext Paris became anonymous. For CAC40 stocks, we find that following this change, the average quoted spreads declined significantly while the average quoted depth increased significantly.

Keywords: Natural Experiment, Limit Order Book, Anonymity, Price signaling.
1 Introduction

In the last decade, the security industry has witnessed a proliferation of electronic trading systems (for instance ECNs’ such as Island or Instinet in the U.S.). Several of these trading venues are organized as limit order markets where investors can either post quotes (submit limit orders) or trade at posted quotes (submit market orders). In some of these markets (e.g. the Hong Kong Stock Exchange), the identities of the traders with orders standing in the limit order book are disclosed whereas in other markets (e.g. Island), these identities are concealed. Is market liquidity affected by the disclosure of limit order traders’ identities? Our objective in this paper is to provide an answer to this question.

In principle, two types of information regarding market participants’ identities can be disclosed before a transaction occurs: (i) information on the identities of the traders setting prices (liquidity suppliers), or (ii) information on the identities of the traders who demand immediate execution at standing quotes (liquidity demanders).¹ We refer to the first type of information as supply side information and to the second type as demand side information.

In practice, the provision of supply and demand side information is constrained by the timing of the trading process. In limit order markets, liquidity suppliers first set their prices and then liquidity demanders submit their orders. This timing enables disclosure of liquidity suppliers’ identities but it precludes disclosure of liquidity demanders’ identities. In contrast, in dealership markets (e.g. Nasdaq) or in floor markets (for instance the CBOT or the NYSE), liquidity demanders often indicate first the direction and the size of their desired trade and then liquidity suppliers propose prices. In this case, disclosure of liquidity demanders’ identities before the determination of quotes is possible.

Several authors have analyzed the effects of providing demand side information (e.g. Seppi (1990), Admati and Pfleiderer (1991), Forster and Georges (1992), Benveniste et al. (1992), Madhavan and Cheng (1997) or Theissen (2001)). In general, these authors conclude that demand side information reduces trading costs, at least for uninformed traders. Intuitively, non-anonymity enables liquidity suppliers to screen

¹The identities of the contraparties to each transaction constitute a third type of information that can be provided to market participants. Obviously, this information can only be distributed after the transaction took place.
informed and uninformed traders. Much less is known on the effects of providing supply side information in limit order markets.

Our paper fills this gap in two ways. First we develop a simple theoretical model in which we show why and how market liquidity can be affected by information on liquidity suppliers’ identities. Second, using a natural experiment, we test the prediction that market liquidity is affected by the disclosure of limit order traders’ IDs. This experiment takes opportunity of a change in the anonymity of the trading system owned by the French Stock Exchange (Euronext Paris). Euronext Paris operates an electronic limit order market (called NSC) where brokerage firms (henceforth broker-dealers) can place orders for their own account or on behalf of their clients. Until April 23, 2001 the identification codes for broker-dealers submitting limit orders were displayed to all brokerage firms. Since then, the limit order book is anonymous. Thus, using Euronext Paris data, we can empirically test whether concealing liquidity suppliers’ identities impacts market liquidity or not.

Obviously, a non-anonymous limit order book enables traders to adopt bidding strategies which are contingent on the identities of broker-dealers with orders standing in the book. This is worthwhile if some broker-dealers have a superior expertise in choosing their quotes, that is in assessing the cost of providing liquidity. In our model, this expertise stems from knowledge of whether or not an information event will occur. Intuitively traders placing orders for their own account should bid less aggressively when they observe broker-dealers with more expertise doing so. Actually cautious bidding by expert traders constitutes a signal that the cost of liquidity provision is large. For this reason this bidding behavior deters non-expert brokers from improving upon the offers posted in the book. This effect provides expert broker-dealers with some leverage when the true cost of liquidity provision is small. In order to capture larger rents, they can try to “fool” non-expert brokerage firms by behaving as if the cost of liquidity provision were large (by posting steep limit order schedules).

2NSC is an acronym which stands for “Nouveau Système de Cotation”. Many electronic limit order markets (e.g. the Toronto Stock Exchange, the Stockholm Stock Exchange or Island) have a design which is very similar to Euronext Paris.

3There are other markets in which non-experts extract information from the bids placed by more expert bidders. Roth and Ockenfels (2002) notice that bidders in internet auctions of antiques on e.Bay (which is non-anonymous) closely monitor the bids placed by experts (dealers of antiques).

4This line of reasoning is reminiscent of Milgrom and Roberts (1982)’s analysis of limit pricing.
These strategic interactions exist whether limit order traders’ IDs’ are concealed or not since the shape of the limit order book remains informative even when trading is anonymous. In this case as well, a thin limit order book signals that the cost of liquidity provision is large. However, the informational content of the book is lessened when trading is anonymous as non-expert traders cannot distinguish informative orders (those placed by expert traders) from non-informative orders (those placed by non-expert traders or for liquidity reasons). Thus, other things equal, the deterrent effect of steep limit order schedules is smaller with anonymous trading. Expert brokers react by posting aggressive offers (i.e., flatter price schedules) more frequently when their identities are concealed. Building on this effect, we show that concealing liquidity suppliers’ identities should affect measures of liquidity such as (i) the size of the inside spread and/or (ii) the quantities offered at the best quotes.

The empirical analysis strongly supports this prediction. In the case of Euronext Paris, we find a significant decrease in various measures of the quoted spreads after the switch to an anonymous limit order book for most of the stocks in our sample. This result is robust even after controlling for changes in other variables which are known to affect market spreads (such as volatility and trading volume). Furthermore, we document a significant increase in the quoted depth (the quantities offered at the best ask and bid prices) after the switch to market anonymity. Thus the switch to an anonymous limit order book has improved various measures of market liquidity for Euronext Paris.

Our findings highlight the complex nature of the issues related to anonymity in financial markets. We have already stressed that the extant literature focuses on the effect of concealing information regarding the identities of liquidity demanders. The consensus is that concealing this type of information impairs market liquidity. In contrast, our theoretical and empirical findings show that concealing information on the identities of liquidity suppliers can be beneficial to market liquidity.

The provision of information on traders’ identities contributes to market transparency. For this reason our paper adds to the longstanding controversy regarding the desirability of transparency in security markets (see O’Hara (1995) for a review). Recent papers have analyzed theoretically and empirically the effect of providing information regarding the prices and sizes of limit orders standing in the book (respectively Baruch (1999), Madhavan, Porter and Weaver (2002) and Boehmer, Saar and Yu (2003)). However, none of these papers analyze the effect of disclosing infor-
information on limit order traders' identities, holding information on limit order sizes and prices constant.\(^5\)

Few authors have analyzed the effect of providing supply side information. Albanesi and Rindi (2000) study the effect of a switch to trading anonymity in the Italian treasury bond market. The organization of this market is quite different from a pure limit order market. Furthermore, they focus on the time-series properties of transaction prices before and after the switch but not on direct measures of market liquidity such as quoted spreads and depth, as we do in this paper. Rindi (2002) considers a rational expectations model in which informed and uninformed traders simultaneously submit demand functions. In the non-anonymous market, uninformed traders can make their offers contingent on the demand of informed traders whereas they cannot in the anonymous market. She establishes that market liquidity depends on whether or not the market is anonymous.

Our paper is more closely related to the literature on price formation in limit order markets. Our baseline model has the flavor of Glosten (1994) model. The main difference with Glosten (1994) is that we assume that some traders posting limit orders are better informed about the cost of liquidity provision. It is worth stressing that information about the cost of liquidity provision is distinct from information about asset payoffs (i.e. fundamentals). In particular, it cannot be used to pick off mispriced limit orders standing in the book. We show that the depth of the limit order book is smaller (on average) when there is asymmetric information among limit order traders \((\text{compared to the benchmark case in which bidders have symmetric information})\). Furthermore, information on limit order traders' identities matters only when limit order traders have asymmetric information. Sandas (2001) tests Glosten (1994) model and finds that actual price schedules are too steep (i.e. actual limit order books are too thin) relative to those predicted by Glosten (1994). Our model suggests to consider asymmetric information among limit order traders as an explanation for this empirical finding.

The paper is organized as follows. Section 2 describes the theoretical model that we use to compare trading outcomes when the identities of liquidity suppliers are disclosed and when they are not. This comparison is performed after solving for equilibrium bidding strategies in Section 3. In Section 4, we empirically analyze the

\(^5\)In Euronext Paris, intermediaries can observe all limit orders standing in the book (except hidden orders). This feature of the market has not been altered by the switch to anonymous trading.
Section 5 concludes. The proofs which do not appear in the text are collected in the appendix. The notations used in the paper are listed in Table 2 just before the Appendix.

2 The Model

2.1 Timing and Market Structure

We consider the following model of trading in a security market. There are 3 dates. At date 2, the final value of the security, which is denoted $\tilde{V}_2$, is realized. It is given by

$$\tilde{V}_2 = v_0 + \tilde{\epsilon}_1 + \tilde{\epsilon}_2,$$

where $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ are independent random variables with zero means. If an information event occurs at date 1, a trader (henceforth a speculator) observes the innovation, $\epsilon_1$, with probability $\alpha$. Upon becoming informed, the speculator can decide to trade or not. If no information event occurs or if, as happens with probability $(1 - \alpha)$, no trader observes $\epsilon_1$, a liquidity trader submits a buy or a sell market order with equal probabilities. Each order must be expressed in terms of a minimum unit (a round lot) which is equal to $q$ shares. The order size submitted by a liquidity trader is random and can be equal to 1 or 2 round lots with equal probabilities.

Liquidity suppliers (described below) post limit orders for the security at date 0. A sell (buy) limit order specifies a price and the maximum number of round lots a trader is willing to sell (buy) at this price. Following Easley and O’Hara (1992), we assume that there is uncertainty on the occurrence of an information event at date 1. Specifically, we assume that the probability of an information event is $\pi_0 = 0.5$. Figure 1 depicts the tree diagram of the trading process. Now we describe in more details the decisions which are taken at dates 1 and 0.

Speculators. The speculator submits a buy or a sell order depending on the direction of his information. For simplicity we assume that $\tilde{\epsilon}_1$ takes one of two values: $+\sigma$ or $-\sigma$ with equal probabilities. If $\epsilon_1$ is positive (negative), the speculator submits a
buy (sell) market order so as to pick off all sell (buy) limit orders with a price below (resp. above) \( v_0 + \sigma \) (resp. \( v_0 - \sigma \)).\(^6\)

**Liquidity Suppliers.** There are two kinds of liquidity suppliers: (a) risk-neutral value traders who post limit orders so as to maximize their expected profits and (b) pre-committed traders whose limit orders are determined exogenously. In Euronext Paris, limit orders are posted by intermediaries who act on behalf of investors (agency trades) or who trade for their own account (principal trades). Value traders can be viewed as intermediaries who post limit orders for their own account whereas pre-committed traders are brokers who exclusively submit agency orders. Henceforth we will refer to the value traders as being “the dealers”\(^7\).

We assume that dealers are not equally informed on the likelihood of an information event. There are two types of dealers: (i) an informed dealer who knows whether or not an information event will take place at date 1 and (ii) an uninformed dealer who does not have this knowledge. It is worth stressing that the informed dealer does not observe the direction of the information event. Hence his information is different from the information possessed by speculators at date 1 (more on this in Section 2.2). We show in Section 2.3 (the benchmark case) that the cost of providing liquidity depends on the likelihood of an information event. For this reason, the schedule of limit orders posted by the informed dealer constitutes a signal of the true cost of providing liquidity.

The uninformed dealer can use this signal if she has the possibility to choose her limit orders *after* observing the informed dealer’s price schedule. For this reason, we consider a model in which, at date 0, dealers post their limit orders sequentially, in

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\(^6\)An information event can be seen, for instance, as the arrival of public information (earnings announcement, price movements in related stocks, headlines news etc...). In this case, the probability \( \alpha \) is the probability that a trader reacts to the new information before mispriced limit orders disappear from the book (either because a market order arrived or because limit order traders cancelled their orders). This probability depends on the intensity with which traders monitor the flow of information (as in Foucault, Roëll and Sandás (2002) for instance).

\(^7\)Harris and Hasbrouk (1996) argue that there are two types of limit order traders: (i) traders who are precommitted to buy or sell the security for portfolio rebalancing and (ii) passive or value traders who act as dealers and submit limit orders to make positive expected profits. Precommitted traders have a very different objective function than value traders since they have to trade. Hence for these traders the risk of being picked off is not a concern since not trading is not an option. See Glosten (1998) for a related point.
2 stages denoted $L$ (first stage) and $F$ (second stage). Figure 2 describes the timing of the bidding game. With probability $(1-\beta)$, the price schedule posted in the first stage (the limit order book) is posted by the informed dealer. Otherwise the limit order book is established by precommitted liquidity suppliers. In the second stage, the uninformed dealer observes the limit order book, updates her beliefs on the cost of liquidity provision (i.e. the likelihood of an information event) and decides to add limit orders to the book or not. In the rest of the paper, we sometimes refer to the liquidity supplier acting in stage $L$ as being the leader and to the liquidity supplier acting in stage $F$ as being the follower.

At date 1, the incoming buy (sell) order is filled against the sell (buy) limit orders posted in the book. Price priority is enforced and each limit order executes at its price (execution is discriminatory). Furthermore, time priority is enforced. That is, at a given price, the limit order placed by the leader is executed before the limit order placed by the follower. Table 1 below summarizes the different types of traders in our model.

<table>
<thead>
<tr>
<th>Liquidity Suppliers (date 0)</th>
<th>Liquidity Demanders (date 1)</th>
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<tbody>
<tr>
<td>Precommitted Limit Order Traders</td>
<td>Liquidity Traders</td>
</tr>
<tr>
<td>Uninformed Dealer</td>
<td>Speculators</td>
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<tr>
<td>Informed Dealer</td>
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**Limit Order Book.** Modeling price formation in limit order markets quickly becomes very complicated. In order to keep the model tractable, we make the following assumptions. Liquidity suppliers can post sell limit orders at prices $A_1$ and $A_2$. We assume that

$$A_2 - A_1 = A_1 - v_0 = \Delta. \quad (1)$$

In financial markets, such as Euronext, bidders are often required to position their quotes on a grid. Parameter $\Delta$ can be seen as the tick size, i.e. the difference between two consecutive prices on the grid. In this case $A_1$ is the first price on the grid above the asset unconditional expected value and $A_2$ is the second price on the grid above the asset unconditional expected value. We describe a price schedule posted
by liquidity supplier $j$ by the pair $(x_{1j}, x_{2j})$ where $x_{kj}$ denotes the quantity offered by liquidity supplier $j$ at price $A_k$, $k \in \{1, 2\}$. We also assume that

$$A_1 < v_0 + \sigma < A_2.$$  

(2)

This assumption implies that only limit orders posted at price $A_1$ are exposed to the risk of an information event. Hence dealers (informed or uninformed) can safely offer to trade any quantity at a price larger than $A_1$. Furthermore they will never supply more than 2 round lots at price $A_1$ since this is the maximal demand of a liquidity trader.\(^8\) Thus, we can restrict our attention to the case in which the leader chooses one of 3 price schedules on the sell side: (a) $(0, 2q)$, (b) $(q, 2q)$ and (c) $(2q, 2q)$ that we denote $T$, $S$ and $D$, respectively. At the end of the first stage, the limit order book can be in one of 3 states: (a) “thin” if the leader posts schedule $T$, (b) “shallow” if the leader posts schedule $S$ or (c) “deep” if the leader posts schedule $D$. We assume that a pre-committed liquidity supplier chooses schedule $K \in \{T, S, D\}$ with probability $\Phi_K > 0$. Given the state of the book, the uninformed dealer decides to add 1 or 2 round lots at price $A_1$ or does nothing. Thus she chooses one of the following price schedules: (a) $(q, 0)$, (b) $(2q, 0)$ and (c) $(0, 0)$.\(^9\) We denote by $Q_1 \in \{0, q, 2q\}$ the number of shares offered at price $A_1$ at the end of the bidding stage.

We make symmetric assumptions on the buy side. This symmetry implies that the equilibrium price schedules on the buy side are the mirror image of the equilibrium price schedules on the sell side. Thus from now on we focus on the sell limit orders chosen by the dealers exclusively.

Now let $\tilde{Q}_s$ and $\tilde{Q}_l$ be the size of buy market orders submitted at date 1 by a speculator and a liquidity trader, respectively. As $A_1 < v_0 + \sigma < A_2$, the optimal trading strategy of a speculator if $\epsilon_1 = \sigma$ is to submit a buy market which exhausts the quantity offered at price $A_1$. Hence $\tilde{Q}_s = Q_1$. Consequently the size of the buy market order which is submitted at date 1, that we denote $\tilde{Q}$, can be written as:

$$\tilde{Q}(Q_1) = I(Q_1) + (1 - I)\tilde{Q}_l,$$  

(3)

\(^8\)Suppose that 2 round lots are offered at price $A_1$. A third round lot would only execute against orders sent by a speculator and therefore would lose money.

\(^9\)An order placed by the follower at price $A_2$ has a zero execution probability since 2 round lots are offered by the leader at this price. Hence we assume, without loss of generality, that the follower never submits a limit order at this price.
where \( J \) is an indicator variable equal to 1 if the trader who submits a market order at date 1 is informed and zero otherwise. It is important to note that the probability distribution of \( \tilde{Q} \) depends on the depth of the book, \( Q_1 \).

**Anonymous and Non-Anonymous Limit Order Books.** We shall distinguish two different trading systems: (i) the **anonymous limit order market** and (ii) the **non-anonymous limit order market**. In the non-anonymous trading system, the follower observes the identity of the leader, that is she knows whether the book has been established by an informed trader or by an uninformed trader. In the anonymous market, she cannot distinguish between informative and non-informative limit orders. In both cases, however, the follower observes the price schedules posted in the first stage (i.e. the book is “opened”).

Thus, in the non-anonymous market, we assume that the uninformed dealer is able to distinguish an informed trader from an uninformed trader. This is a reasonable assumption in markets where traders interact repeatedly. In Euronext Paris, quotes are posted by a stable set of brokerage firms. Hence, we conjecture that over time, brokerage firms have been able to identify those who have a greater expertise in choosing their limit orders.

We will compare the liquidity of these two trading systems for **fixed values** of the exogenous parameters \((\sigma, \alpha, \beta, \Delta)\). To this end, we consider 2 different measures of market liquidity: (a) the quoted (half-) spread which is the difference between the best ask price and the unconditional expected value of the security and (c) the quoted depth which is the number of shares offered at the best ask price. Market liquidity unambiguously decreases when both the quoted spread increases and the quoted depth decreases.

### 2.2 Discussion.

**Informed Dealers.** Several empirical findings suggest that some liquidity suppliers have a superior expertise in choosing their quotes. For instance, Blume and Goldstein (1997) find that the NYSE specialist establishes the best quotes much more frequently than regional exchanges’ specialists. The latter tend to match the NYSE specialist’s quotes and rarely improve upon his spread even when it is larger than the tick size. This leader-follower type of behavior suggests that regional exchanges’ specialists view
the NYSE specialist as knowledgeable for the determination of the “right” spread.

Anand and Martell (2001) find that limit orders placed by institutional investors on the NYSE perform better than those placed by individuals, even after controlling for order characteristics (such as order aggressiveness or order size). They argue (page 2) that institutional investors are better able “to predict at least the flow of information and use this knowledge to submit trades, which avoid adverse selection associated with limit orders”. Finally, for Euronext Paris, Declerk (2001) shows that there are substantial variations in the trading profits of the intermediaries who actively trade for their own account. One possible explanation is that some intermediaries (those with superior profits on average) have more expertise.

In line with these empirical observations, we assume that one dealer (the informed dealer) has information on the true cost of liquidity provision. This information gives him an edge in the choice of his limit orders. There are many ways to model this idea. Here we assume that the informed dealer has information on the likelihood of an information event. Intuitively this information helps the dealer to assess the expected loss associated with the risk of being picked off and therefore to better position his quotes (see Section 2.3 for a formal statement). It is intuitive that superior information on the (a) magnitude of the innovation (i.e. $\sigma$) or (b) on the probability of informed trading (i.e. $\alpha$) would have the same effect. The results in these cases are qualitatively similar to those we obtain here. Alternatively the informed dealer might have information on the final payoff of the security. The problem with this approach is that the informed dealer may then find it optimal to submit market orders (act as a speculator) rather than limit orders. This difficulty does not arise in our set-up because the expected value of the security at date 0 is the same (and equal to $v_0$) for the informed and the uninformed dealers alike. In other words, information on the likelihood of an information event is useful to position quotes but useless for the decision to trade at these quotes.

**Timing.** At first glance, the timing of our model (the uninformed dealer is always the follower) might look artificial. A more general formulation would assume that the

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10 Kavajecz (1998) finds that NYSE specialists have superior information on the direction of earnings announcements. Kyle (1989) or Calcagno and Lovo (2001) consider models in which liquidity suppliers possess private information on the final value of a security.

11 Actually the occurrence of an information event determines whether or not dealers are exposed to the risk of being picked off by speculators but is independent of the realizations in the innovations $\varepsilon_1$ and $\varepsilon_2$. 

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sequence in which the informed and the uninformed dealer choose their price schedules is random. This formulation however would obscure the presentation of our results without adding new insights. Actually, the follower’s bidding strategy depends on the identity of the leader only when (i) the leader has a chance to be informed and (ii) the follower is uninformed. This configuration is therefore the only case in which concealing the leader’s identity has an effect, if any. The timing of our model is a way to focus the attention on the interesting case.

**Pre-committed Traders.** Uninformed dealers’ bidding strategy will depend on the informativeness of the price schedule posted in the first stage. When trading is anonymous, the limit order book constitutes a noisy observation of the limit orders chosen by expert brokers since many orders are likely to be submitted by investors who are not concerned by the risk of being picked off (liquidity traders submitting limit orders). In particular, for Euronext Paris, we expect a large fraction of agency orders to be non-informative on the risk of being picked off. In our model, this source of noise is captured by the existence of precommitted liquidity suppliers. Submission of limit orders by these liquidity traders reduce the informational content of equilibrium prices. In this sense, they play the role ascribed to noise traders in Noisy Rational Expectations models (e.g. Hellwig (1980)).

Finally we assume that the informed dealer always trades for his own account. In reality, all intermediaries operating in Euronext Paris conduct agency trades. Even when the market was non-anonymous, liquidity suppliers were not able to distinguish agency from principal orders. The model can easily be amended to accomodate this uncertainty, however. For instance the informed dealer might post quotes on behalf of another investor (acts as a precommitted liquidity supplier) with probability $\gamma < 1$. In this case, the results are qualitatively unchanged since limit orders placed by the informed dealer remain more informative than those placed by precommitted liquidity suppliers.

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12 In auctions with fixed end times, expert bidders may choose to place their bids in the closing seconds of the auction to avoid revealing their information (see Roth and Ockenfels (2002)). In limit order markets, the notion of fixed end time does not apply since the times at which market orders arrive are random. Thus an informed bidder who chooses to wait in order to avoid revealing his information runs the risk of missing the next trade. In addition, he cannot be certain that an uninformed bidder will not react before the arrival of the next market order. In these conditions, the natural modelling strategy is to assume that bidders’ arrival times are random.

13 There are few intermediaries who actively trade for their own account in Euronext. Declerk (2001) report that there were 59 intermediaries posting orders in CAC40 stocks in 1999, of which 6 accounted for more than 71% of all principal orders.
suppliers. This is the feature of our model which drives the results.

2.3 A Benchmark

It is useful to analyze the benchmark case in which there is no asymmetric information among dealers. This analysis helps to clarify why information on the likelihood of an informational event affects dealers’ bidding strategies. Furthermore it sets the ground for the case in which dealers have asymmetric information. We first study the follower’s optimal reaction in each possible state of the book for given, but arbitrary, beliefs, $\pi_K$ regarding the occurrence of an information event ($\pi_K$ will be endogenized in the next section).

Suppose that the book is thin ($K = T$) at the end of the first stage. The follower can profitably add one round lot at price $A_1$ if this price is larger than her valuation of the asset, conditional on execution of one round lot or:

$$A_1 - E_{\pi_T}(V \mid \tilde{Q} \geq q) \geq 0. \quad (4)$$

In case of execution, the follower knows that the size of the market order is at least equal to $q$ but she does not know the exact size of the order. This explains why the follower’s valuation is given by an “upper-tail expectation” (see Glosten (1994)). Computations yield:

$$E_{\pi_T}(V \mid \tilde{Q} \geq q) = v_0 + \pi_T \alpha \sigma. \quad (5)$$

Using the same logic, we conclude that the follower can profitably offer another round-lot at price $A_1$ if:

$$A_1 - E_{\pi_T}(V \mid \tilde{Q} \geq 2q) \geq 0.$$ 

Computations yield\(^{14}\)

$$E_{\pi_T}(V \mid \tilde{Q} \geq 2q) = v_0 + \left(\frac{2\pi_T}{\pi_T \alpha + 1}\right) \alpha \sigma. \quad (6)$$

More generally, $E_{\pi}(V \mid \tilde{Q} \geq q)$ can be interpreted as the “cost” of providing 1 round lot at price $A_1$ for a dealer who assigns a probability $\pi$ to the occurrence of an

\(^{14}\)Notice that $\tilde{Q}$ (given in Equation (3)) is a function of the depth posted at price $A_1$, $Q_1$. Hence its probability distribution depends on $Q_1$. This must be taken into account in computing the upper-tail conditional expectations in the various possible cases ($Q_1 = q$ or $Q_1 = 2q$).
information event. Similarly $E_\pi(V \mid \tilde{Q} \geq 2q)$ is the cost of providing one additional round lot at price $A_1$ when one is already offered (this is a marginal cost).\footnote{Notice that, for a given $\pi^c$, the cost of providing a second round lot at price $A_1$ does not depend on whether the trader offering the second round lot is the same as the trader offering the first round lot. Actually if the two traders are different, the first one has time priority. Thus the first round lot will be executed before the second. This means that execution of the second round lot implies that the market order size is larger than or equal to 2 round lots.} For this reason, we refer to the cost schedule defined by Equations (5) and (6) as being the expected cost of liquidity provision.\footnote{The actual cost is either high if an information event occurs or low (and equal to zero) if there is no information event.} This schedule is increasing (in the quantity) since

$$(\frac{2\pi}{\pi \alpha + 1})\alpha \sigma > \pi \alpha \sigma \quad \forall \pi.$$ 

One implication is that it may be optimal (depending on parameter values) to offer 1 round lot but not more at price $A_1$. The informed trader always exhausts the depth available at price $A_1$. In contrast, a liquidity trader always trades at least 1 round lot but trades 2 round lots with probability 0.5 only. Thus the second round lot offered at price $A_1$ is relatively more exposed to informed trading than the first round lot. This explains why the cost of providing this second round lot is larger than the cost of providing the first one.

The follower’s optimal behavior given the state of the book immediately derives from the previous remarks. For a given state of the book at the end of stage $L$, the follower will fill the book up to the point where an additional round lot offered at price $A_1$ would lose money. This means that the number of shares offered at price $A_1$ at the end of the bidding stage is the largest $Q_1$ such that

$$A_1 - E_{\pi \mathcal{K}}(V \mid \tilde{Q} \geq Q_1) \geq 0. \quad (7)$$

For instance, suppose that she observes a shallow book. Then the follower adds one additional round lot at price $A_1$ if $(\frac{2\pi}{\pi \alpha + 1}) \leq \Delta$ and does not otherwise. The same reasoning yields the follower’s optimal behavior in each possible state of the book. This behavior is described in the next lemma.

**Lemma 1 :**
1. When the follower observes a thin book, she submits a limit order at price $A_1$ for 2 round lots if $\frac{2\pi\alpha\sigma}{\pi\alpha+1} < \Delta$, 1 round lot if $\pi_T\alpha\sigma < \Delta < \frac{2\pi\alpha\sigma}{\pi\alpha+1}$ and does nothing otherwise.

2. When the follower observes a shallow book, she submits a limit order at price $A_1$ for 1 round lot if $\frac{2\pi\alpha\sigma}{\pi\alpha+1} < \Delta$ and does nothing otherwise.

3. When the follower observes a deep book, she does nothing.

The risk of being picked off is large when the likelihood of an information event is large. For this reason the expected cost of liquidity provision increases with the likelihood of an information event (see Equations (5) and (6)). Hence the follower’s inclination to add depth to the book is smaller when she assigns a large probability to the occurrence of an information event. This mechanism explains why, for a given state of the book, the follower acts less and less aggressively as the likelihood of an information event, $\pi_K$, increases (consider the case in which the book is thin for instance).

When dealers have symmetric information, the state of the book at the end of the first stage does not convey any information on the actual cost of liquidity provision. Furthermore the follower’s beliefs regarding the cost of liquidity provision are identical in the anonymous and in the non-anonymous trading system since the leader’s identity (combined with his bidding strategy) does not convey information on the cost of liquidity provision. Therefore $\pi_S^c = \pi_T^c = \pi_0 \overset{\text{def}}{=} 0.5$ in both the anonymous and the non-anonymous trading systems. It follows that the equilibrium is not affected by the anonymity of the trading system.

**Proposition 1 (Benchmark):** When dealers have symmetric information, the depth of the book and the quoted spread are identical in the anonymous and in the non-anonymous trading system.

This result will not hold when there is asymmetric information among dealers, as shown in the next section. In the rest of the paper we concentrate on the case in which

$$\frac{2\pi_0\alpha\sigma}{\pi_0\alpha+1} = \frac{2\alpha\sigma}{\alpha+2} < \Delta.$$  \hspace{1cm} (8)
This assumption implies that, in absence of asymmetric information, the follower always fills the book so that the maximal possible depth (2 round lots) is offered at price $A_1$ (Lemma 1). Anticipating this behavior, the dealer acting in the first stage (who also has belief $\pi_0$) must fill the book so as to leave no profit opportunity at price $A_1$, that is he must choose schedule $D$. Suppose to the contrary that he chooses schedule $S$. This schedule can yield a strictly larger profit to the dealer since the second round lot is offered at price $A_2$ rather than at price $A_1$. However, this profit will never materialize since the follower fills the book in such a way that orders placed at price $A_2$ have a zero execution probability. Thus we obtain the following result.

**Proposition 2 (Benchmark):** Suppose that (i) dealers have symmetric information and (ii) Condition (8) is satisfied. The unique subgame perfect equilibrium is as follows: (i) the dealer acting in stage $F$ chooses schedule $D$ and (ii) the follower acts as described in Lemma 1 for $\pi_S = \pi_T = 0.5$. In equilibrium, the book obtained at the end of the second stage is always deep (2 round lots are offered at price $A_1$). Furthermore the quoted spread is equal to $\Delta$.

With sequential bidding and time priority, the book fills up so as to leave no profit opportunities at each price level (as in Seppi (1997) or Sandas (2001)).\footnote{This property does not hold if (i) time priority is not enforced or if (ii) limit orders are placed simultaneously (as in Biais, Martimort and Rochet (2000)). See Glosten (1998) for a discussion.} Here the zero profit condition for the follower does not exactly hold because orders are for discrete quantities.

### 3 Competition in Anonymous and Non-Anonymous Limit Order Markets

We first analyze the possible equilibria in the anonymous limit order market. The equilibria in the non-anonymous market will obtain as special cases. Throughout we focus on the Perfect Bayesian Equilibria of the bidding game as it is usual for the analysis of signaling games. We denote by $\Psi$ an indicator variable which is equal to 1 if there is an information event and zero otherwise.
3.1 The Anonymous Limit Order Market

From now on, to make things interesting, we focus on the case in which:

\[ \frac{2\alpha \sigma}{\alpha + 2} < \Delta < \frac{2\alpha \sigma}{\alpha + 1} \]  

(9)

This condition guarantees that the follower’s reaction is influenced by her belief on the occurrence of an information event. The L.H.S inequality implies that the uninformed dealer will fill the book so that 2 shares are offered at price \( A_1 \) if her posterior belief about the occurrence of an information event is not too different from her prior belief (\( \pi_0 = 0.5 \)). The R.H.S implies that the uninformed dealer will find suboptimal to offer 2 shares at price \( A_1 \) if her posterior belief is large enough compared to her prior belief (consider Equation (6) with \( \pi_T = 1 \)). Furthermore, the R.H.S condition implies that the informed dealer will never establish a deep limit order book when \( \Psi = 1 \).

The schedule chosen by the informed dealer when there is an information event can therefore be schedule \( S \) or \( T \). The case which will be obtained in equilibrium depends on parameter values. Hence in what follows we will distinguish two cases: (1) the case in which the informed dealer posts schedule \( S \) if there is an information event and (2) the case in which the informed dealer posts schedule \( T \) if there is an information event.

The formal treatment of these two cases are very similar. However they will illustrate two distinct mechanisms through which a switch to an anonymous limit order book can affect liquidity. When the informed dealer chooses a shallow book if there is an information event, a switch to an anonymous book will not affect the quoted spread but it will change the depth available at the posted quotes. In contrast, when the informed dealer chooses a thin book if there is an information event, a switch to an anonymous book will affect the quoted spread.

When there is no information event, the informed dealer can clearly post the deep schedule profitably. But he can also try to reap a larger profit by posting the schedule that he would use when there is an information event. For the follower, this

\[^{18}\text{Clearly the set of parameters such that Condition (9) is satisfied is never empty. Recall that we also assume that } \sigma \leq 2\Delta. \text{ This condition combined with the R.H.S of Condition (9) imposes } \alpha \geq \frac{1}{3}. \text{ Intuitively, since } \sigma \text{ cannot be made arbitrarily large, offering two shares at price } A_1 \text{ is suboptimal only if } \alpha \text{ is large enough (here greater than } 1/3).\]

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schedule constitutes a warning and she revises upward the probability she assigns to an information event. If this revision is large enough, she is deterred from adding depth to the book and the informed dealer obtains larger expected profits. If the informed dealer sometimes behaves in this way, we say that he follows a bluffing strategy. In the rest of this section we identify the conditions under which bluffing strategies emerge in equilibrium.

**Remark.** Recall that we focus on Perfect Bayesian equilibria of the bidding game. This means that in equilibrium, the follower’s posterior belief about the occurrence of an information event is determined by Bayes rule whenever this is possible. As usual in signaling games, there is a difficulty if some states of the book are out-of-the-equilibrium path. For these states the follower’s posterior belief cannot be determined by Bayes rule since these states have a zero probability of occurrence in equilibrium. The Perfect Bayesian Equilibrium concept does not put restrictions on how the follower’s beliefs should be formed for such states. Here we assume that the follower does not revise her prior belief about the occurrence of an information event if she observes a state of the book which is out-of-the-equilibrium path. This never happens when the market is anonymous and $\beta > 0$. There are states of the book which are out-of-the-equilibrium path when $\beta = 0$ in the anonymous market or when the leader is informed in the non-anonymous market.

### 3.1.1 Case 1: The informed dealer chooses schedule $S$ if there is an information event.

When there is no information event, the informed dealer can behave competitively by posting the deep schedule. Or he can try to bluff by posting the shallow schedule. In order to take into account this possibility, let $m$ be the probability with which the informed dealer chooses schedule $D$ when $\Psi = 0$. With the complementary probability, he chooses schedule $S$ when $\Psi = 0$. Now consider the following likelihood ratio:

$$ l(\beta, \Phi_S, m) = \frac{\beta \Phi_S + (1 - \beta)}{\beta \Phi_S + (1 - \beta)(1 - m)} \geq 1. $$

The numerator is the likelihood of observing a shallow book at the end of stage $L$ when $\Psi = 1$ to the likelihood of the same event when $\Psi = 0$. It is larger than 1 (the inequality is binding for $m = 0$), reflecting the fact that a shallow book is more likely to occur if $\Psi = 1$. Thus, when she observes a shallow book, the uninformed
dealer revises upward the probability she assigns to an information event. Formally, the uninformed dealer’s posterior belief is

$$\pi_S(m, \beta) \equiv \text{prob}(\Psi = 1 \mid K = S) = \frac{\beta \Phi_S + (1 - \beta)}{\beta \Phi_S + (1 - \beta)(2 - m)} \geq \pi_0 = 0.5$$

As the uninformed dealer revises upward the probability she assigns to an information event, she marks up the cost of liquidity provision. We refer to this effect as being the deterrence effect of cautious bidding by the leader since it reduces the uninformed dealer’s incentive to add depth at price $A_1$.

The deterrence effect increases as $(\pi_S(m, \beta) - \pi_0)$ enlarges, that is when $m$ increases or $\beta$ decreases. Actually these two variables determine the informativeness of the limit order book. If $m = 0$, the book is not informative at all since the informed dealer establishes a shallow book whether an information event will occur or not. As $m$ increases, a shallow book becomes a stronger signal of an information event since the informed dealer chooses it less and less frequently when there is no information event. As $\beta$ increases, the informational content of the book decreases since it is more and more likely that it has been established by a precommitted trader. Hence the follower’s belief about the occurrence of an information event is less and less sensitive to the state of the book ($\pi_S(m, \beta)$ decreases with $\beta$). To sum up the deterrence effect is strong when the informational content of the book is large.

Conditional on the state of the book being $S$, the cost of offering one additional round lot at price $A_1$ for the uninformed dealer is:

$$E_{\pi_S}(V \mid Q \geq 2q) = v_0 + \left(\frac{2\pi_S(m, \beta)}{\pi_S(m, \beta)\alpha + 1}\right)\alpha\sigma$$

(10)

A graphical representation of this conditional expectation as a function of $m$ is given in Figure 3. The perceived cost of offering one additional round lot at price $A_1$ for the uninformed dealer becomes larger as $m$ enlarges. This reflects the fact that the deterrence effect is strong when $m$ is large. Figure 3 also offers an intuitive view of the 2 possible cases which arise in equilibrium. We discuss these cases now.

The uninformed dealer submits a limit order for 1 round lot at price $A_1$ iff

$$A_1 - E_{\pi_S(m)}(V \mid Q \geq 2q) = \Delta - \left(\frac{2\pi_S(m, \beta)}{\pi_S(m, \beta)\alpha + 1}\right)\alpha\sigma \geq 0.$$  

(11)
The L.H.S of this inequality decreases with \( m \). Thus if it is satisfied for \( m = 1 \) then it is satisfied for all smaller values. For \( m = 1 \), the condition is

\[
A_1 - E_{\pi_S(m)}(V \mid \bar{Q} \geq 2q) = \Delta - \left( \frac{2\pi_S(1, \beta)}{\pi_S(1, \beta)\alpha + 1} \right)\alpha\sigma \geq 0.
\]

Substituting \( \pi_S(1, \beta) \) by its expression (see Equation (??)), this condition rewrites as

\[
\beta \geq \beta^* \overset{\text{def}}{=} \left( \frac{(2 - r)\alpha - r}{(2 - r)\alpha - r + \Phi_S(2r - (2 - r)\alpha)} \right).
\]

with \( r \overset{\text{def}}{=} \frac{\Delta}{\sigma} \). Under this condition on \( \beta \), the follower cannot be deterred from improving upon the shallow book. Intuitively the signal sent by a shallow book is too noisy to influence the follower’s perceptions of the profitability of adding depth to the book. Thus the latter behaves as in the benchmark case, that is she fills the book so that eventually 2 round lots are offered at price \( A_1 \). Anticipating this behavior, the leader establishes a deep book whenever this is profitable. Thus there is an equilibrium in which \( m = 1 \) as claimed in the next proposition.

**Proposition 3**: When \( \beta^* \leq \beta < 1 \) and \( \alpha\sigma < \Delta < \frac{2\alpha\sigma}{\alpha+1} \), then the following bidding strategies constitute an equilibrium:

1. When \( \Psi = 1 \), the informed dealer chooses schedule \( S \). When \( \Psi = 0 \), the informed dealer chooses schedule \( D \).
2. The uninformed dealer always fills the book so that eventually 2 round lots are offered at price \( A_1 \).
3. All measures of market liquidity are unchanged compared to the benchmark case.

It is straightforward that \( \beta^* > 0 \) since \( \Delta < \frac{2\alpha\sigma}{\alpha+1} \). The condition \( \alpha\sigma < \Delta \) guarantees that choosing schedule \( S \) is a best response for the informed dealer if \( \Psi = 1 \).

For \( 0 < \beta < \beta^* \), the previous equilibrium cannot be sustained. If \( m = 1 \), a shallow book induces a large revision in the follower’s estimation of the cost of liquidity provision. So large that she does not find it optimal to add depth to the book. But then the informed dealer should always choose schedule \( S \), whether an information event
took place or not. This deviation precludes the existence of the previous equilibrium. Therefore we now consider the possibility of a mixed strategy equilibrium in which the informed dealer randomly chooses schedules $S$ or $D$ ($0 < m < 1$).

In this mixed strategy equilibrium the informed dealer must be indifferent between schedules $S$ and $D$ when $\Psi = 0$. Suppose that the follower adds 1 round lot at price $A_1$ with probability $u^*$ when she observes a shallow book. When $\Psi = 0$, the informed dealer obtains an expected profit equal to

$$ (A_1 - v_0)q + \left(\frac{1 - u^*}{2}\right)(A_2 - v_0)q, $$

if he chooses schedule $S$ since the limit order placed at price $A_2$ is hit only if (a) the liquidity trader needs to buy 2 round lots and (b) the follower does not offer a second round lot at price $A_1$. If he posts schedule $D$, when $\Psi = 0$, the informed dealer obtains an expected profit equal to

$$ (A_1 - v_0)E(\tilde{Q}_u) = \frac{3(A_1 - v_0)q}{2} > 0. $$

The informed dealer is therefore indifferent between schedules $S$ and $D$ price if

$$ u^* = \frac{A_1 - v_0}{A_2 - v_0} = 0.5. $$

The follower must be indifferent between improving upon the shallow book or doing nothing. Otherwise she would either always add 1 round lot ($u = 1$) or never add 1 round lot ($u = 0$). This requires

$$ A_1 - E_{\pi_S}(V | \tilde{Q} \geq 2q) = \Delta - \left(\frac{2\pi_S(m)}{\pi_S(m)\alpha + 1}\right)\alpha\sigma = 0, $$

that is (solving this equation for $m$)

$$ m^* = \left(\frac{1 - \beta + \beta\Phi_S}{1 - \beta}\right)\left(2 - \left(\frac{2 - r}{r}\right)\alpha\right). $$

It is easily checked that $m^* < 1$ since $0 < \beta < \beta^*$ and $m^* > 0$ if $\alpha\sigma < \Delta$. The previous analysis yields the following proposition.

**Proposition 4**: When $0 \leq \beta < \beta^*$ and $\alpha\sigma \leq \Delta < \frac{2\alpha\sigma}{\alpha + 1}$, the following bidding strategies constitute an equilibrium.
1. When $\Psi = 1$, the informed dealer posts schedule $S$. When $\Psi = 0$, the informed dealer posts schedule $D$ with probability $m^*(\beta)$ and schedule $S$ with probability $(1 - m^*(\beta))$, where $m^*(\beta) = \left(\frac{(1-\beta + \beta \Phi_S)}{1-\beta}\right)(2 - \frac{(2-\tau)\alpha}{\tau})$ and $0 < m^* < 1$.

2. When the book is shallow, the follower adds 1 round lot at price $A_1$ with probability $u^* = \frac{1}{2}$ and else does nothing. When the book is thin, the follower adds 2 round lots at price $A_1$ and when the book is deep, the follower does nothing.

3. Compared to the benchmark case: (i) the expected depth at price $A_1$ is smaller, (ii) the average quoted spread is unchanged.

In this case, asymmetric information between dealers has a negative effect on market liquidity (last part of the proposition). This reflects the fact that both the informed dealer and the uninformed dealer act less aggressively: (i) the informed dealer does not necessarily post a deep book ($m^* < 1$) and (ii) the uninformed dealer does not necessarily improve upon a shallow book ($u^* < 1$). Nevertheless the informed dealer bids more aggressively when $\beta$ enlarges ($m^*(\beta)$ increases with $\beta$).

The intuition is as follows. Other things equal ($m^*$ fixed), the larger is $\beta$, the smaller is the informational content of the book (a shallow book is less and less indicative of the occurrence of an information event). As we already explained, this relaxes the deterrence effect. In order to restore his influence on the follower’s belief, the informed dealer must choose schedule $D$ more frequently ($m^*$ increases) when $\Psi = 0$ so as to strengthen the informational content of the book.

To sum up, we have shown in this section that there exists a bluffing strategy in which the informed dealer chooses a shallow schedule when there is no information event if and only if

$$0 \leq \beta < \beta^* \quad \text{and} \quad \alpha \sigma \leq \Delta < \frac{2\alpha \sigma}{\alpha + 1}.$$ 

Furthermore an exogenous decrease in the informational content of the book induces the informed dealer to bid more aggressively.

### 3.1.2 Case 2: The informed dealer chooses schedule $T$ when there is an information event.

Now we turn to the case in which a thin book constitutes the signal that the cost of liquidity provision is large. The analysis is qualitatively similar to that supporting the
two previous propositions. One difference is noteworthy: in this case, the expected quoted spread is also affected by asymmetric information among dealers. The next proposition describes the conditions under which there exists an equilibrium with bluffing.

**Proposition 5**: Let \( \beta^{**} \triangleq \frac{(\alpha-r)}{(\alpha-r)+47(2r-\alpha)} \). When \( 0 \leq \beta < \beta^{**} \) and \( \frac{2\alpha\sigma}{\alpha+2} < \Delta < \alpha\sigma \), the following bidding strategies constitute an equilibrium:

1. When \( \Psi = 1 \), the informed dealer posts schedule \( T \). When \( \Psi = 0 \), the informed dealer posts schedule \( D \) with probability \( m^{**}(\beta) = (1-\beta^{**}(\beta))(1-(\alpha-r)) \) and schedule \( T \) with probability \( (1-m^{**}(\beta)) \) with \( 0 < m^{**} < 1 \).

2. When the book is thin, the follower adds 1 round lot at price \( A_{1} \) with probability \( u^{*} = \frac{3}{4} \) and else does nothing. When the book is shallow, the follower adds 1 round lots at price \( A_{1} \) and when the book is deep, the follower does nothing.

3. Compared to the benchmark case: (i) the expected depth at price \( A_{1} \) is smaller and (ii) the average quoted spread is larger.

Observe that the condition \( \Delta < \alpha\sigma \) guarantees that \( \beta^{**} > 0 \). Furthermore the condition \( \beta < \beta^{**} \) guarantees that the probability \( m^{**}(\beta) \) is strictly smaller than 1. The intuition and the interpretation of the previous proposition are exactly as those given for Proposition 4. In particular the informed dealer bids more aggressively when \( \beta \) increases in order to maintain his influence on the follower’s belief. The average quoted spread is larger than in the benchmark case because (i) the informed dealer sometimes chooses to make no offer at price \( A_{1} \) and (ii) the uninformed dealer does not necessarily improve upon price \( A_{2} \). When \( \beta \geq \beta^{**} \), the informational content of the book is too small to deter the follower from adding at least one round lot at price \( A_{1} \) when the book is thin. Thus there is no equilibrium with bluffing and we obtain the following result.

**Proposition 6**: When \( \beta \geq \beta^{**} \) and \( \frac{2\alpha\sigma}{\alpha+2} < \Delta < \alpha\sigma \) then the following bidding strategies constitute an equilibrium:

1. When \( \Psi = 1 \), the informed dealer chooses schedule \( T \). When \( \Psi = 0 \), the informed dealer chooses schedule \( D \).
2. When the book is thin, the follower adds only 1 round lot at price $A_1$ if $\beta^{**} \leq \beta \leq \beta$ and 2 round lots if $\beta < \beta \leq 1$ with $\beta \overset{\text{def}}{=} \frac{(2-r)\sigma^r}{(2-r)\sigma^r+4\gamma(2-r)\alpha}$. When the book is shallow, the follower adds 1 round lots at price $A_1$ and when the book is deep, the follower does nothing.

3. Compared to the benchmark case: (i) the expected depth at price $A_1$ is smaller if $\beta^{**} \leq \beta \leq \beta$ and unchanged if $\beta < \beta \leq 1$ and (ii) the average quoted spread is unchanged.

Thus for $\beta$ large enough (here $\beta \geq \beta^{**}$), there is no bluffing equilibrium when $\frac{2\alpha\sigma}{\alpha+2} < \Delta < \alpha\sigma$. Thus the conclusions of this subsection are qualitatively similar to those we obtained in the previous subsection. Bluffing strategies emerge only when $\beta$ is small enough in the anonymous market.

3.2 The Non-Anonymous Trading Mechanism

Intuitively, the equilibria in the non-anonymous market can be derived by considering a special case of the equilibria in the anonymous market. Consider the polar situation in which $\beta = 0$. In this case, in the anonymous market, the uninformed dealer knows that the leader is an informed dealer, even though she does not directly observe his identity. Thus the equilibrium path in the non-anonymous market when the leader is an informed dealer must be exactly as in the anonymous market when $\beta = 0$. This remark yields the next corollary.

**Corollary 1**: In the non-anonymous market, when the leader is informed, there is an equilibrium in which dealers’ equilibrium strategies are:

1. as given in Proposition 4 when $\alpha\sigma < \Delta$ and $\beta = 0$. In particular, when $\Psi = 0$, the informed dealer chooses schedule $D$ with probability $m^*(0)$.
2. as given in Proposition 5 when $\alpha\sigma \geq \Delta$ and $\beta = 0$. In particular, when $\Psi = 0$, the informed dealer chooses schedule $D$ with probability $m^{**}(0)$.

As $m^*(\beta)$ and $m^{**}(\beta)$ increases with $\beta$, we obtain

$$m^*(\beta) > m^*(0) \text{ and } m^{**}(\beta) > m^{**}(0).$$
This means that the informed dealer behaves more competitively in the anonymous market than in the non-anonymous market. A switch to anonymity decreases the informational content of the book, other things equal. From the point of view of the informed dealer, it is as if $\beta$ was increased. As explained in the previous section, this effect induces the informed dealer to post more aggressive limit orders in absence of an information event so as to maintain the deterrence effect of a shallow or a thin book.

Now consider the case in which the leader is a precommitted liquidity supplier. In this case, the limit orders posted in the first stage contains no information and the uninformed dealer optimally behaves as in the benchmark case. In all cases, she fills the book so that 2 round lots are offered at price $A_1$. Notice that in this case, the uninformed dealer bids more aggressively in the non-anonymous market if $\beta < \beta^*$ or if $\beta < \beta^{**}$. Actually in the anonymous market, the uninformed dealer might decide not to improve upon a shallow or a thin book, as pointed out in the previous section.

Overall, the effect of anonymity on the uninformed dealer’s bidding behavior is ambiguous. For instance consider the case in which $\alpha \sigma < \Delta$ and the book is shallow. In the non-anonymous market, the uninformed dealer adds depth to the book with probability $u^* = 0.5$ if the leader is informed and with probability 1 if the leader is a precommitted trader. Thus the unconditional probability that the follower will offer one additional round lot at price $A_1$ when the book is shallow is:

$$\text{Prob}_{\text{na}}(\text{not deterred} | K = S) = \frac{1 - \beta}{2} + \beta = \frac{1 + \beta}{2},$$

in the non-anonymous environment. In the anonymous market, if $\beta < \beta^*$, the uninformed dealer adds depth to the book with probability $u^* = 0.5$ whether the leader is informed or not. In contrast, if $\beta \geq \beta^*$, the uninformed dealer adds depth to the book at price $A_1$ with probability 1 in the anonymous market. As $0.5 < \text{Prob}_{\text{na}}(\text{not deterred} | K = S) < 1$, we conclude that the uninformed dealer can bid more or less aggressively in the anonymous market depending on the value of $\beta$.

### 3.3 Anonymity and Market Liquidity

In this section, we analyze the impact of anonymity on our measures of market liquidity. We have shown in the previous section that a switch to anonymity induces
the informed dealer to behave more aggressively but has an ambiguous impact on the follower’s aggressiveness. Thus the effect of anonymous trading on liquidity is unclear.

**Proposition 7 :**

1. If $\alpha \sigma > \Delta$, a switch to an anonymous limit order book decreases the expected spread and increases the expected depth of the limit order book if and only if $\beta > \beta^*$. 

2. If $\alpha \sigma \leq \Delta$, a switch to an anonymous limit order book increases the expected depth of the limit order book if and only if $\beta > \beta^*$. For all values of $\beta$, it leaves the expected spread unchanged.

This proposition shows that a switch from non-anonymous to anonymous trading should have an impact on market liquidity because it alters uninformed and informed dealers’ bidding strategies. It yields 2 testable hypotheses:

**H.1:** The average quoted spread in a trading system where liquidity suppliers’ identities are concealed is significantly different from the average quoted spread in a trading system where liquidity suppliers’ identities are disclosed.

**H.2:** The average quoted depth in a trading system where liquidity suppliers’ identities are concealed is significantly different from the average quoted depth in a trading system where liquidity suppliers’ identities are disclosed.

We will test these predictions by considering the switch to an anonymous order book which took place on Euronext Paris in April 2001. Notice that according to the previous proposition, the direction of the change in market liquidity depends on the proportion of precommitted liquidity suppliers. Market liquidity improves only if this proportion is large enough. We cannot directly test this prediction since we do not observe principal and agency orders. However, empirical studies of the French market suggest that for CAC40 stocks, the order flow stems mainly from agency orders. In particular, Declerck (2001) reports that the 6 intermediaries which handled 71% of all principal principal trades accounted for only 39% of all orders during her study period. Furthermore, on average principal trading accounted for 27% of the trading volume. This suggests that $\beta$ is relatively high for CAC40 stocks that we consider in
our empirical analysis. Thus for these stocks we expect a decrease in the spread and an increase in the depth following the switch to non-anonymity.

4 Empirical Analysis

4.1 Institutional Background and Dataset

4.1.1 Euronext Paris

In March 2000, the Amsterdam Stock Exchange, the Brussel Stock Exchange and the Paris Bourse decided to merge. This merger (which took place in September 2000) gave birth to Euronext, a holding with 3 subsidiaries: Euronext Amsterdam, Euronext Brussels and Euronext Paris. Since the merger, the 3 exchanges have strived to create a unique trading platform (called NSC). This goal is achieved since October 29, 2001 and members of one exchange can directly trade on the other exchanges. However, as of today, the 3 exchanges still have separate limit order books for each stock (mainly because clearing houses for French, Dutch and Belgian stocks still differ). Euronext Paris was first to adopt the new trading platform on April 23, 2001, soon followed by Brussels on May 21, 2001 and Amsterdam on October 29, 2001. For Euronext Paris, the trading rules were very similar before and after the switch to NSC. Indeed, for very liquid stocks, the switch to an anonymous limit order book was the only significant change (see below).

NSC is an electronic limit order market which is very similar to other limit order markets (e.g. Island, the Toronto Stock Exchange or the Stockholm Stock Exchange).\(^\text{19}\) Trading occurs continuously from 9:00 a.m. to 5:25 p.m. for most of the stocks.\(^\text{20}\) The opening and the closing prices are determined by a call auction. All orders are submitted through brokers who trade for their own account or on behalf of other investors. Traders mainly use two types of orders: (a) limit orders and (b) market orders. Limit orders specify a limit price and a quantity to buy or to sell at the limit price. Limit orders are stored in the limit order book and executed in

\(^{19}\)Description of the trading process in the Paris Bourse can be found in recent empirical papers; for instance Biais, Hillion and Spatt (1995), Harris (1996) or DeJong, Nijman and Röell (1996).

\(^{20}\)Less liquid stocks trade in call auctions which take place at fixed point in time during the trading day. No stocks trade in this way in our sample.
sequence according to price and time priority. If the limit price crosses a limit on the opposite side of the book (so called “marketable limit orders”) then the limit order is immediately executed (entirely or partially depending on its size). Market orders execute upon arrival against the best price on the opposite side of the book. Any quantity in excess of the depth available at this price is transformed in a limit order at that price. Consequently, in Euronext, market orders are not allowed to walk up or down the book (they can be viewed as marketable limit order at the best price on the opposite side of the book).

All limit orders must be priced on a pre-specified grid. The tick size is a function of the stock price level. At the time of our study, the tick size is 0.01 euros for prices below 50 euros, 0.05 euros for prices between 50.05 and 100 euros, 0.1 euros for prices between 100.1 euros and 500 euros and 0.5 euros for prices above 500 euros.

The transparency of the market is quite high. Brokers observe (on their computer terminals) all the visible limit orders (price and associated depth) standing in the book at any point in time. The 5 best limits on each side of the book, the total depth available at these limits and the number of orders placed at each limit are disclosed to the public. The depth available in the book can be larger than the visible depth. Actually NSC enables traders to display only a portion of their limit order by submitting hidden orders. The hidden portion retains price priority but loses time priority. A fraction of the hidden quantity becomes visible only when the quantity initially disclosed is fully executed.

Until April 23, 2001, **but not after**, the identification code of the issuing broker was also displayed for each order standing in the book. We refer to this change in the trading organization as **a switch to supply side anonymity**. Euronext Paris classifies stocks in different categories based on their market capitalization and trading rules vary from one category to another. The switch to supply side anonymity affected all categories. However, for medium-capitalization stocks, it was accompanied by another major change. For these stocks, contraparties IDs’ used to be disclosed immediately after completion of their transaction until April 23, 2001. This is not the case anymore since this date. Thus medium capitalization stocks have experienced a change in both supply side and post-trade anonymity. For this reason, it is difficult to isolate the effects of supply side anonymity on measures of market liquidity for these stocks. Fortunately, contraparties IDs’ have always been concealed for large caps’.

---

21In April 2001, 1 euro was approximately worth 0.86 dollar.
Consequently, our empirical analysis focuses on stocks with large capitalization only.

4.1.2 The Dataset

We use data provided by Euronext Paris. Our dataset contains a time stamped record of all transactions and orders (price and quantities) submitted to the market from March 1 to May 30, 2001. As explained previously we focus on a subset of large capitalization stocks. More precisely our sample includes the stocks which compose the CAC40 index.\(^{22}\) We exclude stocks which dropped from or were added to the index during the period of our study. Furthermore, in order to avoid contamination of our findings due to the proximity of the event date, we drop two weeks of observations around April 23, 2001. Finally we drop all observations after May 20, 2001 in order to avoid confounding effects due to the adoption of NSC by Euronext Brussels.\(^{23}\) Eventually our data set contains 39 stocks and 28 trading days: (i) 14 trading days before the event from March 26 to April 12, 2001 and (ii) 14 trading days after the event from April 30 to May 20, 2001.

Additional but minor changes in trading rules took place for the stocks in our sample on April 23, 2001. Firstly, the Bourse changed some of the criteria which are used to select the opening price when there is a multiplicity of clearing prices at the opening. Secondly, it advanced of 5 minutes the end of the continuous trading session in order to facilitate the organization of the call auction determining the closing price. In our empirical analysis, we exclude observations collected during the first and the last 5 minutes of the continuous trading period. Thus our findings should not be contaminated by changes which affect the determination of opening and closing prices.

The Bourse also changed the treatment of orders which can trigger a trading halt. In Euronext Paris (as in many other exchanges) trading halts occur when price changes exceed pre-specified thresholds. Before April 23, 2001 traders had the possibility to submit marketable limit orders resulting in a halt without partial execution of their order. Thus traders could suspend the trading process without

\(^{22}\)These stocks represent 74% of the capitalization of the French market. Furthermore they concentrate a large fraction of the trading activity (approximately 84% of the average daily trading volume). See Decklerck (2001).

\(^{23}\)Arguably, this switch facilitated the access of Belgian traders to the French market. Thus it may have increased the number of participants to Euronext Paris.
bearing any direct cost. In contrast, since April 23, 2001 marketable limit orders triggering a halt are partially executed up to the threshold price. This change in the handling of trading halts applied to all stocks. Hence there is no obvious way to control for its possible effects. The empirical literature (e.g. Lee and Seguin (1994)) finds that volatility and trading volume are affected by the resumption of trading after a trading halt. As we want to capture the effect of concealing traders' IDs in normal trading conditions, we exclude observations collected during days in which there is a trading halt. We lose 4 observations in the pre-event period and 7 in the post-event period.

Table 1 presents some summary statistics (trading volumes, average trade sizes, prices and daily return volatility) for the whole sample (Panel C). The figures reveal a high level of trading activity for the stocks in our sample. The average daily number of transactions per stock is equal to 2320 and the average trading volume (in number of shares) is larger than 1.3 million shares per stock. The average price is equal to approximately 87 euros. Panel A contains information on trading volumes, average trade sizes, prices and volatility for the pre-event period (14 trading days in March 2001) whereas Panel B contains the same information for the post-event period (14 trading days in May 2001). As it can be observed, transaction sizes have increased in the post-event period whereas the number of trades is smaller, however. Overall these findings suggest that traders place larger orders and trade less frequently in the post-event period.

4.2 Empirical Findings

4.2.1 Spreads and Anonymity

Our first hypothesis states that the average quoted spread for the stocks in our sample should be significantly different before and after the switch to supply side anonymity in Euronext Paris. In order to test this hypothesis, we consider 3 standard measures of the quoted spread: (i) the quoted spread in euros, (ii) the quoted percentage spread (the quoted spread divided by the mid-quote) and (iii) the time weighted quoted spread in euros. For both the pre-event and the post-event periods and for each stock, we record the value of these different measures of the quoted spread each time a new order is submitted to the market. Then, for each stock, we compute
daily averages. In this way we obtain one observation of the spread per day and per stock. Finally we average the observations in Panels A and B separately, across stocks (equally weighted average) and across days. This procedure yields a measure of the average spread in the pre-event and the post-event periods, respectively. The results are given in Table 2.

The third column in Table 2 indicates the difference between the various measures of the quoted spread after the switch to supply side anonymity and before the switch. Bold types mean that this difference is significant at the 1% level for both parametric and non parametric tests (namely t-tests for both equal and unequal variances, the Wilcoxon 2-sample test and the Kruskal-Wallis test). Clearly, we reject the hypothesis that the switch to supply side anonymity had no effect on the spread. On average, the quoted spread and the time weighted quoted spread have decreased by 0.03 euros in the post-event period.

We also ran a similar test for each stock separately. For brevity, we just provide a summary of our findings. For all the stocks but one, we find a decrease in the spread following the switch to pre-trade anonymity. This decrease is significant for 23 stocks, (either at the 1% level or the 5% level for all parametric and non-parametric tests). For the remaining stocks, the decrease in the spread is not significant at the 5% level for at least one test.

Table 1 revealed that measures of trading activity, price levels and return volatility have changed over our study period. The decrease in the spread that we document in Tables 2 might be due to the changes in these variables (which are known to influence the spread, see Harris (1994)) rather than a change in supply side anonymity. In order to account for this possibility, following Madhavan et al. (2002), we run the following pooled regression:

\[ \text{Spread}_{i,t} = a_0 + a_1 P_{i,t} + a_2 Volu_{i,t} + a_3 Volat_{i,t} + a_4 Dum_i, \]  

(12)

where \( \text{Spread}_{i,t} \) is a measure of the average quoted spread for stock \( i \) on day \( t \), \( P_{i,t} \) is the average price level for stock \( i \) on day \( t \), \( Volu_{i,t} \) is a measure of the average trading volume of stock \( i \) on day \( t \), \( Volat_{i,t} \) is a measure of daily return volatility for stock \( i \) on day \( t \) and \( Dum_i \) is a dummy variable which is equal to 1 in the post-event period and zero in the pre-event period. The price level is a crude way to account for the effect of the tick size. The tick size constrains the minimum quoted spread.

\(^{24}\text{Complete results are available upon request.}\)
As the tick size increases with price level, we expect the coefficient on the price level to be positive when the dependent variable is the quoted spread in euros. When the dependent variable is the quoted percentage spread, Harris (1994) argues that price discreteness should cause the coefficient on price level to be negative. We expect spreads to decrease with volume if there are fixed costs in supplying liquidity. Finally, spreads should increase with volatility because the costs of being picked off for limit order traders are larger in volatile markets (see Foucault (1999)).

We first estimate Equation (12) using OLS. Table 3a reports the results of this regression when the dependent variable is (a) the quoted spreads or (b) the quoted percentage spread and when trading volume is measured in number of shares. For robustness, we also considered other specifications for Equation (12) where the trading volume is measured in number of trades or in euros. The findings are very similar. Hence we omit to report them for brevity.

All the coefficients are significant and have the expected sign. In particular, the quoted spread (in euros and in relative terms) decreases with trading volume and increases with volatility. Controlling for changes in prices, trading volume and volatility, the findings in Table 4 show that the quoted spread (in euros and in percentage) significantly decreased after the switch to supply side anonymity. The coefficient estimate on the dummy variable indicates that the switch triggered a decrease of 0.02 euros for the quoted spread.

OLS might not be adequate to estimate Equation (12) since there disturbances are likely to be correlated across stocks and over time. In this case, we may overestimate the statistical significance of our estimations. In order to take into account this concern, we also estimate the regression using Feasible Generalized Least Squares with corrections for (AR(1)) autoregressive, contemporaneously heteroscedastic, and contemporaneously correlated errors (Parks-Kwenta method). The results are given in Table 3b. They confirm that the switch to supply side anonymity triggered a significant decrease in the quoted spread and the quoted percentage spread.

### 4.2.2 Quoted Depth and Anonymity

Now we turn to our second testable hypothesis and we examine the impact of the switch to an anonymous order book on the number of shares supplied at the best quotes. To this end, we divide each trading day into intervals of 30 minutes. Then
for each trading day and for each stock, we compute the average number of shares (quoted depth) offered at the best ask price and at the best bid price at the end of each interval.  

We also compute the average quoted depth as the equally weighted average of the quoted depth at the bid and the quoted depth at the ask. In this way we obtain one observation per day of the quoted depth for each stock. Finally we average these observations across days and across stocks. We report the mean for each panel and each measure of the quoted depth in Table 4.

The third column of Table 4 shows that the number of shares offered at the bid and the ask has increased after the switch to the anonymous order book. This increase is statistically significant at the 5% level for both parametric and non parametric tests (namely t-tests for both equal and unequal variances, the Wilcoxon 2-sample test and the Kruskal-Wallis test). Furthermore it represents a substantial fraction of the depth at the bid and at the ask (the depth at the ask increased by 35% and the depth at the bid increased by 37%).

We also considered the evolution of the quoted depth for each stock separately. For all the stocks, but 3, the average quoted depth has increased after the switch. Furthermore this increase is significant at the 1% level for 12 stocks and the 5% level for 5 stocks for all parametric and non-parametric tests.

Finally, in order to control for possible effects of a change in volatility and trading activity on quoted depth, we also ran the following pooled regression:

\[
\text{Depth}_{i,t} = a_0 + a_1 P_{i,t} + a_2 \text{Vol}_{i,t} + a_3 \text{Volat}_{i,t} + a_4 \text{Dum}_i, \tag{13}
\]

where \( \text{Depth}_{i,t} \) is a measure of the average quoted depth for stock \( i \) on day \( t \) and the independent variables are defined as in Equation (12).

We estimate Equation (13) using Feasible Generalized Least Squares to allow for cross and serial correlations in disturbances. The results are given in Table 5. We find that the switch to supply side anonymity triggered an increase in the quoted depth of approximately 560 shares and that this increase is statistically significant at the 5% level.

\[\text{\footnotesize 25}\] Another procedure consists in recording the quoted depth each time a new order is submitted to the market. The findings in this case are very similar to those we report in the paper.

\[\text{\footnotesize 26}\] As a point of comparison, Madhavan et al. (2002) find a decrease of about 2% in the quoted depth after the change in the transparency of the Toronto Stock exchange in 1990.

\[\text{\footnotesize 27}\] Complete results are available upon request.
Overall the findings suggest that the switch to an anonymous limit order book has improved the liquidity of the market: (i) the quoted spread and (ii) the depth quoted at the best offers have significantly improved after the switch to an anonymous order book. Notice that this evolution of depth and spreads is consistent with our model when $\beta$ is large enough. In this case our model predicts that both the spread and the depth posted at the best quotes should increase.

There are two limitations, however. First, we do not take into account the hidden depth in our computation of the quoted depth. Second, our data do not allow us to investigate the evolution of the quantities offered away from the best quotes. If the hidden depth or the quantities offered at prices behind the best quotes have declined after the switch to an anonymous limit order book then the impact of this switch on market liquidity is ambiguous.

In future work, we plan to investigate the robustness of our empirical findings by (i) considering a larger sample of stocks and (ii) by comparing measures of market liquidity in March 2001 with measures of market liquidity in November 2002, rather than simply April-May 2002.

5 Conclusions

In this paper, we have analyzed the effect of concealing limit order traders’ identities on market liquidity. In our theoretical model limit order traders have asymmetric information on the true cost of limit order trading (which is determined by the risk of being picked off). Uninformed limit order traders draw inferences on this cost from the state of the book. A thin book can be due to untapped profit opportunities or a high cost of placing limit orders because the risk of being picked off is large. The last possibility reduces uninformed bidders’ inclination to add depth when the book is thin. Informed bidders exploit this effect by bidding less aggressively than when bidders have symmetric information. These strategic interactions have two implications:

- On average, the depth of the book is smaller and the quoted spread is larger when there is asymmetric information among limit order traders.
- Information on limit order traders’ IDs affects market liquidity. On the one
hand, informed limit order traders bid more aggressively when the limit order book is anonymous than when it is non-anonymous. On the other hand, uninformed limit order traders bid less aggressively (on average) when the limit order book is anonymous. The net effect on market liquidity depends on model parameters but in all the cases the depth of the book and the quoted spread are different when limit order traders' identities are concealed or disclosed.

We test this prediction using a natural experiment. On April 23, 2001, the limit order book for stocks listed on Euronext Paris became anonymous. For CAC40 stocks, we find that following this change, the average quoted spreads declined significantly while the average quoted depth increased significantly. This suggests that anonymity of the limit order book is an important pillar of the architecture of a limit order market.
References


Table 2: Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_2$</td>
<td>Final value of the security at Date 2</td>
</tr>
<tr>
<td>$\epsilon_t$</td>
<td>Innovation at date $t$</td>
</tr>
<tr>
<td>$v_0$</td>
<td>Unconditional expected value of the security</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Probability of order submission by a speculator if information event</td>
</tr>
<tr>
<td>$q$</td>
<td>Size of 1 round lot</td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>Prior probability of an information event</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Size of an innovation</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Probability that the leader is a precommitted trader</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Tick size</td>
</tr>
<tr>
<td>$A_j$</td>
<td>$j^{th}$ ask price on the grid above the unconditional expected value</td>
</tr>
<tr>
<td>$K$</td>
<td>State of the book at the end of the first stage</td>
</tr>
<tr>
<td>$\Phi_K$</td>
<td>Probability that the state of the book is $K$ if the leader is a pre-commited trader</td>
</tr>
<tr>
<td>$Q_{A_1}$</td>
<td>Depth of the book at price $A_1$</td>
</tr>
<tr>
<td>$Q_s$</td>
<td>Size of the market order submitted by a speculator</td>
</tr>
<tr>
<td>$Q_l$</td>
<td>Size of the market order submitted by a liquidity trader</td>
</tr>
<tr>
<td>$\pi_K$</td>
<td>Follower’s belief about the occurrence of an information event</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>Indicator variable (0 if there is no information event; 1 otherwise)</td>
</tr>
</tbody>
</table>
6 Appendix

6.1 Proofs

Preliminary Remarks. Let $\Pi^F(n, K)$ be the follower’s expected profit if she offers $n$ round lots at price $A_1$ conditional on the state of the book being $K$ at the end of stage $L$ and conditional on the arrival of a buy order at date 1 (this is the expected profit on a sell limit order). Obviously $\Pi^F(0, K) = 0$. Furthermore, it cannot be optimal for the follower to fill the book in such a way that more than 2 round lots are eventually offered at price $A_1$ (any round lot in excess of the second would be hit only in case of information arrival). Therefore we just need to compute $\Pi^F(2q, T)$, $\Pi^F(q, T)$ and $\Pi^F(q, S)$. We have

$$\Pi^F(2q, T) = [\pi_T[2\alpha(A_1 - (v_0 + \sigma))] + \frac{3}{2}(1 - \alpha)(A_1 - v_0)] + \frac{3}{2}(1 - \pi_T)(A_1 - v_0)] q,$$

which rewrites (using the expressions for $E_{\pi_T}(V \mid \tilde{Q} \geq q)$ and $E_{\pi_T}(V \mid \tilde{Q} \geq 2q)$ given in Equations (5) and (6)):

$$\Pi^F(2q, T) = [A_1 - E_{\pi_T}(V \mid \tilde{Q} \geq q) + \text{Prob}(\tilde{Q} \geq 2q \mid K = T)(A_1 - E_{\pi_T}(V \mid \tilde{Q} \geq 2q))] q,$$

(14)

where $\text{Prob}(\tilde{Q} \geq 2q \mid K = T) = \frac{\alpha \pi + 1}{2}$. Using the same type of reasoning we also obtain:

$$\Pi^F(q, T) = [A_1 - E_{\pi_T}(V \mid \tilde{Q} \geq q)] q$$

(15)

and

$$\Pi^F(q, S) = [\text{Prob}(\tilde{Q} \geq 2q \mid K = T)(A_1 - E_{\pi_S}(V \mid \tilde{Q} \geq 2q))] q.$$  

(16)

These expressions will be used in some of the proofs below.

**Proof of Lemma 1.** The proof follows directly from the arguments in the text. The reader can also check the claim by using the follower’s expected profits given in Equations (14), (15) and (16).■

**Proof of Proposition 1.** It follows from the argument before the proposition. ■

**Proof of Proposition 2.** We denote by $\Pi^L(K)$, the leader’s expected profit if he posts schedule $K$ conditional on the arrival of a buy order at date 1. When the dealers
have symmetric information, in equilibrium, the follower always fills the book so that 2 round lots are offered at price $A_1$ (since $\frac{2\alpha\sigma}{\alpha + 2} < \Delta$). We deduce that

$$\Pi^L(T) = 0,$$

$$\Pi^L(S) = \left[\pi_0 \left[\alpha(A_1 - (v_0 + \sigma)) + (1 - \alpha)(A_1 - v_0)\right] + (1 - \pi_0)(A_1 - v_0)\right]q = [A_1 - E_{\pi_0}(V \mid \tilde{Q} \geq q)]q,$$

$$\Pi^L(D) = \left[\pi_0 \left[2\alpha(A_1 - (v_0 + \sigma)) + \frac{3}{2}(1 - \alpha)(A_1 - v_0)\right] + \frac{3}{2}(1 - \pi_0)(A_1 - v_0)\right]q,$$

which rewrites (using the expressions for $E_{\pi_0}(V \mid \tilde{Q} \geq q)$ and $E_{\pi_0}(V \mid \tilde{Q} \geq 2q)$):

$$\Pi^L(D) = [A_1 - E_{\pi_0}(V \mid \tilde{Q} \geq q) + \text{Prob}(\tilde{Q} \geq 2q)(A_1 - E_{\pi_0}(V \mid \tilde{Q} \geq 2q))]q,$$

where $\text{Prob}(\tilde{Q} \geq 2q) = \frac{\alpha\pi_0 + 1}{2}$ is the probability that a buy order at date 2 is larger than 2 round lots (given that 2 round lots are offered at price $A_1$). Recall that

$$E_{\pi_0}(V \mid \tilde{Q} \geq 2q) = v_0 + \frac{2\pi_0 \alpha \sigma}{\pi_0 \alpha + 1} = v_0 + \frac{2\alpha \sigma}{\alpha + 2} < v_0 + \Delta,$$

and that $E_{\pi_0}(V \mid \tilde{Q} \geq q) < E_{\pi_0}(V \mid \tilde{Q} \geq 2q)$. Therefore we conclude that

$$E_{\pi_0}(V \mid \tilde{Q} \geq q) < E_{\pi_0}(V \mid \tilde{Q} \geq 2q) < A_1.$$

It immediately follows that

$$\Pi^L(T) < \Pi^L(S) < \Pi^L(D),$$

which proves that the dealer acting in stage $L$ chooses schedule $D$. The rest of the proposition directly follows from this result and Lemma 1. ■

**Proof of Proposition 3.** First we show that the follower’s bidding strategy is a best response to the informed dealer’s bidding strategy. First consider the case in which the book is shallow at the end of the first stage. As explained in the text,

$$A_1 - E_{\pi_s(t)}(V \mid \tilde{Q} \geq 2q) = \Delta - \left(\frac{2\pi_s(1, \beta)}{\pi_s(1, \beta)\alpha + 1}\right)\alpha \sigma \geq 0,$$

when $\beta \geq \beta^*$. We conclude (using Equation (16) in the preliminary remarks of the Appendix) that:

$$\Pi^F(q, S) > \Pi^F(0, S) = 0.$$

Thus if the book is shallow at the end of the first stage, the follower adds 1 round lot and eventually 2 round lots are offered at price $A_1$. The bidding strategy of the
informed leader is such that it never chooses a thin book. Thus a thin book does not lead the follower to revise her beliefs regarding the occurrence of an information event, i.e. \( \pi_T(1) = \pi_0 = 0.5 \). Therefore the follower behaves as in the benchmark case when she observes a thin book: she places a limit order for 2 round lots at price \( A_1 \). It ensues that the follower always fills the book so that eventually 2 round lots are offered at price \( A_1 \) if \( m = 1 \) and \( \beta \geq \beta^* \).

Next we show that the informed dealer’s bidding strategy is a best response. We denote by \( \Pi^L(K, \Psi) \), the leader’s expected profit in state \( \Psi \) if he posts schedule \( K \) conditional on the arrival of a buy order at date 1. When \( \Psi = 0 \), straightforward computations yield (taking into account the follower’s reaction):

\[
\Pi^L(T, 0) = 0,
\]

and

\[
\Pi^L(S, 0) = A_1 - v_0,
\]

and

\[
\Pi^L(D, 0) = E(\tilde{Q}_u)(A_1 - v_0) = \frac{3}{2}(A_1 - v_0).
\]

Thus when \( \Psi = 0 \), the leader optimally chooses schedule \( D \).

Now we consider the informed dealer’s optimal reaction when \( \Psi = 1 \). Recall that the follower’s reaction is such that she fills the book so that ultimately 2 shares are offered at price \( A_1 \). We deduce that:

\[
\Pi^L(T, 1) = 0,
\]

and

\[
\Pi^L(S, 1) = A_1 - E_1(V | \tilde{Q} \geq q),
\]

and

\[
\Pi^L(D, 1) = [A_1 - E_1(V | \tilde{Q} \geq q) + \text{Prob}(\tilde{Q} \geq 2q)(A_1 - E_1(V | \tilde{Q} \geq 2q))]q
\]

Observe that \( E_1(V | \tilde{Q} \geq q) = v_0 + \alpha \sigma \) and that \( E_1(V | \tilde{Q} \geq 2q) = v_0 + \frac{2\alpha \sigma}{\alpha + 1} \). Hence when

\[
\alpha \sigma < \Delta < \frac{2\alpha \sigma}{\alpha + 1},
\]

we have:

\[
\Pi^L(S, 1) > \max\{\Pi^L(T, 1), \Pi^L(D, 1)\}. 
\]
Thus when $\Psi = 1$, the leader optimally chooses schedule $S$. The last part of the proposition is immediate.$\blacksquare$

**Proof of Proposition 4.**

**Part 1.** We first show that the bidding strategy of the follower is a best response. First consider the case in which the book is shallow. In this case, $m^*(\beta)$ is such that

$$A_1 - E_{\pi_S(m^*)}(V \mid \tilde{Q} \geq 2q) = \Delta - \left(\frac{2\pi_S(m^*)}{\pi_S(m^*)\alpha + 1}\right)\alpha\sigma = 0.$$ 

It follows that

$$\Pi^F(q, S) = \Pi^F(0, S) = 0.$$ 

Thus when she observes a shallow book, the follower is indifferent between adding 1 round lot or doing nothing. It ensues that choosing to add 1 round lot with probability 0.5 or doing nothing with probability 0.5 is a best response for the follower. Second consider the case in which the book is thin. In equilibrium, the informed dealer never chooses a thin book (whether $\Psi = 1$ or not). Thus when she observes a thin book, the follower does not update her beliefs and behaves as in the benchmark case.

Now we show that the informed dealer’s bidding strategy is a best response. When $\Psi = 0$, taking into account the follower’s optimal reaction, we obtain:

$$\Pi^L(T, 0) = 0,$$

$$\Pi^L(S, 0) = (A_1 - v_0)q + \frac{1}{4}(A_2 - v_0)q,$$

$$\Pi^L(D, 0) = E(\tilde{Q}_u)(A_1 - v_0) = \frac{3}{2}(A_1 - v_0).$$

As $A_1 - v_0 = \Delta$ and $A_2 - v_0 = 2\Delta$, we have

$$\Pi^L(S, 0) = \Pi^L(D, 0) > \Pi^L(T, 0).$$

Thus when $\Psi = 0$, the informed dealer is indifferent between schedules $S$ and $D$. It follows that choosing schedule $S$ with probability $(1 - m^*)$ and schedule $D$ with probability $m^*$ is a best response.

When $\Psi = 1$, we have *(given the follower’s reaction)*:

$$\Pi^L(T, 1) = 0.$$
Furthermore
\[ \Pi^L(S, 1) = [\alpha(A_1 - (v_0 + \sigma)) + (1 - \alpha)((A_1 - v_0) + \frac{1}{4}(A_2 - v_0))]q, \]
which rewrites
\[ \Pi^L(S, 1) = [A_1 - E_1(V | \tilde{Q} \geq q)]q + \frac{(1 - \alpha)}{4}(A_2 - v_0)q. \]
Finally
\[ \Pi^L(D, 1) = [A_1 - E_1(V | \tilde{Q} \geq q) + \text{Prob}(\tilde{Q} \geq 2q)(A_1 - E_1(V | \tilde{Q} \geq 2q))]q. \]
Observe that \( E_1(V | \tilde{Q} \geq q) = v_0 + \alpha \sigma \) and that \( E_1(V | \tilde{Q} \geq 2q) = v_0 + \frac{2\alpha \sigma}{\alpha + 1} \). As \( \alpha \sigma \leq \Delta < \frac{2\alpha \sigma}{\alpha + 1} \) we have:
\[ \Pi^L(S, 1) \geq \text{Max}\{\Pi^L(T, 1), \Pi^L(D, 1)\}. \]
Thus when \( \Psi = 1 \), the leader optimally chooses schedule \( S \).

**Part 2.** On the equilibrium path, there is at least 1 round lot offered at price \( A_1 \). Thus the quoted spread is \( (A_1 - v_0) \) with certainty when a market order arrives at date 1, as in the benchmark case. Recall that \( \tilde{Q}_1 \) is the quantity offered at price \( A_1 \) when a market order arrives at date 1. In equilibrium, this quantity is either \( q \) or \( 2q \). Thus
\[ E(\tilde{Q}_1) = q(1 + \text{Prob}(\tilde{Q}_1 = 2q)). \]
In equilibrium, the leader (informed or not) sometimes chooses to post a shallow book. Furthermore the follower does not necessarily add depth to a shallow book. It ensues that \( \text{Prob}(\tilde{Q}_1 = 0) < 1 \), which implies \( E(\tilde{Q}_1) < 2q \).

**Proof of Proposition 5** The proof of this proposition is very similar to the proof of Proposition 4. Thus we omit it for brevity. It can be obtained upon request. We just prove that in equilibrium, the expected spread is larger than in the benchmark case when \( \beta < \beta^{**} \) and \( \alpha \sigma > \Delta \). The expected spread in equilibrium is given by:
\[ EA - v_0 = \Delta(1 + \text{Prob}(\tilde{Q}_1 = 0)), \]
where \( \text{Prob}(\tilde{Q}_1 = 0) \) is the probability that no depth is offered at price \( A_1 \) in equilibrium. In equilibrium, the leader (informed or not) sometimes chooses to post a thin book (when \( \beta < \beta^{*} \) and \( \alpha \sigma > \Delta \)). Furthermore the follower does not necessarily submit a limit order at price \( A_1 \) when the book is thin. It ensues that \( \text{Prob}(\tilde{Q}_1 = 0) > 0 \), which implies \( EA - v_0 > \Delta \).
Proof of Proposition 6

Part 1. For $\beta^{**} < \beta \leq \overline{\beta}$, it is easily checked that $\pi_T(1)$ (which depends on $\beta$) is such that
\[
A_1 - E_{\pi_T(1)}(V | \tilde{Q} \geq 2q) = \Delta - \left( \frac{2\pi_T(1)}{\pi_T(1)\alpha + 1} \right)\alpha\sigma < 0.
\]
Thus when the book is thin and $\beta^{**} < \beta \leq \overline{\beta}$, the follower never submit a limit order for two round lots at price $A_1$ since he would lose money on the second round lot. Furthermore, it is easily shown that $m^{**}(\beta)$ is such that
\[
A_1 - E_{\pi_T(1)}(V | \tilde{Q} \geq q) = \Delta - \pi_T(1)\alpha\sigma > 0.
\]
Thus when the book is thin, the follower is better off submitting a limit for one round lot at price $A_1$ than doing nothing. For $\beta > \overline{\beta}$,
\[
A_1 - E_{\pi_T(1)}(V | \tilde{Q} \geq 2q) = \Delta - \left( \frac{2\pi_T(1)}{\pi_T(1)\alpha + 1} \right)\alpha\sigma > 0,
\]
Thus when the book is thin and $\overline{\beta} < \beta$, the follower submits a limit order for two round lots at price $A_1$. In equilibrium, the informed dealer never chooses a shallow book (whether $\Psi = 1$ or not). Thus when she observes a shallow book, the follower does not update her beliefs and behaves as in the benchmark case. These arguments establish the second part of the proposition.

Now we show that the informed dealer’s bidding strategy is a best response. When $\Psi = 0$, taking into account the follower’s optimal reaction, we obtain:
\[
\Pi^L(T, 0) \leq \frac{1}{2}(A_2 - v_0).
\]
(The inequality is binding only when $\beta^{**} < \beta \leq \overline{\beta}$). Furthermore,
\[
\Pi^L(S, 0) = (A_1 - v_0),
\]
and
\[
\Pi^L(D, 0) = E(\tilde{Q}_u)(A_1 - v_0) = \frac{3}{2}(A_1 - v_0).
\]
Hence $\Pi^L(D, 0) > \text{Max}\{\Pi^L(S, 0), \Pi^L(T, 0)\}$. This means that the informed dealer establishes a deep book when $\Psi = 0$. Now consider the case in which $\Psi = 1$. We have (given the follower’s reaction):
\[
\Pi^L(T, 1) \geq 0.
\]
Furthermore
\[ \Pi^L(S, 1) = [\alpha(A_1 - (v_0 + \sigma)) + (1 - \alpha)((A_1 - v_0))]q, \]
which rewrites
\[ \Pi^L(S, 1) = [A_1 - E_1(V | \tilde{Q} \geq q)]q. \]
Finally
\[ \Pi^L(D, 1) = [A_1 - E_1(V | \tilde{Q} \geq q) + \text{Prob}(\tilde{Q} \geq 2q)(A_1 - E_1(V | \tilde{Q} \geq 2q))]q. \]

Observe that \( E_1(V | \tilde{Q} \geq q) = v_0 + \alpha \sigma \) and that \( E_1(V | \tilde{Q} \geq 2q) = v_0 + \frac{2\alpha \sigma}{\alpha + 1} \). As \( \Delta < \alpha \sigma < \frac{2\alpha \sigma}{\alpha + 1} \) we have:
\[ \Pi^L(T, 1) \geq 0 > \max\{\Pi^L(T, 1), \Pi^L(D, 1)\}. \]
Thus when \( \Psi = 1 \), the leader optimally chooses schedule \( T \).

**Part 2:** The last part of the proposition is straightforward and can be proved using the same steps as Part 2 in Proposition 4. Thus we omit this proof for brevity.

**Proof of Corollary 1.** It follows immediately from the arguments in the text.

**Proof of Proposition 7.**

**Part 1.** The case in which \( \alpha \sigma \leq \Delta \). A superscript “a” (resp. “na”) indexes the value of a variable in the anonymous (resp. non-anonymous) market. When \( \alpha \sigma < \Delta \),
\[ E(\tilde{Q}_1^j) = q(1 + \text{Prob}(\tilde{Q}_1^j = 2q)), \quad \text{for } j \in \{a, na\} \]
as explained in the proof of Proposition 4. We deduce that the difference between the expected depth in the anonymous market and the expected depth in the non-anonymous markets is:
\[ E(\tilde{Q}_1^a) - E(\tilde{Q}_1^{na}) = (\text{Prob}(\tilde{Q}_1^a = 2q) - \text{Prob}(\tilde{Q}_1^{na} = 2q))q. \]

When \( \beta > \beta^* \), we have \( \text{Prob}(\tilde{Q}_1^a = 2q) = 1 \). This follows from Proposition 3. Furthermore, using the bidding strategies described in Section 3.2,
\[ \text{Prob}(\tilde{Q}_1^{na} = 2q) = \beta + (1 - \beta)(\frac{2 + m^*}{4}) < 1. \]
Thus for $\beta > \beta^*$, $E(\tilde{Q}^a_1) - E(\tilde{Q}^{na}_1) > 0$. When $\beta \leq \beta^*$, using the equilibrium bidding strategies described in Proposition 4 and in Corollary 1 we obtain:

$$\Pr(\tilde{Q}^a_1 = 2q) = \beta(\Phi_T + \Phi_D + \frac{\Phi_S}{2}) + \frac{(1 - \beta)}{2}(1 + \frac{m^*(\beta)}{2}),$$

and

$$\Pr(\tilde{Q}^{na}_1 = 2q) = \beta(\Phi_T + \Phi_D + \Phi_S) + \frac{(1 - \beta)}{2}(1 + \frac{m^*(0)}{2}).$$

We deduce that

$$E(\tilde{Q}^a_1) - E(\tilde{Q}^{na}_1) = \frac{\beta\Phi_S}{2}(m^*(0, \Phi_S) - 1)q < 0.$$ 

When $\alpha\sigma \leq \Delta$, the follower always fills the book so that at least 1 round lot is offered at price $A_1$, for all values of $\beta$. Thus the expected quoted spread is not affected by anonymity in this case.

**Part 2.** The case in which $\alpha\sigma > \Delta$. We can follow the same reasoning as in Part 1 to show that the expected depth is larger in the anonymous market iff $\beta \geq \beta^*$. Now we consider the effect of a switch to an anonymous trading environment on the expected spread. In equilibrium the expected spread is given by:

$$EA^j - v_0 = \Delta(1 + \Pr(\tilde{Q}^j_1 = 0)), \text{ for } j \in \{a, na\}$$

where $\Pr(\tilde{Q}^j_1 = 0)$ is the probability that no depth is offered at price $A_1$ in equilibrium. We deduce that the difference between the expected spread in the anonymous market and the expected depth in the non-anonymous markets is:

$$EA^a - EA^{na} = (\Pr(\tilde{Q}^a_1 = 0) - \Pr(\tilde{Q}^{na}_1 = 0))\Delta.$$ 

When $\beta \geq \beta^{**}$, we have $\Pr(\tilde{Q}^j_1 = 0) = 0$. This follows from Proposition 6. Furthermore, using the bidding strategies described in Section 3.2,

$$\Pr(\tilde{Q}^{na}_1 = 0) = \frac{(1-\beta)}{8}(2 - m^{**}(0)) > 0.$$ 

Thus for $\beta \geq \beta^*$, $EA^a - EA^{na} < 0$. For $\beta < \beta^{**}$, using the equilibrium bidding strategies described in Proposition 5, we obtain:

$$\Pr(\tilde{Q}^a_1 = 0) = \frac{\beta\Phi_T}{4} + \frac{(1-\beta)}{8}(2 - m^{**}(\beta)),$$

Hence

$$EA^a - EA^{na} = \frac{\beta\Phi_T}{4}(1 - \frac{m^{**}(0)}{2})\Delta > 0.$$ 

We conclude that a switch to anonymity decreases the expected spread iff $\beta \geq \beta^*$ when $\alpha\sigma > \Delta$. As $\beta^* < \beta^{**}$, we have proved the first part of the proposition.
Figure 1

Date 1: Tree Diagram of the Trading Process.
Figure 2: The Bidding Game

Stage L

1 - $\beta$

$\beta$

Informed Dealer

Pre-Committed Trader

Uninformed Dealer

Stage F

Uninformed Dealer

Figure 3

$E_{\pi_S}(V|Q \geq 2q)$

$A_i = V_0 + \Delta$

$V_0 + (2\alpha \sigma)/(\alpha + 2)$

$\beta < \beta^*$

$\beta > \beta^*$
### Table 1

**Summary Statistics**

This table reports the mean daily averages of the number of trades, the transaction prices, the trading volume in shares and in euros (in 000s) and the trade size as well as the volatility (daily standard deviation of mid-quote returns) and the market capitalization for the 39 stocks in our sample. Panel A reports these numbers for 14 trading days from March 26, 2001 to April 12, 2001 and Panel B reports these numbers for 14 trading days from April 30, 2001 to May 20, 2001. Panel C reports the figures for the whole sample.

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Number of trades</td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
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<tr>
<td>Price</td>
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<td>85.43</td>
<td>90.11</td>
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<td>Trading Volume (Nber of Shares)</td>
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<td>1 358 679</td>
<td>1 439 810</td>
<td>81 131</td>
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<tr>
<td>Trading Volume (Euros '000)</td>
<td>92 287</td>
<td>85 683</td>
<td>98 928</td>
<td>13244</td>
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<tr>
<td>Trade Size</td>
<td>489</td>
<td>461</td>
<td>518</td>
<td>57</td>
</tr>
<tr>
<td>Intraday volatility (in %)</td>
<td>0.5224%</td>
<td>0.6060%</td>
<td>0.4384%</td>
<td>-0.1676%</td>
</tr>
<tr>
<td>Market capitalization in Euros (millions)</td>
<td>27 021</td>
<td>26 239</td>
<td>27 807</td>
<td>1 568</td>
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Table 2
Quoted spread

For each trading day and for each stock in our sample, we compute the Quoted Spread in Euros, the Quoted Percentage Spread (the quoted spread divided by the midquote), and the Time Weighted Quoted Spread (the quoted spread weighted by its length before a new revision in quotes). We thus obtain 14 observations in the pre-event period (Panel A) and 14 observations in the post-event period (Panel B) for each of the 39 stocks. We average these observations across days and stocks and we report the mean for each panel and each measure of the spread. We then test whether each measure of the spread is significantly different in panels A and B (last column) using parametric and non parametric tests (namely t-tests for both equal and unequal variances, the Wilcoxon 2-sample test and the Kruskal-Wallis test) on the basis of our daily observations for all stocks. Bold types indicate that the difference between the measure of the spread in the pre-event and the post-event periods is significantly different from zero at the 1% level. In the last column, we report the t-statistics of the t-test for unequal variances.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
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<tr>
<td>Quoted Spread (Euros)</td>
<td>0.1775</td>
<td>0.1454</td>
<td>-0.0320</td>
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<tr>
<td>Quoted Percentage Spread</td>
<td>0.2198</td>
<td>0.1696</td>
<td>-0.0503</td>
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<tr>
<td>Time Weighted Quoted Spread</td>
<td>0.1772</td>
<td>0.1466</td>
<td>-0.0307</td>
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Table 3a
Regression Model for the Quoted Spread

We run the following regression: $S_{i,t} = a_0 + a_1 P_{i,t} + a_2 V_{u,i,t} + a_3 V_{a,i,t} + a_4 D_{u,i,t} + \epsilon_{i,t}$ where $S_{i,t}$ is the average spread (quoted or percentage) for stock $i$ on day $t$, $P_{i,t}$ is the average closing price for stock $i$ on day $t$, $V_{u,i,t}$ is the average daily trading volume in Euros of stock $i$ on day $t$, $V_{a,i,t}$ is the midquotes’ returns volatility of stock $i$ on day $t$ (in percent), and $D_{u,i,t}$ is a dummy variable which is equal to 1 in the post-event period and zero in the pre-event period.

<table>
<thead>
<tr>
<th></th>
<th>Quoted Spread</th>
<th>Quoted Percentage Spread</th>
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<tbody>
<tr>
<td></td>
<td>Coefficients</td>
<td>t-stat</td>
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<tr>
<td>Intercept</td>
<td>0.000</td>
<td>0.040</td>
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<tr>
<td>Price</td>
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<td>70.43</td>
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<tr>
<td>Volume</td>
<td>-2.63E-07</td>
<td>-20.60</td>
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<tr>
<td>Volatility</td>
<td>9.283</td>
<td>13.94</td>
</tr>
<tr>
<td>Dummy</td>
<td>-0.021</td>
<td>-7.13</td>
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</table>
Regression Model for the Quoted Spread

We run the following regression: \[ S_{i,t} = a_0 + a_1 P_{i,t} + a_2 V_{o i u,t} + a_3 V_{o i l,i,t} + a_4 D_{u i m,t} + \varepsilon_{i,t} \] where \( S_{i,t} \) is the average spread (quoted or percentage) for stock \( i \) on day \( t \), \( P_{i,t} \) is the average price for stock \( i \) on day \( t \), \( V_{o i u,t} \) is the average daily trading volume in Euros of stock \( i \) on day \( t \), \( V_{o i l,i,t} \) is the midquotes' returns volatility of stock \( i \) on day \( t \) (in percent), and \( D_{u i m,t} \) is a dummy variable which is equal to 1 in the post-event period and zero in the pre-event period. We run the Prais-Winsten model with panel-corrected standard errors to allow for contemporaneous cross-correlation and for autocorrelation. We estimate the regression using Feasible Generalized Least Squares with corrections for \( (AR(1)) \) autoregressive, contemporaneously heteroscedastic, and contemporaneously correlated errors (Parks-Kwenta method).

<table>
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<tr>
<th></th>
<th>Quoted Spread</th>
<th>Quoted Percentage Spread</th>
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<tbody>
<tr>
<td></td>
<td>Coefficients</td>
<td>t-stat</td>
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<tr>
<td>Intercept</td>
<td>0.008</td>
<td>1.100</td>
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<tr>
<td>Price</td>
<td>1.7E-03</td>
<td>24.52</td>
</tr>
<tr>
<td>Volatility</td>
<td>7.791</td>
<td>12.14</td>
</tr>
<tr>
<td>Dummy</td>
<td><strong>-0.024</strong></td>
<td><strong>-5.38</strong></td>
</tr>
</tbody>
</table>
Table 4
Quoted depth

For each trading day and each stock, we compute the average number of shares (quoted depth) offered at the best ask price and at the best bid price every 30 minutes. We also compute the average quoted depth as the equally weighted average of the quoted depth at the bid and the quoted depth at the ask. Then we average each measure of the quoted depth across stocks in each day. We thus obtain 14 observations in the pre-event period (Panel A) and 14 observations in the post-event period (Panel B). We average these observations across days and we report the mean for each panel and each measure of the quoted depth. We then test whether each measure of the quoted depth is significantly different in panel A and B (last column) using parametric and non parametric tests (namely t-tests for both equal and unequal variances, the Wilcoxon 2-sample test and the Kruskal-Wallis test). Bold types indicate that the difference between the measure of the quoted depth in the pre-event and the post-event periods is significantly different from zero at the 5% level. The last column gives the t-statistics for the t-test with unequal variances.

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<tr>
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<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
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<tr>
<td>Depth at the Bid</td>
<td>1 361</td>
<td>1 873</td>
<td>512</td>
</tr>
<tr>
<td>Depth at the Ask</td>
<td>1 395</td>
<td>1 895</td>
<td>501</td>
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<tr>
<td>Quoted Depth</td>
<td>1 378</td>
<td>1 884</td>
<td>507</td>
</tr>
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</table>
Table 5

Regression Model for the Quoted Depth

We run the following regression: \( D_{i,t} = a_0 + a_1 P_{i,t} + a_2 Volu_{i,t} + a_3 Volat_{i,t} + a_4 Dum_{i,t} + \varepsilon_{i,t} \) where \( D_{i,t} \) is the average quoted depth (daily average of the quoted depth observed every 30 minutes, in number of shares) for stock \( i \) on day \( t \), \( P_{i,t} \) is the average closing price for stock \( i \) on day \( t \), \( Volu_{i,t} \) is the average daily trading volume in Euros of stock \( i \) on day \( t \), \( Volat_{i,t} \) is the midquotes' returns volatility of stock \( i \) on day \( t \) (in percent), and Dum_{i,t} is a dummy variable which is equal to 1 in the post-event period and zero in the pre-event period. We estimate the regression using Feasible Generalized Least Squares with corrections for (AR(1)) autoregressive, contemporaneously heteroscedastic, and contemporaneously correlated errors (Parks-Kwenta method).

<table>
<thead>
<tr>
<th>Quoted Depth</th>
<th>Coefficients</th>
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<tbody>
<tr>
<td>Intercept</td>
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<td>Price</td>
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<td>-9.17</td>
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<tr>
<td>Volume</td>
<td>6.46E-03</td>
<td>8.18</td>
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<tr>
<td>Volatility (divided by 1E5)</td>
<td>-1.193</td>
<td>-3.06</td>
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<tr>
<td>Dummy</td>
<td>242.378</td>
<td>2.18</td>
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