Arbitraging Arbitrageurs

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Abstract

This paper develops a theory of strategic trading in markets with large influential arbitrageurs. If arbitrageurs are not very well-capitalized, margin requirements or capital constraints make their trades predictable. Other market participants can exploit this by trading against them. Competitors may even find it optimal to lend to arbitrageurs that are financially fragile; additional capital makes the arbitrageurs more viable, and lenders can reap profits from trading against them for a longer time. The strategic behavior of these market participants has implications for the functioning of financial markets. Strategic trading may produce significant price distortions, increase price manipulation activities, and trigger forced liquidations of large traders.

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1 Introduction

In many financial and commodity markets a few large players account for a significant fraction of the trading volume. These large players are often arbitrageurs because their primary activity involves taking large positions to profit from small mispricings.

The presence of large players is usually well known to market participants. In some cases this is the result of reputation built over time. Examples include Yasuo Hamanaka, the Sumitomo trader once known as Mr. Five Percent among copper traders to reflect its holdings of 5% of the total copper market; Long-Term Capital Management, which was referred to as the Central Bank of Volatility because of its large short positions in equity derivatives; Metallgesellschaft, which was known as the largest Wall Street Refiner after it built positions in oil futures contracts equivalent to the annual production of Kuwait; and more recently, Enron, which was named Gas Bank for its overwhelming trading and risk management activities in electricity and gas markets. In other cases, the holding of large positions is known because of reporting by exchanges through which the positions are held. Derivatives exchanges, for example, regularly publish information on large positions held in the contracts traded on the exchange. An examination of a cross-section of reports published by the London Metal Exchange shows a concentration of open positions in many contracts held by a single market participant with a net position exceeding 50% of the open interest. Although position reports do not identify holders, identities are often known to other market participants. Finally, institutions that interact with large players, such as counterparties, clearing brokers and lenders, know of the presence and identities of the large traders.

In normal circumstances, well-capitalized arbitrageurs act as suppliers of liquidity, and their presence is key to the smooth functioning of markets. When arbitrageurs become financially constrained, however, they cannot take full advantage of mispricings and may even become partly paralyzed – the constraint limits their ability to provide liquidity, and also makes it less likely they can quickly reduce exposure by rebalancing their portfolios. Severely constrained arbitrageurs may have an incentive to move prices in an attempt to ensure their own survival. And when they finally liquidate, they are forced to dump large positions on the market. In this case, arbitrageurs' actions can temporarily cause prices to diverge considerably from values, and their presence destabilizes markets.

Sophisticated traders can use knowledge of the financial condition of an arbitrageur to benefit from the predictable price deviations caused by a constrained arbitrageur's trades. When the arbitrageur's financial condition is very weak, other traders provide liquidity and are beneficial to the arbitrageur when they increase the amount realized from liquidation of the arbitrageur's positions. For a healthier but still financially fragile arbitrageur, the trades of sophisticated traders can be detrimental enough to tip the balance against recovery of the arbitrageur, pushing it into insolvency. Actions of these traders push markets against the arbitrageur, once they determine its vulnerability and finish it off.

We model an arbitrageur who trades a single asset. The arbitrageur interacts with other traders, whom we refer to as strategic traders, who attempt to gain from knowledge of the arbitrageur's financial condition. When the arbitrageur is well-capitalized, strategic traders find it optimal to remain inactive. But, as the arbitrageur's financial condition weakens, strategic traders become active. The optimal actions of the arbitrageur and the strategic traders depend on the characteristics of the market in which the asset is traded, and on the arbitrageur's financial condition and the size of its positions.

If an arbitrageur is in danger of violating minimum capital requirements, liquidation of some outstanding positions is the most obvious way to regain control. In less than perfectly liquid markets, this is costly, as liquidation trades have an adverse impact on price and, through price, on the arbitrageur's capital. The price impact creates an incentive for strategic traders to take the other side of the trade, and by absorbing part of the trade help improve the arbitrageur's financial condition.

In thin markets, or for arbitrageurs with large positions, the effect of the adverse price impact can be so great that partial liquidation of positions is no longer viable. Then, the arbitrageur must either fully liquidate its positions, or move the asset price in an attempt to remain solvent. Price manipulation is costly. It involves trading at an unfavorable price – an arbitrageur with a long position needs to buy more of the asset to keep the price high, while an arbitrageur with a short position needs to sell more to keep the price low. Again, deviations of price from value create an incentive for strategic traders to enter the market and take the other side of the arbitrageur's trade. In this case, strategic traders raise the cost to the arbitrageur of manipulating the price. The arbitrageur chooses to fully liquidate positions when the cost to manipulating price exceeds the cost associated with full liquidation.

A strategic trader bases trades on its expectation of the arbitrageur's actions. In thin markets, when there is considerable uncertainty regarding the arbitrageur's choice of action, because it may either support the price to stay afloat or fully liquidate, the strategic trader prefers to stay inactive. A particular choice of action results in a profit for the strategic trader if the arbitrageur chooses to trade one way, but in a loss if the arbitrageur decides to trade the other way. In this case, the strategic trader may find it optimal to invest in the arbitrageur, even when the investment per se has a negative expected return. Such an investment by reducing uncertainty regarding the arbitrageur's actions increases the strategic trader's trading profit. The strategic trader gains from selling the asset at the artificially high prices that result as an arbitrageur with a long position buys more of the asset in a gamble to survive. If the arbitrageur's gamble for survival does not pay off, the strategic trader benefits from the collapse in prices that result when the arbitrageur is forced to dump the resulting larger positions. Even if the arbitrageur succeeds in ensuring its survival, the strategic trader benefits from the lower prices that result as the arbitrageur trims positions back to more normal levels. Providing finance to and simultaneously trading against the arbitrageur makes the strategic trader behavior predatory.

Our work relates to research that recognizes the importance of financial constraints for trading in financial markets. Shleifer and Vishny (1997) argue that, in general, potential investors of trading institutions observe only returns but not the investment opportunities available. Traders are then not always able to raise additional capital, and financial constraints become binding. In a liquid market, this leads to the inefficient liquidation of positions. Attari and Mello (2001) and Xiong (2001) take into account the fact that in illiquid markets individual traders influence asset prices. They show that financial constraints on traders in such markets can be responsible for periods of excessive volatility and even liquidity crises. All these papers ignore the possibility that other traders can take advantage of the restricted financial flexibility of large arbitrageurs, which significantly increases the risks that arbitrageurs face. Cai (2002) documents strategic trading by participants in T-bond futures markets during the collapse of LTCM in the second half of 1998. She shows that market participants, by selling ahead of LTCM, drove prices down and benefited from closing out the resulting short positions at lower prices. The model of predatory behavior in security markets in Brunnermeier and Pedersen (2002) is closest to the one presented here. While some of the implications of their model, that strategic behavior on the part of competitors adds to the risk of arbitrage, are similar to ours, the focus of the paper is on price dynamics and its resulting consequences and is thus quite different from ours.

Foster and Viswanathan (1994) explicitly address the interaction of smaller traders with large market participants when the degree of private information differs. Smaller traders balance between trading to make profits from their own information and waiting to learn from order flow about the information of large investors. Financial constraints do not play a role in their analysis, and differences in traders' information are not our focus.

Taking advantage of the financial fragility of competitors has been analyzed in papers that link product markets and corporate finance. Bolton and Scharfstein (1990) show that a firm may engage in predatory price reductions if its competitor is financially constrained. This occurs when an increase in future profits more than compensates for temporarily low current profits. We also link the plan of forcing a collapse and the importance of financing constraints for the success of such a trading strategy. In our case, the motivations and the actions of participants are different. In Bolton and Scharfstein, the predator benefits from increased market power after the collapse of the rival. In our approach, the predator takes advantage of depressed prices *during* the time of the collapse, as well as the arbitrageur's price manipulation efforts *before* the collapse. We also see different behaviors of the aggressors. Strategic traders may decide to lend to a financially fragile arbitrageur for it to continue trading for a longer period. We relate features typically seen in corporate finance to trading environments, and thus take account of aspects often overlooked in explaining the behavior of traders.

Section 2 describes the model used in Sections 3.1 and 3.2 to analyze market activity in two polar cases: when the arbitrageur has complete financial flexibility, and when it does not have any financial flexibility. In Section 3.3 the results are derived for an arbitrageur whose flexibility is between the two polar cases. In Section 3.4 we analyze strategic trader lending to the arbitrageur to improve its trading profit. A number of extensions are presented in Section 4. Section 5 concludes.

2 The Model

The model assumes a market for an asset that is less than perfectly liquid. The market price of the asset in this case is affected by the trading decisions of market participants. At the initial date, $t = 0$, traders are assumed to have positions in the asset accumulated from previous actions. We analyze trading decisions at dates $t = 1$ and $t = 2$.

The fundamental value of the asset is assumed to be constant over time and is denoted by I. The market price of the asset at time t is denoted by P_t. Each period, the price of the asset is set by a market clearing condition and may deviate from its value. More complex value processes can be accommodated without affecting the main results.

There are three groups of traders in the market: liquidity traders, a single arbitrageur, and a single strategic trader.

The aggregate demand of a large number of small liquidity traders is given by $D_t^L = \varepsilon_t - \beta(P_t - I)$, where D_t^L is the flow of liquidity trades in the interval $(t-1, t)$, and $\beta > 0$. Liquidity demand is composed of two parts. A random component, ε_t, which is independent and identically uniformly distributed on the interval $[-R, R]$, represents pure noise trading activity. The second term, $-\beta(P_t - I)$, takes into account that liquidity traders respond to deviations of the market price of the asset from its value. As prices fall, more buyers are drawn to the market, while as prices rise, more sellers appear. We refer to this group of liquidity traders as value traders, since they sell the asset when $P_t > I$ and purchase it when $P_t < I$. The coefficient $\beta < \infty$ is an indicator of market depth; the larger β is, the greater the sensitivity of value trader demand to price deviations.

Prices are determined by market clearing. In the absence of additional market participants, $P_t = I + \frac{\varepsilon_t}{\beta}$. That is, the price at t, P_t, is independent and identically distributed with a mean of I. We implicitly assume that the impact of the liquidity shock dies out completely in one period and has no impact on future prices, which is a reasonable specification for pure liquidity shocks. It is also possible to model liquidity shocks and price deviations as following persistent or mean-reverting processes. Our results are based on the predictable deviations of prices caused by the trades of a constrained arbitrageur. The presence of other sources of predictability in price deviations does not qualitatively affect our results.

2.1 The Arbitrageur

A risk-neutral arbitrageur will trade the asset with the objective of maximizing its expected wealth at $t = 2$. Let θ_{t-1} be the arbitrageur's position in the asset after trading at time $t-1$; $\theta_t - \theta_{t-1}$ is the quantity that the arbitrageur trades at time t. In addition to holding θ_t units of the asset, the arbitrageur has debt outstanding of B_t. The initial condition of the arbitrageur is characterized by θ_0 and B_0.

Each period the arbitrageur observes noise trader activity, ε_t, and uses its knowledge of the behavior of value traders and the trading strategies of the strategic trader to determine the optimal quantity to trade to maximize its expected wealth at $t = 2$. The arbitrageur's objective function is

$$\max_{\theta_1,\theta_2} E_0\left[\theta_2 V_2 - B_2\right] \tag{1}$$

where V_2, the value of the asset at $t = 2$, can be set at its fundamental value, I, or may reflect some continuation value of the asset if the problem considered here is a sub-problem of some longer horizon problem. Note that we are assuming unlimited liability on the part of the arbitrageur. We do this to ensure that our results are not driven by the risk-taking incentives that limited liability would induce.

The arbitrageur's debt, B_t, changes from one period to the next as the arbitrageur's position in the asset changes. Without loss of generality we assume a zero interest rate. If there are no intermediate additions or withdrawals of wealth, we can write the evolution of the amount owed by the arbitrageur as

$$B_t = B_{t-1} + \left(\theta_t - \theta_{t-1}\right) P_t \ . \tag{2}$$

The arbitrageur's objective function thus becomes

$$\max_{\theta_1,\theta_2} E_0\left[\left(\theta_2 - \theta_1\right)\left(V_2 - P_2\right) + \left(\theta_1 - \theta_0\right)\left(V_2 - P_1\right) + \theta_0 V_2 - B_0\right] \ . \tag{3}$$

The first two terms in the objective function are the profits from trading, quantity times value minus price, at dates 1 and 2, while the final term is the value of the initial position.

The arbitrageur faces a capital constraint that requires it to fund positions partially with its own capital. Let M be the amount of own capital required per unit position. The arbitrageur must satisfy the constraint

$$\theta_t P_t - B_t \geq \left|\theta_t\right| M. \tag{4}$$

The left-hand side of equation (4) is the net value of the arbitrageur's position at market prices, while the right-hand side is the total capital required.

Assume that the strategic trader trades x_t units at t. The price of the asset, set by the market clearing condition $\theta_t - \theta_{t-1} + x_t + D_t^L = 0$, is

$$P_t = I + \frac{\theta_t - \theta_{t-1}}{\beta} + \frac{x_t + \varepsilon_t}{\beta} \tag{5}$$

On substituting for P_t and B_t using (5) and (2) and rearranging terms, (4) simplifies to

$$\theta_t\left(I + \frac{x_t + \varepsilon_t}{\beta}\right) - B_t \geq \left|\theta_t\right| M - \theta_t\left(\frac{\theta_t - \theta_{t-1}}{\beta}\right).$$

The left-hand side is the arbitrageur's capital evaluated at the market price of the asset that would prevail if the arbitrageur did not trade. The second term on the right-hand side reflects the impact of the arbitrageur's own trades on the capital constraint, and depends on the illiquidity of the market. This term has the effect of increasing the total capital required when the arbitrageur liquidates positions and of reducing the total capital required when the arbitrageur builds positions. This can be seen by examination of an arbitrageur long in the asset, $\theta_t > 0$. If $\theta_{t-1} > \theta_t$ ($\theta_t > \theta_{t-1}$), this term is positive (negative), causing an increase (a reduction) in the total capital required to stay in business. The sensitivity of prices to the extent of arbitrage trading in the illiquid asset forces the arbitrageur to pay attention to the level of market liquidity when evaluating potential arbitrage trades.

For $\theta_t > 0$, the capital constraint can be rewritten as[1]

$$\beta\left(\theta_{t-1}I - B_{t-1}\right) - \theta_{t-1}\left(\theta_{t-1} - x_t - \varepsilon_t\right) \geq \theta_t\left(\beta M - \theta_{t-1}\right).$$

For initial positions that are small relative to the product of market depth and the per unit capital requirement, $\beta M > \theta_{t-1}$, the constraint places an upper bound on the arbitrageur's position at t

$$\frac{\beta\left(\theta_{t-1}I - B_{t-1}\right) - \theta_{t-1}\left(\theta_{t-1} - x_t - \varepsilon_t\right)}{\left(\beta M - \theta_{t-1}\right)} \geq \theta_t. \tag{6}$$

A negative value for the expression to the left of the inequality sign indicates that the constraint cannot be met, and the arbitrageur is forced to fully liquidate its positions and exit the market, $\theta_t = 0$. Otherwise, the arbitrageur can satisfy a binding capital constraint by limiting the size of its positions. Because of the price impact of its trades, the arbitrageur has to liquidate more than what it would have to if the market were perfectly liquid. As a matter of comparison, consider $x_t = \varepsilon_t = 0$. For a given θ_{t-1} and B_{t-1}, the upper bound on the arbitrageur's position in a market with no price impact, $\beta \to \infty$, is $\frac{\theta_{t-1}I - B_{t-1}}{M}$. The upper bound on the arbitrageur's position in an illiquid market, $\beta < \infty$, is $\frac{(\theta_{t-1}I - B_{t-1}) - \frac{\theta_{t-1}^2}{\beta}}{\left(M - \frac{\theta_{t-1}}{\beta}\right)} = \frac{\theta_{t-1}I - B_{t-1}}{M} - \theta_{t-1}\left(\frac{B_{t-1} - \theta_{t-1}(I-M)}{\beta M - \theta_{t-1}}\right)$. When the arbitrageur is close to the constraint, $B_{t-1} > \theta_{t-1}\left(I - M\right)$, the maximum position that it can hold in an illiquid market is smaller than the maximum position that it can hold in a liquid market.

For positions that are large relative to the product of market depth and the capital required, $\theta_{t-1} > \beta M$, the capital constraint places a lower bound on the arbitrageur's position of:

$$\theta_t \geq \frac{\theta_{t-1}\left(\theta_{t-1} - x_t - \varepsilon_t\right) - \beta\left(\theta_{t-1}I - B_{t-1}\right)}{\left(\theta_{t-1} - \beta M\right)}. \tag{7}$$

When the arbitrageur is close to the constraint, $B_{t-1} > \theta_{t-1}\left(I - M\right)$, the lower bound increases with market depth, β. In a highly liquid market, however, a capital constraint will not constitute a lower bound but rather an upper bound on the arbitrageur's holdings, since a sufficiently large β always implies that $\beta M > \theta_{t-1}$.

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[1] For $\theta_t < 0$ we get $\beta\left(\theta_{t-1}I - B_{t-1}\right) - \theta_{t-1}\left(\theta_{t-1} - x_t - \varepsilon_t\right) \geq -\theta_t\left(\beta M + \theta_{t-1}\right)$.

2.2 The Strategic Trader

The strategic trader has information on the arbitrageur's financial condition and knowledge of the trading strategy of the arbitrageur and value traders. Unlike the arbitrageur and value traders, however, the strategic trader does not observe the liquidity shock experienced by noise traders. The strategic trader trades the asset when it perceives opportunities to profit from trading on information regarding the arbitrageur's financial condition.

Specifically, the strategic trader knows the arbitrageur's position in the asset as well as its financial condition (borrowings). The extent of the arbitrageur's positions may be known because the arbitrageur is known to hold large positions in the asset, because the size of its trades make it visible to others, or because the exchange publishes information on large positions. The arbitrageur's financial condition can become known in a number of different ways. Observed adverse price movements along with knowledge of the arbitrageur's positions may reveal financial difficulties; or information may become available if the arbitrageur attempts to raise additional capital; or it may be revealed by an attempt to reduce positions. For arbitrageurs that are part of publicly held companies, financial condition may also become known through the release of accounting information.

Strategic traders include banks that trade with and lend to arbitrageurs, institutions that act as clearing brokers for arbitrageurs, and more generally traders in assets frequently traded by the arbitrageur. These institutions understand the workings of financial markets well – represented by their knowledge of trading strategies, but may not have a high level of expertise in all assets – which we capture through their lack of knowledge of the liquidity shock. Another group of potential strategic traders are competitors of the arbitrageur who would have the same information as the arbitrageur. We consider the strategic behavior of a competing arbitrageur in Appendix A. The results are similar to those presented in the main text.

We assume that strategic traders have less information than arbitrageurs because we want to show that the pool of potential strategic traders is wider than at first thought, and not just the very few with the best trading platforms, computer programs, and talented traders. According to Cai (2002), a good number of market makers and futures brokers took advantage of LTCM.[2] If we can show that informationally disadvantaged strategic traders can profit from trading with better informed arbitrageurs, it is straightforward to extend the results to strategic traders who have the same information as the arbitrageur.

For a similar reason we consider the case where the strategic trader is limited to simple trading

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[2] Also, Lowenstein (2000) writes that institutions who hadn't been trading in LTCM's markets before entered as they learned about LTCM's financial difficulties: "American International Group, which hadn't shown any interest in equity volatility before, was suddenly bidding for it." Lowenstein goes on to claim that this sudden interest could only be aimed at exploiting LTCM's distress. He also mentions that it was well known during the crisis that Salomon Smith Barney "was, and had been, pounding the fund's positions for months.

strategies – strategies where the strategic trader puts on a position at $t = 1$ and has to liquidate the position at $t = 2$. Again, allowing the strategic trader a richer set of trading strategies will improve the profitability of trading. In this sense we bias the model against finding a role for strategic traders, making our results more robust.

The strategic trader sets up positions that are reversed one period later. If it trades x at $t = 1$, its profits are given as $x\left(P_2 - P_1\right)$. Substituting for P_1 and P_2 using (5) allows us to write the strategic trader's objective function as

$$\max_x \frac{x}{\beta}\left(\theta_0 - 2E\left[\theta_1\right] + E\left[\theta_2\right] - 2x\right). \tag{8}$$

The expected profit has a maximum at x^* that solves

$$\theta_0 - 2E\left[\theta_1\right] + E\left[\theta_2\right] - 4x^* = 0. \tag{9}$$

The maximized value of the strategic trader's profit is greater than zero if $E\left[\theta_2\right] - 2E\left[\theta_1\right] + \theta_0 - 2x^*$ is of the same sign as x^*. This requires that $\frac{\theta_0 + E[\theta_2]}{2} - E\left[\theta_1\right] > x^*$.

The strategic trader may also consider lending to the arbitrageur. Lending is common among traders who provide collateral to one another for trading purposes. The loan is for a single period, after which it is either rolled over or cancelled. The strategic trader lends b_1 prior to trading at $t = 1$ in return for a promised repayment of $\overline{b}_2$ at $t = 2$. This is assumed to be a risky loan since the arbitrageur may default. After having to liquidate its positions and repaying the original senior debt, B_2, the arbitrageur may have insufficient funds to fully repay amount $\overline{b}_2$. If the arbitrageur gets credit from the strategic trader, it can continue to trade in a regular way, as if the capital constraint is temporarily relaxed.

Why would a strategic trader ever lend to a financially constrained and risky arbitrageur? The strategic trader knows about the arbitrageur's financial condition, but does not observe its trades. If the arbitrageur is financially weak, the likelihood of full liquidation is high. This causes great uncertainty regarding its trading strategy, since it either tries to liquidate or tries to purchase more to support prices. This uncertainty reduces the expected profit of the strategic traders. A loan-to-trade extended to the arbitrageur, by reducing the likelihood of liquidation at $t = 1$, reduces the uncertainty of the arbitrageur's trade and increases the profitability of the strategic trader. Therefore, when uncertainty regarding the arbitrageur's trade is high, lending offers the strategic trader an opportunity to increase its gains.

The strategic trader has three variables to choose from: the loan amount, b_1; the repayment amount, $\overline{b}_2$; and the quantity to trade, x. With lending, the strategic trader's objective function is given as:

$$\max_{x, b_1, \overline{b}_2} \frac{x}{\beta}\left(\theta_0 - 2E\left[\theta_1\right] + E\left[\theta_2\right] - 2x\right) + E\left[b_2\right] - b_1. \tag{10}$$

We work with a simple version of the problem assuming that the arbitrageur liquidates its holdings over two periods and chooses the position at $t = 1$, θ_1, to maximize its wealth. This eliminates the problem of specifying V_2 and gives us an intuitive framework to analyze the factors driving the results. The results remain unchanged if we allow for a general value of θ_2, which can, for example, be derived as an optimal value by an arbitrageur with a long horizon. We discuss this in Section 4. All proofs are in an Appendix B.

3 Optimal Liquidation by the Arbitrageur

As stated we consider the problem outlined above where the arbitrageur has to liquidate its initial position over two periods, so that $\theta_2 = 0$. Also, we assume that the arbitrageur's initial position is positive and large enough to avoid short positions at date 1, $\theta_0 > \frac{1}{2}R$, where R is the upper bound of the liquidity shock produced by the actions of noise traders. Restricting our attention to positive values of θ_0 is not critical since the model setup is symmetric, while assuming $\theta_0 > \frac{1}{2}R$ serves to simplify the notation.

With $\theta_2 = 0$ the strategic trader's optimal trade, x^*, can be determined using (9) as the solution to

$$\theta_0 - 2E\left[\theta_1\right] - 4x^* = 0\ . \tag{11}$$

The arbitrageur's objective function obtained after substituting for P_1 and P_2 using (5) in (3) is

$$\max_{\theta_1} \theta_0 I - B_0 - (\theta_1 - \theta_0)\left(\frac{\theta_1 - \theta_0}{\beta} + \frac{x_1 + \varepsilon_1}{\beta}\right) - \theta_1\left(\frac{\theta_1}{\beta} + \frac{x_1}{\beta}\right) \tag{12}$$

which can be thought of as the arbitrageur minimizing the cost of liquidating its position over the two periods, $(\theta_1 - \theta_0)\left(\frac{\theta_1-\theta_0}{\beta} + \frac{x_1+\varepsilon_1}{\beta}\right) + \theta_1\left(\frac{\theta_1}{\beta} + \frac{x_1}{\beta}\right)$.

We start by considering two polar cases: an unconstrained arbitrageur, and an arbitrageur that is forced to liquidate at $t = 1$. We then analyze intermediate cases, where the constraint is sometimes binding and where the arbitrageur is sometimes forced to fully liquidate at $t = 1$. In this we need to separately consider an arbitrageur with a small initial position and with a large initial position. Finally, we consider that the strategic trader can lend to the arbitrageur in addition to trading with it. To keep the notation simple, we use lower case letters for a small initial position and capital letters to indicate a large initial position.

3.1 Strategic Trading When the Arbitrageur is Financially Unconstrained

If the arbitrageur is unconstrained, we obtain that the arbitrageur's best response to a given liquidity shock and an order by the strategic trader at $t = 1$ is

$$\theta_1 = \frac{1}{2}\left(\theta_0 - x - \frac{1}{2}\varepsilon_1\right). \tag{13}$$

The expected value of θ_1 is $E\left[\theta_1\right] = \frac{1}{2R}\int_{-R}^{R}\theta_1 d\varepsilon_1 = \frac{1}{2}\left(\theta_0 - x\right)$. Using (11) we obtain $x^* = 0$. The strategic trader decides not to trade against the arbitrageur. The strategic trader's need to reverse the trade at $t = 2$, combined with the price impact of its trades, makes it costly for it to trade against the arbitrageur. In response to a trade by the strategic trader, an arbitrageur with financial flexibility can adjust the quantities it trades at $t = 1$ and $t = 2$ to reduce the cost of liquidating the portfolio.

The equilibrium is characterized by $x^* = 0$ and $\theta_1^* = \frac{1}{2}\left(\theta_0 - \frac{1}{2}\varepsilon_1\right)$. The equilibrium price of the asset at date 1 is given by $P_1^* = I - \frac{\theta_0}{2\beta} + \frac{3\varepsilon_1}{4\beta}$. The unconstrained arbitrageur must have enough capital so that the constraint is not binding for all values of ε_1. Using the price computed at the most adverse value of ε_1, $-R$ in the capital constraint, $\theta_1^* P_1^* - B_1^* \geq \theta_1^* M$, and simplifying, we get

$$\theta_0 \left(I - \frac{M}{2} - \frac{3R}{4\beta} - \frac{\theta_0}{2\beta} \right) - \frac{RM}{4} \geq B_0 \tag{14}$$

as the condition necessary for the arbitrageur to be completely unconstrained at $t = 1$.

This result is important since it shows that a well-capitalized arbitrageur, even when it has to fully liquidate positions over two periods, can trade around a strategic trader. The arbitrageur can use any trade by the strategic trader to lower the cost of liquidating its positions.

3.2 Strategic Trading When the Arbitrageur Fully Liquidates at $t = 1$ ($\theta_1 = 0$)

When the arbitrageur's financial situation is very precarious, it always fully liquidates its positions and exits the market at $t = 1$. This happens either because it is forced to do so due to an inability to satisfy the constraint or because it optimally chooses to do so because the cost of meeting the constraint at $t = 1$ exceeds that of liquidation at $t = 1$.

In this case, the equilibrium is characterized by $x^* = \frac{\theta_0}{4}$ and $\theta_1^* = 0$. The equilibrium price of the asset at date 1 is given by $P_1^* = I - \frac{3\theta_0}{4\beta} + \frac{\varepsilon_1}{\beta}$.

The strategic trader has information about the dire financial condition of the arbitrageurs, and anticipates the rapid liquidation and the resulting downward price pressure. To take advantage of the temporary crash in the price of the asset, it builds up a long position at date 1. The larger the arbitrageur's position in the asset, the more dramatic the expected mispricing caused by the liquidation, and consequently the greater is the strategic trader's demand for the asset.

The strategic trader's expected profit in this case is $\frac{2}{\beta}\left(\frac{\theta_0}{4}\right)^2$, which is strictly larger than zero. For the strategic trader we can also compute a paper gain or loss in each of the two periods using the intrinsic value of the asset to value positions held. The strategic trader has an expected gain of $\frac{3}{\beta}\left(\frac{\theta_0}{4}\right)^2$ in the first period and an expected loss of $\frac{1}{\beta}\left(\frac{\theta_0}{4}\right)^2$ in the second period. The second period loss reflects the fact that the strategic trader's liquidation of its position in the second period exerts a downward pressure on the market price of the asset.

3.3 Strategic Trading When the Arbitrageur is Financially Constrained

In more interesting cases, the arbitrageur is constrained, but does not always fully liquidate at $t = 1$. In these cases, B_0 is between the levels discussed already. For values of B_0 that lie in this range, the extent of the liquidity shock, ε_1, determines whether the arbitrageur's trade (i) is unaffected by the constraint, (ii) is affected by the constraint, or (iii) is a full liquidation.

The constraint has a different effect depending on the size of the position held by the arbitrageur. First, we consider the case where the arbitrageur has a small position, that is, $\beta M > \theta_0$. Then, we consider the case where the arbitrageur has a large position, that is, $\theta_0 > \beta M$.

3.3.1 Arbitrageur with small initial position, $\beta M > \theta_0$

When the arbitrageur has a small initial position, $\beta M > \theta_0$, the constraint places an upper bound on the position that it can hold after trading at $t = 1$, θ_1. In this case, the arbitrageur can fulfill the capital requirement by liquidating a portion of its holdings. If the constraint is not binding, the arbitrageur trades as an unconstrained arbitrageur would trade, yielding $\theta_1^U = \frac{1}{2}\left(\theta_0 - x - \frac{1}{2}\varepsilon_1\right)$. If the constraint is binding, the arbitrageur trades to satisfy the constraint, giving $\theta_1^C = \frac{\beta(\theta_0 I - B_0) - \theta_0(\theta_0 - x - \varepsilon_1)}{(\beta M - \theta_0)}$. The values of ε_1 for which the constraint is binding can be obtained by comparing θ_1^C and θ_1^U; the constraint is binding for all ε_1 that satisfy $l_1 \equiv 2\frac{(\beta M + \theta_0)(\theta_0 - x) - 2\beta(\theta_0 I - B_0)}{(\beta M + 3\theta_0)} > \varepsilon_1$. Note that for some values of ε_1 it is not possible for the arbitrageur to meet the constraint, $\theta_1^C < 0$, and it must trade down to a flat position and exit the market immediately. The values of ε_1 for which this is the case satisfy $l_2 \equiv \frac{\theta_0(\theta_0 - x) - \beta(\theta_0 I - B_0)}{\theta_0} > \varepsilon_1$. This leads to

$$\theta_1 = \begin{cases} \frac{1}{2}\left(\theta_0 - x - \frac{1}{2}\varepsilon_1\right) & \text{if} \quad \varepsilon_1 \geq l_1 \\ \frac{\beta(\theta_0 I - B_0) - \theta_0(\theta_0 - x - \varepsilon_1)}{(\beta M - \theta_0)} & \text{if} \quad l_1 > \varepsilon_1 \geq l_2 \\ 0 & \text{if} \quad l_2 > \varepsilon_1 \end{cases} . \tag{15}$$

We solve for the optimal quantity of the strategic trader, x^*, by first obtaining the expected size of the arbitrageur's position after trading at $t = 1$, $E[\theta_1]$, and using this in (11) to obtain the following result.

Proposition 1 *Suppose that the arbitrageur holds a small initial position, $\beta M > \theta_0$. Then, ordered in increasing levels of the arbitrageur's initial debt, B_0, we have five regions:*

(i) Region 1, where the probability that the arbitrageur is unconstrained is one, and the strategic trader does not trade;

(ii) Region 2, where the probability that the constraint is binding and the probability that the arbitrageur is unconstrained are both greater than zero and add up to one, and the strategic trader trades x_2^*;

(iii) Region 3, where (a) the constraint on the arbitrageur binds with probability one and the strategic trader trades x_{3a}^**; or (b) the probabilities that the arbitrageur is unconstrained, constrained, or forced to fully liquidate are all greater than zero and add up to one, and the strategic trader trades* x_{3b}^**;*

(iv) Region 4, where the probability that the constraint is binding and the probability that the arbitrageur is forced to fully liquidate are both greater than zero and add up to one, and the strategic trader trades x_4^**; and*

(v) Region 5, where the probability that the arbitrageur is forced to fully liquidate is equal to one, and the strategic trader trades $\frac{\theta_0}{4}$*.*

The expressions for x_2^**,* x_{3a}^**,* x_{3b}^**,* x_4^**, and the cutoff debt levels that separate the regions are presented in Appendix B.1.*

The characteristics of Region 3 depend on the support of the distribution of ε_t. If the support is relatively wide, (b) prevails and (a) otherwise.

Using the proposition, we get additional results.

Corollary 1 *Suppose that the arbitrageur holds a small initial position,* $\beta M > \theta_0$*. In the regions labelled 2, 3, and 4 in Proposition 1, the strategic trader's optimal quantity increases with the arbitrageur's debt,* $\frac{\partial x^*}{\partial B_0} > 0$*.*

When the strategic trader faces an arbitrageur with a small initial position, $\beta M > \theta_0$, its expected profit increases with the arbitrageur's initial borrowing, B_0, and has a maximum value of $\frac{2}{\beta}\left(\frac{\theta_0}{4}\right)^2$ for levels of B_0 where the arbitrageur is always forced to fully liquidate. For B_0 such that the arbitrageur is always unconstrained, the strategic trader does not trade, and its expected profit is 0. For all B_0 where the arbitrageur is constrained with probability greater than zero, the strategic trader buys the asset.

Figure 1 plots the optimal quantity the strategic trader trades as a function of the arbitrageur's borrowings. When the arbitrageur's debt is small, the strategic trader is unable to benefit from its information about the arbitrageur's financial situation. This changes as soon as the debt level is high enough to impair the arbitrageur's financial flexibility. The strategic trader then buys the asset in anticipation of a liquidation of the arbitrageur's position to ensure that the constraint is met, and benefits from the resulting price pressure.

Corollary 2 *Suppose that the arbitrageur holds a small initial position,* $\beta M > \theta_0$*. Then, given* θ_0 *and* B_0 *for the arbitrageur, the strategic trader's trading (i) reduces the probability of complete* *forced liquidation of the arbitrageur's holdings at* $t = 1$*; (ii) reduces the amount the arbitrageur is forced to liquidate to meet the constraint; and (iii) results in higher prices.*

By buying, the strategic trader provides price support and thus diminishes the probability that the arbitrageur violates the capital constraint. This reduces the likelihood that the arbitrageur will have to liquidate its entire position. Thus, the strategic trader helps to prevent disruptions in the market by allowing the arbitrageur to liquidate positions over a longer period. The arbitrageur that has a small position benefits from the presence of the strategic trader, since fulfilling the capital constraint requires a lower level of liquidation. The strategic trader only seemingly acts altruistically, since its trades maximize its profit.

Figure 2 plots the strategic trader's expected trading profit as a function of the arbitrageur's initial debt level. It documents that the strategic trader's profits stem entirely from gains at $t = 1$, when it takes advantage of the liquidation of positions by the arbitrageur in an attempt to meet the constraint that leads to downward pressure on prices. At $t = 2$, the strategic trader closes out the positions accumulated at $t = 1$ and sells, and this has a downward impact on the price, which represents a cost to it of trading the illiquid asset.

3.3.2 Arbitrageur with large initial position, $\theta_0 > \beta M$

When the arbitrageur has a large initial position, $\theta_0 > \beta M$, the fall in the value of its holdings caused by the downward pressure on prices resulting from a liquidation can be large enough that it makes it impossible for the arbitrageur to meet the constraint. This implies that the arbitrageur can fulfill the constraint only if its position remains sufficiently large. If the constraint is not binding, the arbitrageur trades as an unconstrained arbitrageur would, which yields $\theta_1^U = \frac{1}{2}\left(\theta_0 - x - \frac{1}{2}\varepsilon_1\right)$. When the constraint is binding, the arbitrageur has two alternatives: (i) it either trades to satisfy the constraint, or (ii) it liquidates right away and exits the market. The capital requirement (7) can be solved for θ_1. Meeting it implies

$$\theta_1^C = \frac{\theta_0\left(\theta_0 - x - \varepsilon_1\right) - \beta\left(\theta_0 I - B_0\right)}{\left(\theta_0 - \beta M\right)}. \tag{16}$$

The capital constraint is binding only if the arbitrageur intends to hold a smaller position than it would in the absence of the capital requirement. Comparing θ_1^C to θ_1^U shows that the constraint is binding for $L_1 \equiv 2\frac{(\beta M+\theta_0)(\theta_0-x)-2\beta(\theta_0 I-B_0)}{(\beta M+3\theta_0)} > \varepsilon_1$ (the capital letters refer to the arbitrageur holding a large position).

An arbitrageur with a large position can always meet the capital requirement on its position by placing a sufficiently large order. Meeting the constraint is costly because the arbitrageur has to liquidate a larger position at $t = 2$. This cost keeps the arbitrageur from pursuing such a strategy without restriction. If the arbitrageur liquidates its entire position at $t = 1$, the net value given in (3) is $\theta_0\left(I - \frac{\theta_0}{\beta} + \frac{x+\varepsilon_1}{\beta}\right) - B_0$. Comparing this value with the value for $\theta_1 > 0$ yields a critical

θ_1 above which meeting the constraint results in a value lower than full liquidation. Concretely, satisfying the constraint yields an equal or higher value than immediately liquidating, while

$$\theta_0 - x - \frac{\varepsilon_1}{2} \geq \theta_1^C. \tag{17}$$

Using (16) for θ_1^C in (17) yields an explicit expression for ε_1 below which full liquidation at $t = 1$ is the optimal choice, $L_2 \equiv 2\frac{\beta M(\theta_0 - x) - \beta(\theta_0 I - B_0)}{\beta M + \theta_0} > \varepsilon_1$.

This leads to

$$\theta_1 = \begin{cases} \frac{1}{2}\left(\theta_0 - x - \frac{1}{2}\varepsilon_1\right) & \text{if} \quad \varepsilon_1 \geq L_1 \\ \frac{\theta_0(\theta_0 - x - \varepsilon_1) - \beta(\theta_0 I - B_0)}{(\theta_0 - \beta M)} & \text{if} \quad L_1 > \varepsilon_1 \geq L_2 \\ 0 & \text{if} \quad L_2 > \varepsilon_1 \end{cases}. \tag{18}$$

The first region $\varepsilon_1 \geq L_1$ is the unconstrained region. In the intermediate region, for values of ε_1 between L_2 and L_1, the arbitrageur liquidates less than it would if unconstrained to keep the price high. This is the region where price manipulation allows it to meet the capital constraint.[3]

We solve for the optimal quantity of the strategic trader, x^*, by first obtaining the expected size of the arbitrageur's position after trading at $t = 1$, $E[\theta_1]$, and using this in (11) to obtain the result.

Proposition 2 *Suppose the arbitrageur holds a large initial position, $\theta_0 > \beta M$. Then, ordered in increasing levels of the arbitrageur's initial debt, B_0, we have the five regions:*

(i) Region 1, where the probability that the arbitrageur is unconstrained is one, and the strategic trader does not trade;

(ii) Region 2, where the probability that the constraint is binding and the probability that the arbitrageur is unconstrained are both greater than zero and add up to one, and the strategic trader trades x_2^*;

(iii) Region 3, where (a) the constraint on the arbitrageur is binding with probability one, and the strategic trader trades x_{3a}^*; or (b) the probabilities that the arbitrageur is unconstrained, constrained, or chooses to fully liquidate are all greater than zero and add up to one, and the strategic trader trades x_{3b}^*;

(iv) Region 4, where the probability that the constraint is binding and the probability that the arbitrageur chooses to fully liquidate are both greater than zero and add up to one, and the strategic trader trades x_4^*; and

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[3] Note that this is different from other models of price manipulation. In Allen and Gale (1992) and Allen and Gorton (1992), for example, manipulators trade to appear informed and to induce other market participants to trade.

(v) Region 5, where the probability that the arbitrageur chooses to fully liquidate is equal to one, and the strategic trader trades $\frac{\theta_0}{4}$.

The expressions for x_2^*, x_{3a}^*, x_{3b}^*, x_4^*, and the cutoff debt levels that separate the regions are presented in Appendix B.4.

From Proposition 2 we have that $x^* = 0$ in Region 1, that $x^* < 0$ in Region 2, and that $x^* = \frac{\theta_0}{4} > 0$ in Region 5. Thus, the direction of the strategic trader's trades changes as the arbitrageur's borrowings increase. Continuity of the strategic trader's trade size implies that the strategic trader does not trade, $x^* = 0$, for one initial debt level that falls either in Region 3 or Region 4.

Corollary 3 *Suppose that the arbitrageur holds a large initial position, $\theta_0 > \beta M$. Then, (i) while the probability of full liquidation by the arbitrageur is zero, the strategic trader's optimal quantity decreases with the arbitrageur's initial debt level, $\frac{\partial x^*}{\partial B_0} < 0$, and when the probability of full liquidation by the arbitrageur is greater than zero and smaller than one, the strategic trader's optimal quantity everywhere increases with the arbitrageur's initial debt level, $\frac{\partial x^*}{\partial B_0} > 0$; and (ii) for one level of the arbitrageur's initial debt, B_0, in regions where the probability of full liquidation is greater than zero and smaller than one, the strategic trader's optimal quantity is zero.*

This result implies that the strategic trader's expected profit first rises, then falls, and finally again rises with the debt level, B_0, of a large arbitrageur. The strategic trader's expected profit has two maxima, the first at the debt level where the probability of a full liquidation is zero and the second for debt levels where the arbitrageur always fully liquidates. At one B_0 between these levels, the strategic trader has zero trading profit. Denote this value of B_0 as $B_0(0)$.

Figure 3, which plots the strategic optimal trading quantity as a function of the arbitrageur's initial debt level, illustrates that when the arbitrageur's initial position is small, the strategic trader remains inactive at sufficiently low levels of the arbitrageur's debt. If the arbitrageur's initial debt level is above a threshold, the strategic trader sells, knowing that the arbitrageur is likely to keep the price high by liquidating only a small fraction of its position or even increasing it.

When the arbitrageur is highly levered, the probability of complete liquidation is high enough to make the strategic trader buy the asset. Buying by the strategic trader makes the strategy of supporting prices more attractive for the arbitrageur.

Corollary 4 *Suppose that the arbitrageur holds a large initial position, $\theta_0 > \beta M$. Then, trading by the strategic trader (i) increases the probability of complete liquidation for $B_0(0) > B_0$, and diminishes the probability of complete liquidation for $B_0 > B_0(0)$; (ii) diminishes the probability* *of price manipulation for $B_0(0) > B_0$, and increases the probability of price manipulation for $B_0 > B_0(0)$.*

The impact on prices of trading by the strategic trader is more complicated. In regions where, depending on the ε_1 drawn, the arbitrageur may be either unconstrained or constrained (but does not ever fully liquidate), and the strategic trader sells the asset, $x^* < 0$, the asset price is

$$P_1 = \begin{cases} I - \frac{\theta_0}{2\beta} + \frac{3\varepsilon_1 + 2x^*}{4\beta} & \text{if} \quad \varepsilon_1 \geq 2\frac{(\beta M+\theta_0)\theta_0 - 2\beta(\theta_0 I - B_0)}{(\beta M + 3\theta_0)} - 2\frac{(\beta M+\theta_0)x^*}{(\beta M+3\theta_0)} \\ I + \frac{\theta_0\beta M - \beta(\theta_0 I - B_0) - \beta M\varepsilon_1}{\beta(\theta_0 - \beta M)} - \frac{\beta M x^*}{\beta(\theta_0-\beta M)} & \text{if} \quad 2\frac{(\beta M+\theta_0)\theta_0 - 2\beta(\theta_0 I - B_0)}{(\beta M + 3\theta_0)} - 2\frac{(\beta M+\theta_0)x^*}{(\beta M+3\theta_0)} > \varepsilon_1. \end{cases} \tag{19}$$

Given B_0 and θ_0, consider that ε_1 is such that the arbitrageur is unconstrained. Since $x^* < 0$, the price of the asset is lower in the presence of the strategic trader. For ε_1 such that the arbitrageur is constrained, the price of the asset is higher in the presence of the strategic trader since the arbitrageur needs to manipulate prices more to meet the constraint. We also have that the cutoff level of ε_1 required for the arbitrageur to be unconstrained is higher with a strategic trader present.

In the region where there is a possibility that the arbitrageur fully liquidates and the strategic trader sells the asset, $x^* < 0$, the asset price is

$$P_1 = \begin{cases} I + \frac{\theta_0\beta M - \beta(\theta_0 I - B_0) - \beta M\varepsilon_1}{\beta(\theta_0 - \beta M)} - \frac{\beta M x^*}{\beta(\theta_0-\beta M)} & \text{if} \quad \varepsilon_1 \geq 2\frac{\beta M\theta_0 - \beta(\theta_0 I - B_0)}{\beta M + \theta_0} - \frac{2\beta M x^*}{\beta M+\theta_0} \\ I - \frac{\theta_0}{\beta} + \frac{\varepsilon_1}{\beta} + \frac{x^*}{\beta} & \text{if} \quad 2\frac{\beta M\theta_0 - \beta(\theta_0 I - B_0)}{\beta M + \theta_0} - \frac{2\beta M x^*}{\beta M+\theta_0} > \varepsilon_1. \end{cases} \tag{20}$$

Given B_0 and θ_0, consider ε_1 such that the arbitrageur is constrained, but does not fully liquidate. From the above, the price of the asset is higher in the presence of the strategic trader. For ε_1 such that the arbitrageur fully liquidates, the price of the asset is lower in the presence of the strategic trader. We also have that the cut off level of ε_1 below which the arbitrageur fully liquidates is higher with a strategic trader present. In this case, trading by the strategic trader contributes to an increase in the range of prices observed.

In the region where there is a possibility that the arbitrageur fully liquidates and the strategic trader buys the asset, $x^* > 0$, the asset price is the same as given in (20). Given B_0 and θ_0, consider ε_1 such that the arbitrageur trades to satisfy the constraint. From the above, the price of the asset is lower in the presence of the strategic trader. For ε_1 such that the arbitrageur liquidates, the price of the asset is higher in the presence of the strategic trader. We also have that the cutoff level of ε_1 required for the arbitrageur to fully liquidate is lower with a strategic trader present. In this case, trading by the strategic trader reduces the range of prices observed.

Strategic trading impacts prices directly and also has an effect on the arbitrageur's trading volume. When the strategic trader sells to an arbitrageur who stabilizes prices because it cannot go the other way and liquidate as it would do in a liquid market, the trader makes it more expensive for the arbitrageur to keep the price high, because the arbitrageur must maintain an even larger position to compensate this additional supply. Buying at a high price from strategic traders counting on the fact that prices are being artificially manipulated by an arbitrageur fighting for its survival can be very costly. This reduces the advantage of trading to support the price, and the arbitrageur may end up deciding instead to liquidate its entire position. In this case, the presence of strategic traders increases the likelihood of the arbitrageur's failure. This possibility adds an additional component of risk to the activities of arbitrageurs. Keynes once remarked that markets can be out of line for longer than one might stay solvent. Markets can in fact do worse, by precipitating the downfall of those trying to align prices with fundamentals and riding along on their backs to the point of collapse. And the larger the trader, the more helpless it may be. To pull off his trick, John Rusnak, the foreign exchange arbitrageur who lost $700 million for Allied Irish Banks, had to deal with other traders who were well aware of trades and his weak financial condition.[4]

For debt levels for which the arbitrageur has to buy the asset to avoid liquidation and the strategic trader sells it, the ex-ante probability of price manipulation is reduced. If the arbitrageur must meet the capital constraint, however, it has to push the price higher when the strategic trader is present in the market selling to it. Just compensating for the strategic trader's added supply is not sufficient to fulfill the capital constraint, because the constraint itself depends on the extent of the arbitrageur's position. This means that the possible range of prices widens when a strategic trader is present and is willing to sell to a constrained arbitrageur, who is forced into buying more simply because it cannot sell due to market thinness. Figure 4 documents this finding in a plot of the arbitrageur's probability of early liquidation both when strategic traders are present and when they are absent. The presence of the strategic trader lowers the cutoff debt level above which the arbitrageur finds full liquidation optimal. For a range of debt levels, the probability of full liquidation is already positive without a strategic trader, but increases in its presence.

Figure 5 shows that the strategic trader's profitability varies significantly with the arbitrageur's initial debt and is not monotonic. The expected profits are highest when trading by the strategic trader is more aggressive. The peak in profits occurs when the arbitrageur's debt level is high enough to require the arbitrageur to support the price to survive and low enough to make it worthwhile doing so instead of giving up and liquidating its entire position. When the strategic trader sells at date 1, it is willing to take a loss at that date. Selling at date 1 is strategic insofar as it increases the expected quantity liquidated by the arbitrageur at date 2, because the arbitrageur has to intensify its trading to support – indeed manipulate – the price as a response to the additional supply by the strategic trader. This yields a higher expected gain for the strategic trader at date 2.

3.4 Lending and Strategic Trading When the Arbitrageur is Financially Constrained

Is it optimal for the strategic trader to loosen the constraint on the arbitrageur? For an arbitrageur with a small initial position, $\beta M > \theta_0$, the answer is no. The strategic trader's profit increases

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[4] See Plender (2002).Rusnak accumulated a position in currency forwards much beyond AIB's regular currency positions and quickly became noticed by other traders, who kept trading with him.

with the arbitrageur's debt level. Relaxing the constraint is equivalent to reducing the debt level and causes a drop in the strategic trader's expected profit. For an arbitrageur with a large initial position, $\theta_0 > \beta M$, we have that the strategic trader's expected profit rises, then falls and finally rises again with the arbitrageur's debt. If by relaxing the constraint the strategic trader can move the arbitrageur to a debt level where the strategic trader's trading profit is higher, the strategic trader benefits from relaxing the constraint. This turns out to be the case at debt levels where the arbitrageur may choose to fully liquidate at $t = 1$. For the remainder of this section, we assume that the arbitrageur holds a large position, $\theta_0 > \beta M$, and its debt level is such that the probability of full liquidation at $t = 1$ is greater than zero.

The fall and increase in the strategic trader's expected profit as the arbitrageur's debt level increases is associated with an increase in the likelihood that the arbitrageur will liquidate completely at $t = 1$. As the probability of full liquidation by the arbitrageur at $t = 1$ increases from zero, the uncertainty of the quantity that it trades increases, increasing price uncertainty, and reducing the strategic trader's expected profit. For $B_0 = B_0(0)$, the strategic trader finds it optimal not to trade, giving it a zero expected profit. For $B_0 < B_0(0)$, the strategic trader benefits from relaxing the constraint on the arbitrageur through a reduction in the uncertainty associated with, θ_1, the arbitrageur's position at $t = 1$. For $B_0 > B_0(0)$, the strategic trader benefits only if the constraint is loosened significantly.

The arbitrageur is willing to accept any funds offered since access to additional capital increases its expected wealth. This is true since an increase in capital improves the arbitrageurs's trading possibilities. The intuition can be formalized as follows. Consider two arbitrageurs, A and A', both with an initial position of θ_0, the first has an initial debt level of B_{0A}, and the second has an initial debt level of $B_{0A'}$. Suppose that $B_{0A'} > B_{0A}$, and B_{0A} is such that the constraint binds with a probability greater than zero at $t = 1$. Let $W(\theta_0, B_{0A})$ and $W(\theta_0, B_{0A'})$ be the expected wealth of the two arbitrageurs. Then, it must be that $W(\theta_0, B_{0A}) \geq W(\theta_0, B_{0A'}) + (B_{0A'} - B_{0A})$, as otherwise arbitrageur A could increase its expected wealth by setting aside or withdrawing $(B_{0A'} - B_{0A})$ in capital and then trading like the arbitrageur with a debt level of $B_{0A'}$. This also tells us that an arbitrageur with debt $B_{0A'}$ would, for an injection of capital of $(B_{0A'} - B_{0A})$, be willing to repay an amount with an expected value greater than or equal to $B_{0A'} - B_{0A}$.

The strategic trader can improve its expected profit by providing capital to an arbitrageur with a large position and by trading in the market against the arbitrageur. First consider how an investor can provide capital to the arbitrageur. We have assumed unlimited liability for the arbitrageur. Holders of residual claims in the arbitrageur will have to pay additional amounts if the value of the positions turns out to be negative. When the arbitrageur's debt level is such that it may find it optimal to fully liquidate at $t = 1$, the expected value of the positions is negative. An equity stake in return for the investment would be expensive for an investor since it implies expected losses in excess of the investment.

Yet structuring the investment as risk-free debt is unlikely to be feasible, since this would face opposition from other lenders. To see this, suppose that B_0 is raised through repurchase agreements, and the arbitrageur has insufficient funds to cover the "haircut" at $t = 1$. Raising additional debt of equal seniority is equivalent to an across-the-board reduction in the "haircut" that will be unacceptable to the lenders.

It should be clear that an investor is unlikely to be willing to invest through a residual claim while the current lenders can block the possibility of collateralized debt. This leaves open the possibility of structuring the investment as a junior claim with limited liability. To compensate for the limited liability, the investor would have to accept either (i) a cap on the payoff – giving the claim the payoff profile of risky debt; or (ii) a fractional sensitivity of payoff to value – giving the claim the payoff profile of a call option on the value.

We assume that the investment by the strategic trader is structured as a one-period risky loan that is subordinated to the other debts the arbitrageur has and is committed to prior to trading at $t = 1$. As this is a pre-committed loan, the arbitrageur can use it to show evidence of sufficient capital to obtain financing for positions. Consider an arbitrageur with an initial position θ_0 and debt B_0 large enough that the constraint binds with positive probability at $t = 1$. Let b_1 be the amount that the arbitrageur receives from the strategic trader prior to realization of the liquidity shock at $t = 1$ in return for agreeing to repay an amount $\overline{b}_2$ at $t = 2$. We have that $B_1 = B_0 - b_1 + (\theta_1 - \theta_0) P_1$ and $B_2 = B_1 - \theta_1 P_2$ are the amounts owed to senior lenders after trading at $t = 1$ and $t = 2$. The arbitrageur's objective function is

$$\max_{\theta_1} \theta_0 I - B_0 - (\theta_1 - \theta_0)\left(\frac{\theta_1 - \theta_0}{\beta} + \frac{x + \varepsilon_1}{\beta}\right) - \theta_1 \left(\frac{\theta_1}{\beta} + \frac{x}{\beta}\right) + (b_1 - E_1[b_2]) \tag{21}$$

where the amount that is repaid to the strategic trader, b_2, is given as

$$b_2 = \begin{cases} \overline{b}_2 & \text{if} \quad -\overline{b}_2 > B_2 \\ -B_2 & \text{if} \quad 0 \geq B_2 \geq -\overline{b}_2 \\ 0 & \text{if} \quad B_2 > 0 \end{cases}. \tag{22}$$

Since the amount extended by the strategic trader is considered capital in relation to the original debt, the constraint becomes

$$\theta_1 \geq \frac{\theta_0 (\theta_0 - x - \varepsilon_1) - \beta (\theta_0 I - (B_0 - b_1))}{(\theta_0 - \beta M)}. \tag{23}$$

For all $b_1 > 0$, the constraint is relaxed.

The unconstrained arbitrageur trades to $\theta_1^U = \frac{1}{2}\left(\theta_0 - x - \frac{\varepsilon_1}{2} - \frac{\beta}{2}\frac{\partial E_1[b_2]}{\partial \theta_1}\right)$, and the constrained arbitrageur trades to $\theta_1^C = \frac{\theta_0(\theta_0 - x - \varepsilon_1) - \beta(\theta_0 I - (B_0 - b_1))}{(\theta_0 - \beta M)}$. When ε_1 takes on adverse values, values for which the arbitrageur has to decide between immediate liquidation and price manipulation to continue trading, $E_1[b_2] = 0$ since $B_2 > 0$ for all values of ε_2. This gives a liquidation cutoff of $L_2^l \equiv 2\frac{\beta M(\theta_0 - x) - \beta(\theta_0 I - (B_0 - b_1))}{\beta M + \theta_0} > \varepsilon_1$ as the equivalent of L_2. When ε_1 takes on favorable values, so that the arbitrageur is potentially unconstrained at $t = 1$, $\frac{\partial E_1[b_2]}{\partial \theta_1}$ will be small, giving $L_1^l \equiv 2\frac{(\beta M + \theta_0)(\theta_0 - x) - 2\beta(\theta_0 I - (B_0 - b_1))}{(\beta M + 3\theta_0)} > \varepsilon_1$ as the equivalent of L_1 in this case.

The cutoff debt levels for the regions are now shifted by b_1 relative to those in Proposition 2. The strategic trader's optimal trading strategy is as in Proposition 2 with B_0 replaced by $B_0 - b_1$. Define $\Pi(\theta_0, B_0 - b_1)$ as the expected trading profit of the strategic trader when the arbitrageur has an initial position θ_0 and debt B_0, and the strategic trader extends b_1 to the arbitrageur. We can summarize this analysis in Proposition 3.

Proposition 3 *Suppose the arbitrageur holds a large initial position, $\theta_0 > \beta M$. For initial debt level, B_0, such that the likelihood of complete liquidation by the arbitrageur at $t = 1$ is greater than zero, and where $B_0 < B(0)$ the strategic trader's expected wealth increases if it lends the arbitrageur b_1 in return for an amount $\overline{b}_2$ such that $E_0[b_2] = b_1$. The arbitrageur's expected wealth increases if it accepts the loan.*

Figure 6 plots for values of B_0, the expected loss of the strategic trader on the amount extended to the arbitrageur that makes the trader indifferent between (i) trading and lending to the arbitrageur and (ii) trading but not lending to the arbitrageur. The loan amount b_1 is assumed to bring the arbitrageur back to zero likelihood of complete liquidation at $t = 1$. From Figure 6 we see that for certain values of B_0 the strategic trader may be willing to extend money to the arbitrageur even if it expects to get back a substantially lower amount. This is because for these values of B_0 additional capital reduces the likelihood that the arbitrageur finds immediate full liquidation optimal, which lets the strategic trader profit from trading longer and benefit from a larger liquidation by the arbitrageur later.

Why then does another unrelated institution, different from the strategic trader, not lend to the arbitrageur? Because the loan to the arbitrageur is risky, and will typically have a negative return – the expected repayment will be less than the amount lent. Strategic traders are willing to lend because their gains in trading profits more than offset the loss on the loan. An institution that only lends money but does not trade will find lending unprofitable.

4 Extensions

We present three extensions to the basic framework. The purpose of these extensions is twofold: to provide additional discussion of some of the results, and to show that we can replicate the results at the cost of greater analytical complexity, under more general assumptions.

4.1 General Capital Constraints for the Arbitrageur

The ability of the strategic trader to make profits by entering into trades with the arbitrageur depends on the predictability of the price deviations caused by the financially weak arbitrageurs who are trying to meet some binding constraint. While we have so far considered capital constraints similar to those imposed either explicitly by regulatory agencies or from in-house risk management systems, or implicitly by lenders and counterparties in transactions with low credit rating and insufficiently capitalized traders, the particular type of constraint faced by the arbitrageur is of secondary importance to the mere presence of the constraint. For example, some very specific form of the capital constraint can be designed to be infeasible for the arbitrageur to meet when it tries to manipulate prices, which is what happens when a large arbitrageur trades with the aim of providing price support. Suppose the per unit capital required is a function of traders' positions, that is a capital of $m(\theta_t)$ per unit, instead of the fixed amount considered above. This gives, for $\theta_t > 0$, $\theta_t P_t - B_t \geq \theta_t m(\theta_t)$, which on simplification yields

$$\theta_{t-1}\left(I - \frac{\theta_{t-1}}{\beta} + \frac{x_t + \varepsilon_t}{\beta}\right) - B_{t-1} \geq \theta_t \left(m(\theta_t) - \frac{\theta_{t-1}}{\beta}\right). \tag{24}$$

For $m(\theta_t) = M + N\theta_t$ with $M > 0$ and $N \geq \frac{1}{\beta}$, the arbitrageur can no longer meet the financial constraint by trading to attempt to support prices. All positions are small positions as defined earlier. Although this is possible, it would be very hard for regulators to implement. They would have to know both the condition of the market and the financial situation of the arbitrageur. Nonetheless, in this case there are profitable trading opportunities for the strategic trader. For $\frac{1}{\beta} > N \geq 0$, the arbitrageur can meet the financial constraint by supporting prices, making the analysis for large positions relevant.

Moving the time subscripts forward by one period and setting the right-hand side to zero gives

$$\theta_t \left(I - \frac{\theta_t}{\beta} + \frac{x_{t+1} + z_\alpha}{\beta}\right) - B_t \geq 0 \tag{25}$$

where z_α is such that the probability of the liquidity shock being more adverse than z_α is equal to α, $\Pr(\varepsilon_{t+1} < z_\alpha) = \alpha$. This is in effect equivalent to a forward-looking *Value at Risk* [*VaR*]. Alternatively, it is a credit rating-based constraint on θ_t that adjusts for market illiquidity, which provides a confidence level of $1 - \alpha$ to lenders and trading counterparties that the arbitrageur will be able to meet its debt obligations. In this case the results for small positions apply, but, the arbitrageur cannot meet the constraint by providing price support. The only way the arbitrageur could meet the constraint by manipulating prices is if the computation underestimates the correction factor necessary to account for market illiquidity.

In practice this would easily occur. To see this, consider that to implement a capital constraint, a *VaR* system, or a credit rating-based constraint, the history of price changes up to the current period is what matters. This history depends on two pieces of information. One is the series of shocks drawn and the evolution of the arbitrageur's positions. The other is the level of the arbitrageur's indebtedness. Suppose the shocks to prices cause a buildup in the arbitrageur's position, as well as in its level of indebtedness. This would be the case for a past sequence of shocks with a negative average. As the arbitrageur moves toward the constraint, its ability to absorb negative shocks is progressively weakened. This would be reflected in price changes that would have an increasingly negative conditional mean and higher conditional variance. Volatility estimated using historical prices would then underestimate future volatility. This is the same as incorrectly estimating β, the market depth coefficient, causing lenders and counterparties to incorrectly set N in (24) too low, opening up the possibility that an arbitrageur with a large position will affect prices in order to meet the constraint.

4.2 Arbitrageur with a Longer Horizon

Our results can be extended to an arbitrageur trading to maximize expected terminal wealth at a longer horizon than two periods. Suppose the arbitrageur trades to maximize its expected wealth at the end of $2 < T < \infty$ periods. To maintain comparability, assume that $\theta_T = 0$. For large T, this will only have a small impact on the arbitrageur's trade at $t = 1$. Let $W(\theta_0, B_0, T)$ be the optimized expected wealth of the arbitrageur with an initial position $\theta_0 > \beta M$ and debt level B_0 at $t = 0$. We can write

$$W(\theta_0, B_0, T) = \max_{\theta_1} \left\{ (\theta_1 - \theta_0)(I - P_1) + E\left[W(\theta_1, B_1, T-1) \,|\, \varepsilon_1\right] \right\}. \tag{26}$$

The arbitrageur faces a capital constraint in the current period requiring that

$$\theta_1 P_1 - B_1 \geq |\theta_1| M. \tag{27}$$

The effect of the constraint in future periods shows up through $W(\theta_1, B_1, T-1)$. Depending on the value of ε_1, there are four regions:

(i) The arbitrageur's trades are unaffected by the constraint – $\frac{\partial E[W(\theta_1, B_1, T-1)|\varepsilon_1]}{\partial \theta_1} = -\frac{\theta_0}{T}$, giving $\theta_1^* - \left(\frac{T-1}{T}\right)\theta_0 + \frac{x+\varepsilon_1}{2} = 0$;

(ii) The constraint is not currently binding, but because it might bind in the future it affects the arbitrageur's immediate trades – $\frac{\partial E[W(\theta_1, B_1, T-1)|\varepsilon_1]}{\partial \theta_1} < -\frac{\theta_0}{T}$ for some ε_1, giving for those ε_1 the value of θ_1^* that solves $\theta_1^* - \theta_0 + \frac{x+\varepsilon_1}{2} - \frac{\partial E[W(\theta_1, B_1, T-1)|\varepsilon_1]}{\partial \theta_1} = 0$;

(iii) the constraint is binding in the current period – $\theta_1^* = \theta_C^*$; and

(iv) the arbitrageur liquidates fully at $t = 1$, $-\theta_1^* = 0$. The equality $\frac{\partial E[W(\theta_1, B_1, T-1)|\varepsilon_1]}{\partial \theta_1} = -\frac{\theta_0}{T}$ results from the requirement that at the terminal period T, $\theta_T = 0$.

Note that regions (i), (iii), and (iv) are those described in the problem in the previous section. Region (ii) is the only additional region. For the remainder of the discussion, let θ_{1k}^*, be the θ_1^* for region k.

To solve for x^*, we need to obtain $E[\theta_1^*]$ and $E[\theta_2^*]$, and then plug them into the first-order condition, $\theta_0 - 2E[\theta_1^*] - E[\theta_2^*] - 4x^* = 0$. We first obtain the cutoffs for the regions. Let, L_{10}, L_{11}, and L_{12} be the cutoff points between regions (i) and (ii), (ii) and (iii), and (iii) and (iv), respectively, at $t = 1$. As before, L_{12} is obtained as the level of ε_1 at which the arbitrageur is indifferent between immediate liquidation and buying the asset to support prices and meet the financial constraint. The cost of immediate liquidation is the same as in the previous section. The gains to the arbitrageur if it supports prices and survives are greater in a T-period horizon than with two-periods. Therefore, the arbitrageur is willing to bear greater immediate losses to ensure that the constraint is met and it is able to continue trading. This implies that $L_{12} < L_2$, or that a much more negative shock is required before the arbitrageur decides to fully liquidate. The cutoff level L_{11} is determined by equating θ_1^C to $\theta_{1(ii)}^*$. We have that $\theta_{1(ii)}^* < \theta_{1(i)}^*$ for all ε_1 , and $\frac{\partial E[W(\theta_1,B_1,T-1)|\varepsilon_1]}{\partial\theta_1} < -\frac{\theta_0}{T}$ indicates that the arbitrageur values the flexibility that small positions afford. This causes $L_{11} > L_1$. If T or R are large, the arbitrageur is unlikely to ever be unaffected by the constraint. This implies that $L_{10} > R$, allowing us to ignore region (i).

At $t = 2$, there are again four regions, and the the analysis of the θ_2^* values and the cutoffs are similar. For a given θ_0, consider a debt that $L_{10} > R > -R > L_{11}$ and $L_{20} > R > -R > L_{21}$, implying that the constraint does not bind either at $t = 1$ or at $t = 2$. Since the arbitrageur values financial flexibility, in this case the arbitrageur will adjust its position downward at both dates. On average, more downward adjustment in the arbitrageur's position in the asset will occur at $t = 1$ than at $t = 2$. This implies that $\theta_0 - 2E[\theta_1^*] - E[\theta_2^*] > 0$, giving an optimal trade for the strategic trader $x^* > 0$. For a higher level of debt, such that $L_{11} > R > -R > L_{12}$ and $R > L_{21} > L_{22} > -R$, the constraint is binding at $t = 1$, while at $t = 2$, there is some likelihood of liquidation; some likelihood that the constraint will not bind; and also some likelihood that it will continue to bind in the future. An unconstrained arbitrageur will trade so that $\theta_0 - E[\theta_1^*] = E[\theta_1^*] - E[\theta_2^*] = \frac{\theta_0}{T}$. For a constrained arbitrageur, we have $\theta_0 - E[\theta_1^*] < \frac{\theta_0}{T}$. At $t = 2$, if the region where the constraint binds is smaller than the other two regions, the arbitrageur will liquidate a fraction of its position at $t = 2$, giving $E[\theta_1^*] - E[\theta_2^*] > \frac{\theta_0}{T}$. This gives $\theta_0 - 2E[\theta_1^*] - E[\theta_2^*] < 0$, and the optimal trading amount for the strategic trader is to sell the asset, $x^* < 0$. Finally, if the debt level is so high that $L_2 > R$, the arbitrageur surely liquidates at $t = 1$, and the strategic trader trades an amount equal to $x^* = \frac{\theta_0}{4}$. For intermediate levels of debt, x^* lies between the levels described above.

The more periods the constrained arbitrageur trades the greater the range of initial conditions under which the strategic trader profits from actively trading. This obtains even though a longer horizon allows the arbitrageur greater flexibility to manage its position. When it faces a large arbitrageur with a long position but a constraint that is not immediately binding, the strategic trader's profits are likely to be small. A fixed cost of trading can cause the strategic trader to refrain from trading against the arbitrageur. When the arbitrageur faces a binding financial constraint in the current period or when there is a significant likelihood that it will liquidate its position and exit the market, the strategic trader has a greater incentive to trade and the results are again those in the previous section. Extending the horizon alone then does not change the results much.

4.3 A Market with Several Strategic Traders

So far we have assumed that the arbitrageur faces a single strategic trader. Often there are several, possibly many, traders aware of the financial condition of a visible arbitrageur. Some lenders to the arbitrageur may decide to become active as strategic traders once they realize the weakened financial condition of the arbitrageur. Often too the arbitrageur can to some extent affect the number of strategic traders in the market. The arbitrageur can influence the number of potential strategic traders by different means. It can get funds from relatively few smart institutions, or it may try to borrow from less sophisticated institutions that do not trade or that are prevented by regulatory restrictions from trading the same asset as the arbitrageur. Basically, the arbitrageur can try to select from whom it gets funding to go on with its trading or to whom it reveals information on its positions.

It is always better for small financially constrained arbitrageurs to attract strategic traders to the market. The intuition for this is quite simple. An arbitrageur with a small position sells the asset to meet the constraint. Strategic traders buy the asset and provide liquidity to the arbitrageur, reducing the price impact of its trades and the cost of meeting the constraint. Formally this can be seen in an optimization problem of the strategic trader and the arbitrageur. In the case of n strategic traders, each trading x units at $t = 1$ and a single arbitrageur, an individual strategic trader's objective function that maximizes expected trading profit, defined as $E\left[x\left(P_2 - P_1\right)\right]$, is given as:

$$\max_{x} \frac{x}{\beta}\left(\theta_0 - 2E\left[\theta_1\right] - 2\left(n-1\right)x^* - 2x\right). \tag{28}$$

The optimal size of the trade solves the first-order condition $\theta_0 - 2E\left[\theta_1\left(x^*\right)\right] - 2\left(n+1\right)x^* = 0$. The arbitrageur's objective function is

$$\max_{\theta_1} \theta_0 I - B_0 - \left(\theta_1 - \theta_0\right)\left(\frac{\theta_1 - \theta_0}{\beta} + \frac{nx + \varepsilon_1}{\beta}\right) - \theta_1\left(\frac{\theta_1}{\beta} + \frac{nx}{\beta}\right) \tag{29}$$

which has a first-order condition of $\theta_1^U = \frac{\theta_0}{2} - \frac{nx}{2} - \frac{\varepsilon_1}{4}$. For an arbitrageur with a small initial position, $\beta M > \theta_0 > 0$, the capital constraint requires that $\theta_1^C \leq \frac{\beta(\theta_0 I - B_0) - \theta_0(\theta_0 - nx - \varepsilon_1)}{(\beta M - \theta_0)}$. The capital constraint is binding when

$$l_1^n \equiv 2\frac{\left(\theta_0 + \beta M\right)\left(\theta_0 - nx\right) - 2\beta\left(\theta_0 I - B_0\right)}{\left(3\theta_0 + \beta M\right)} > \varepsilon_1. \tag{30}$$

For $l_1^n < -R$, the constraint on the arbitrageur is never binding, and we get that $x^* = 0$. The debt level below which the constraint on the arbitrageur is never binding remains the same as in the case of a single strategic trader. For a debt level where the constraint is binding, the arbitrageur liquidates some of its position to meet the constraint, and the strategic trader buys the asset, $x^* > 0$. This action by the strategic trader helps reduce the cost to the arbitrageur of meeting the financial constraint. Also, from an examination of the constraint we see that for $x^* > 0$ the constraint is also relaxed as the number of strategic traders, n, increases. Finally, if we plug θ_1^U into the arbitrageur's objective function, we get $\theta_0 I - B_0 + \frac{(\varepsilon_1 + 2nx)^2 + 4\theta_0\varepsilon_1 - 4\theta_0^2}{8\beta}$, which also increases with the number of strategic traders. Thus, for a constrained arbitrageur with a small position in the asset it is always better to face more strategic traders.

For a large arbitrageur, the constraint is $\theta_1^C \geq \frac{\theta_0(\theta_0 - nx - \varepsilon_1) - \beta(\theta_0 I - B_0)}{(\theta_0 - \beta M)}$, so the arbitrageur finds it optimal to fully liquidate at $t = 1$ if

$$L_2^n \equiv 2\frac{\beta M\left(\theta_0 - nx\right) - \beta\left(\theta_0 I - B_0\right)}{\left(\theta_0 + \beta M\right)} > \varepsilon_1. \tag{31}$$

For $L_2^n > R$, the arbitrageur always liquidates at $t = 1$, and the optimal position taken by the strategic trader is to buy $x^* = \frac{\theta_0}{2(n+1)}$. The level of debt where the arbitrageur is always forced to liquidate is $\theta_0 I + \frac{R(\theta_0+\beta M)}{2\beta} - \frac{\beta M(n+2)\theta_0}{2\beta(n+1)}$, which increases with n, the number of strategic traders. When an arbitrageur with a long position faces a high likelihood of immediate forced liquidation, strategic traders as buyers help reduce the cost of liquidation. It is thus better to have more strategic traders active in the market.

When the debt levels are intermediate between these two levels, and the arbitrageur's position is large and when $L_1^n \geq R > -R > L_2^n$, we have

$$E\left[\theta_1\right] = \frac{\theta_0\left(\theta_0 - nx\right) - \beta\left(\theta_0 I - B_0\right)}{\left(\theta_0 - \beta M\right)}. \tag{32}$$

In this intermediate region, the optimal amount traded by a single strategic trader is $x^* = \frac{2\beta(\theta_0 I - B_0) - \theta_0(\theta_0+\beta M)}{2(\theta_0-(n+1)\beta M)}$. In this region, the maximum quantity that the strategic traders as a group trade is $-n\left(\frac{\theta_0}{2} - \frac{R(\theta_0+\beta M)}{2(\theta_0-\beta M)}\right)$ when $B_0 = \theta_0 I - \frac{\beta M(2+n)\theta_0}{2\beta} - R\frac{(\theta_0+\beta M)(\theta_0-(n+1)\beta M)}{2\beta(\theta_0-\beta M)}$. Also, when $E\left[\theta_1\left(0\right)\right] = \frac{\theta_0}{2}$, $x^* = 0$, giving that $B_0^n\left(0\right)$ solves

$$\frac{\left(R - L_2^n\left(x=0\right)\right)\left(2\left(\theta_0^2 - \beta\left(\theta_0 I - B_0\right)\right) - \left(R + L_2^n\left(x=0\right)\right)\theta_0\right)}{2R\theta_0\left(\theta_0 - \beta M\right)} = 1 \tag{33}$$

an expression that does not depend on n. For $B_0\left(0\right) > B_0$ strategic traders sell the asset. An increase in the number of strategic traders increases the cost to the arbitrageur of buying to support prices and meet the constraint. This increases the probability of full liquidation at $t = 1$. For $B_0 > B_0\left(0\right)$, strategic traders buy the asset, and reduce the cost and the likelihood of full liquidation by the arbitrageur. Thus, for $B_0\left(0\right) > B_0$, it is better for the arbitrageur to deal with fewer potential strategic traders. For $B_0 > B_0\left(0\right)$, the arbitrageur would prefer to deal with more strategic traders.

5 Conclusion

This paper presents an analysis of trading in markets when large and prominent traders, such as arbitrageurs do not have access to unlimited amounts of capital. We show that it is not enough to study the trading activity of these traders in isolation, as there are opportunities for other market participants to exploit the large traders' limitations in financial flexibility. When a trader's financial flexibility is limited, its trades, and thus market prices, become predictable. Financial fragility may lead to either excessive liquidation or excessive position holding. Excessively large positions occur when it is feasible to avoid violating capital constraints by manipulating prices.

Trading based on financial constraints of other traders has implications for the functioning of financial markets. As long as the positions of financially fragile traders are not too large, such strategic trading reduces excessive liquidation and smooths price fluctuations. If the positions of a fragile trader are large, however, the effect is reversed if the demise of the trader initially is not yet imminent. In these situations, strategic trading itself increases the probability of the fragile trader's failure. In addition, significant price deviations may occur due to increased price manipulation by the fragile trader.

We identify the link between exploiting a collapse of the arbitrageur and the effect of financing constraints for the success of such trading strategies. Strategic traders act as predators who take advantage of depressed prices during the time of the arbitrageur's collapse, as well as exploit price stabilization efforts made by the fragile arbitrageur before its collapse. Strategic traders may even decide to lend to a financially fragile arbitrageur to allow it to continue trading longer. The inclusion of corporate finance issues into the analysis of capital markets provides novel insights into the behavior of traders. Through our examination of the strategies of predators who attack an arbitrageur's positions until they bring it to its knees, we shed light on the risks that sophisticated traders such as influential arbitrageurs face in financial markets.

Appendix A: Strategic Trader Observes Noise Trader Demand

If the strategic trader observes ε_1 prior to placing its trade the problem is simplified. That is, knowledge of ε_1 and an arbitrageur's trading strategy allows the strategic trader to determine the quantity that the arbitrageur will trade, eliminating one level of uncertainty.

The strategic trader's objective function is now given as $x\left(E\left[P_2\right]-P_1\right)$. Substituting for P_2 and P_1 gives

$$\max_x \quad x\left(\frac{\theta_0}{\beta}-\frac{2\theta_1}{\beta}-\frac{2x}{\beta}-\frac{\varepsilon_1}{\beta}\right)$$

which has a solution of $x^*=\frac{\theta_0}{4}-\frac{\theta_1}{2}-\frac{\varepsilon_1}{4}$. An unconstrained arbitrageur trades to $\theta_1^{U*}=\frac{\theta_0}{2}-\frac{x}{2}-\frac{\varepsilon_1}{4}$. Plugging this into the strategic trader's optimal quantity gives $x^*=-\frac{\varepsilon_1}{6}$ and $\theta_1^{U*}=\frac{\theta_0}{2}-\frac{\varepsilon_1}{6}$. If the arbitrageur is forced to liquidate, we have that $x^*=\frac{\theta_0}{4}-\frac{\varepsilon_1}{4}$. For the intermediate region where the arbitrageur is constrained, we consider separately the two cases of an arbitrageur with a small initial position and one with a large initial position.

For $\theta_1>0$, the constraint is given as $\theta_1 P_1-B_1\geq\theta_1 M$. Substituting for B_1, P_1, and x^* gives $2\beta\left(\theta_0 I-B_0\right)-\frac{3}{2}\theta_0\left(\theta_0-\varepsilon_1\right)\geq\theta_1\left(2\beta M-\theta_0\right)$. Now, small positions are those where $2\beta M>\theta_0$, and large positions are those where $\theta_0>2\beta M$.

Consider first that the arbitrageur has a small initial position. If it is constrained, the arbitrageur trades to $\theta_1^{C*}=\frac{2\beta(\theta_0 I-B_0)-\frac{3}{2}\theta_0(\theta_0-\varepsilon_1)}{(2\beta M-\theta_0)}$. Substituting this in to the strategic trader's optimal quantity and simplifying gives $x^*=-\frac{\beta(\theta_0 I-B_0)}{(2\beta M-\theta_0)}+\frac{(\beta M+\theta_0)(\theta_0-\varepsilon_1)}{2(2\beta M-\theta_0)}$. The cutoffs between regions are $l_1\equiv 3\frac{(\beta M+\theta_0)\theta_0-2\beta(\theta_0 I-B_0)}{(\beta M+4\theta_0)}$ and $l_2\equiv\theta_0-\frac{4}{3}\beta\left(I-\frac{B_0}{\theta_0}\right)$.

Thus, we can summarize trading by the arbitrageur as

$$\theta_1^*=\begin{cases}\frac{\theta_0}{2}-\frac{\varepsilon_1}{6} & \text{if} \quad \varepsilon_1\geq l_1\\ \frac{2\beta(\theta_0 I-B_0)-\frac{3}{2}\theta_0(\theta_0-\varepsilon_1)}{(2\beta M-\theta_0)} & \text{if} \quad l_1\geq\varepsilon_1\geq l_2\\ 0 & \text{if} \quad l_2\geq\varepsilon_1\end{cases}$$

and by the strategic trader as

$$x^*=\begin{cases}-\frac{\varepsilon_1}{6} & \text{if} \quad \varepsilon_1\geq l_1\\ -\frac{\beta(\theta_0 I-B_0)}{(2\beta M-\theta_0)}+\frac{(\beta M+\theta_0)(\theta_0-\varepsilon_1)}{2(2\beta M-\theta_0)} & \text{if} \quad l_1\geq\varepsilon_1\geq l_2\\ \frac{\theta_0}{4}-\frac{\varepsilon_1}{4} & \text{if} \quad l_2\geq\varepsilon_1\end{cases}.$$

For a large initial position, if it is constrained, the arbitrageur trades to $\theta_1^{C*}=\frac{\frac{3}{2}\theta_0(\theta_0-\varepsilon_1)-2\beta(\theta_0 I-B_0)}{(\theta_0-2\beta M)}$. Substituting this in to the strategic trader's optimal quantity and simplifying gives $x^*=\frac{\beta(\theta_0 I-B_0)}{(\theta_0-2\beta M)}-\frac{(\beta M+\theta_0)(\theta_0-\varepsilon_1)}{2(\theta_0-2\beta M)}$. The cutoff point between the unconstrained and the constrained region is $L_1\equiv 3\frac{(\beta M+\theta_0)\theta_0-2\beta(\theta_0 I-B_0)}{(\beta M+4\theta_0)}$. To determine the cutoff between the constrained and the liquidation region, we evaluate the arbitrageur's objective function under full liquidation, $\theta_0 I-B_0-\frac{3\theta_0}{4\beta}\left(\theta_0-\varepsilon_1\right)$, and under the constraint, $\theta_0 I-B_0-\frac{3\theta_0}{4\beta}\left(\theta_0-\varepsilon_1\right)+\frac{\theta_1}{\beta}\left(\theta_0-\theta_1-\frac{\varepsilon_1}{2}\right)$, and equate the two to get $\overline{\theta}_1=\theta_0-\frac{\varepsilon_1}{2}$ giving $L_2=\frac{\frac{1}{2}\theta_0(\theta_0+4\beta M)-2\beta(\theta_0 I-B_0)}{(\theta_0+\beta M)}$.

Thus, we can summarize trading by the arbitrageur as

$$\theta_1^* = \begin{cases} \frac{\theta_0}{2} - \frac{\varepsilon_1}{6} & \text{if} \quad \varepsilon_1 \geq L_1 \\ \frac{\frac{3}{2}\theta_0(\theta_0 - \varepsilon_1) - 2\beta(\theta_0 I - B_0)}{(\theta_0 - 2\beta M)} & \text{if} \quad L_1 \geq \varepsilon_1 \geq L_2 \\ 0 & \text{if} \quad L_2 \geq \varepsilon_1 \end{cases}$$

and by the strategic trader as

$$x^* = \begin{cases} -\frac{\varepsilon_1}{6} & \text{if} \quad \varepsilon_1 \geq L_1 \\ \frac{\beta(\theta_0 I - B_0)}{(\theta_0 - 2\beta M)} - \frac{(\beta M + \theta_0)(\theta_0 - \varepsilon_1)}{2(\theta_0 - 2\beta M)} & \text{if} \quad L_1 \geq \varepsilon_1 \geq L_2 \\ \frac{\theta_0}{4} - \frac{\varepsilon_1}{4} & \text{if} \quad L_2 \geq \varepsilon_1 \end{cases}.$$

In this case, the strategic trader behaves like a competitor to the arbitrageur when the constraint is not binding on the arbitrageur, and trades strategically, taking advantage of the arbitrageur's vulnerability, when the constraint is binding. Taking the initial position for the arbitrageur, θ_0, and the level of debt, B_0, as given, the strategic trader's profits are higher if it can determine the quantity to trade after observing the shock, ε_1, since it now also profits from supplying liquidity to the liquidity traders.

In a more complete equilibrium setting, though the strategic trader's profits may be higher or lower, since the arbitrageur is likely to factor in the better information available to strategic traders when determining positions. This would result less frequently in large values of θ_0 and B_0, the more aggressive positions.

Appendix B: Proofs

B.1: Proof of Proposition 1

To solve the strategic trader's problem, we proceed as follows: (i) Using $\theta_1(x, \varepsilon_1)$, we compute $E[\theta_1(x, \varepsilon_1)]$; and (ii) use $E[\theta_1(x, \varepsilon_1)]$ in the strategic trader's first-order condition to obtain x^* as the solution to $\theta_0 - 2E[\theta_1(x^*, \varepsilon_1)] - 4x^* = 0$. We then plug the optimal x^* into l_1 and l_2 to solve for the boundaries for the regions.

We repeat (15) here for convenience:

$$\theta_1 = \begin{cases} \frac{1}{2}\left(\theta_0 - x - \frac{1}{2}\varepsilon_1\right) & \text{if} \quad \varepsilon_1 \geq l_1 \\ \frac{\beta(\theta_0 I - B_0) - \theta_0(\theta_0 - x - \varepsilon_1)}{(\beta M - \theta_0)} & \text{if} \quad l_1 > \varepsilon_1 \geq l_2 \\ 0 & \text{if} \quad l_2 > \varepsilon_1 \end{cases}.$$

In computing $E[\theta_1]$ we need to consider five regions:

(i) Region 1: $-R > l_1$, where the arbitrageur is always unconstrained. In this case $E[\theta_1] = \frac{1}{2}(\theta_0 - x)$ and $x^* = 0$;

(ii) Region 2: $R > l_1 > -R > l_2$, where the arbitrageur is sometimes constrained and otherwise unconstrained;

(iii) Region 3: (a) $l_1 > R > -R > l_2$, where the arbitrageur is always constrained; or (b) $R > l_1 > l_2 > -R$, where the arbitrageur is unconstrained, constrained, or forced to fully liquidate;

(iv) Region 4: $l_1 > R > l_2 > -R$, where the arbitrageur is sometimes constrained and otherwise is forced to liquidate; and

(v) Region 5: $l_2 > R$, where the arbitrageur is always forced to completely liquidate. In this case, $E[\theta_1] = 0$ and $x^* = \frac{\theta_0}{4}$.

For (ii), where $R > l_1 > -R > l_2$, we have that

$$E[\theta_1] = \frac{(\theta_0 - x)(\beta M - 3\theta_0) + 2\beta(\theta_0 I - B_0)}{4(\beta M - \theta_0)} - \frac{(\beta M + 3\theta_0)}{4(\beta M - \theta_0)}\left(\frac{R^2 + l_1^2}{4R}\right)$$

which on substitution into $\theta_0 - 2E[\theta_1(x^*, \varepsilon_1)] - 4x^* = 0$ gives $\frac{(\beta M + 3\theta_0)}{8R(\beta M - \theta_0)}(R + l_1(x^*))^2 - 3x^* = 0$, with the solution:

$$x_2^* = \theta_0 - \frac{(5\theta_0 - 7M\beta)(M\beta + 3\theta_0)R}{2(M\beta + \theta_0)^2} - \frac{2(I\theta_0 - B_0)\beta}{(M\beta + \theta_0)}$$
$$- \frac{\sqrt{6R(\beta M + 3\theta_0)(\beta M - \theta_0)\left((\theta_0 + \beta M)^2\theta_0 + R(2\beta M - \theta_0)(\beta M + 3\theta_0) - 2\beta(\theta_0 I - B_0)(\theta_0 + \beta M)\right)}}{(\beta M + \theta_0)^2}. \tag{B.1.1}$$

For (iii)(a), where $l_1 > R > -R > l_2$, we have that

$$E[\theta_1] = -\frac{\theta_0}{(\beta M - \theta_0)} l_2$$

which on substitution into $\theta_0 - 2E\left[\theta_1\left(x^*, \varepsilon_1\right)\right] - 4x^* = 0$ gives $\theta_0 + 2\frac{\theta_0}{(\beta M - \theta_0)} l_2\left(x^*\right) - 4x^* = 0$, with the solution:

$$x^*_{3a} = \frac{\theta_0\left(\beta M + \theta_0\right) - 2\beta\left(\theta_0 I - B_0\right)}{2\left(2\beta M - \theta_0\right)}. \tag{B.1.2}$$

For (iii)(b), where $R > l_1 > l_2 > -R$, we have that

$$E\left[\theta_1\right] = \frac{\left(\theta_0 - x\right)\left(R - l_1\right)}{4R} - \frac{\left(R^2 - l_1^2\right)}{16R} + \frac{\left(\beta\left(\theta_0 I - B_0\right) - \theta_0\left(\theta_0 - x\right)\right)\left(l_1 - l_2\right)}{2R\left(\beta M - \theta_0\right)} + \frac{\theta_0\left(l_1^2 - l_2^2\right)}{4R\left(\beta M - \theta_0\right)}$$

which on substitution into $\theta_0 - 2E\left[\theta_1\left(x^*, \varepsilon_1\right)\right] - 4x^* = 0$ gives $\theta_0 - 4x^* - \frac{(\theta_0 - x^*)(R - l_1(x^*))}{2R} + \frac{\left(R^2 - l_1^2(x^*)\right)}{8R} - \frac{\theta_0(l_1(x^*) - l_2(x^*))^2}{2R(\beta M - \theta_0)} = 0$, with the solution:

$$x^*_{3b} = \theta_0 + \frac{7R\left(3\theta_0 + \beta M\right)}{2\left(\beta M + 2\theta_0\right)} - \frac{\beta\left(\theta_0 I - B_0\right)}{\left(\beta M + 2\theta_0\right)}$$
$$- \frac{\sqrt{\theta_0\left(M\beta + 3\theta_0\right)\left(4R\theta_0\left(6\theta_0\left(\beta M + 2\theta_0\right) - 7\beta\left(\theta_0 I - B_0\right)\right) + R^2\theta_0\left(48\beta M + 145\theta_0\right) + 4\beta^2\left(\theta_0 I - B_0\right)^2\right)}}{2\theta_0\left(\beta M + 2\theta_0\right)}. \tag{B.1.3}$$

For (iv), where $l_1 > R > l_2 > -R$, we have that

$$E\left[\theta_1\right] = \frac{\theta_0}{4R\left(\beta M - \theta_0\right)}\left(R - l_2\right)^2$$

which on substitution into $\theta_0 - 2E\left[\theta_1\left(x^*, \varepsilon_1\right)\right] - 4x^* = 0$ gives $\theta_0 - 4x^* - \frac{\theta_0}{2R(\beta M - \theta_0)}\left(R - l_2\left(x^*\right)\right)^2 = 0$, with the solution:

$$x^*_4 = \theta_0 - \frac{R\left(4\beta M - 3\theta_0\right)}{\theta_0} - \frac{\beta\left(\theta_0 I - B_0\right)}{\theta_0} + \frac{\sqrt{2R\left(\beta M - \theta_0\right)\left(4R\left(2\beta M - \theta_0\right) - 3\theta_0^2 + 4\beta\left(\theta_0 I - B_0\right)\right)}}{\theta_0}. \tag{B.1.4}$$

Solving $l_1\left(x^*\right) = -R$ for B_0 using $x^* = 0$ gives $u_1 \equiv \theta_0\left(I - \frac{M}{2} - \frac{3R}{4\beta} - \frac{\theta_0}{2\beta}\right) - \frac{RM}{4}$ for the cutoff debt level below which the arbitrageur is unconstrained. Solving $l_2\left(x^*\right) = R$ for B_0 using $x^* = \frac{\theta_0}{4}$ gives $u_4 \equiv \theta_0\left(I - \frac{3\theta_0}{4\beta} + \frac{R}{\beta}\right)$, the cutoff debt level above which the arbitrageur is always forced to liquidate.

When $l_1\left(x^*\right) - l_2\left(x^*\right) \geq 2R$, the boundaries are crossed in the order as follows: (i) $l_1 = -R$; (ii) $l_1 = R$; (iii) $l_2 = -R$; and (iv) $l_2 = R$. Solving $l_1 = R$ and $l_2\left(x^*\right) = -R$ for B_0 using x^*_{3a} from (B.1.2) gives the cutoff debt levels $u_2 \equiv \theta_0 I - \frac{(\beta M + \theta_0)\theta_0}{2\beta} + \frac{(2\beta M - \theta_0)}{(\beta M - \theta_0)}\frac{(\beta M + 3\theta_0)R}{6\beta}$ and $u_3 \equiv \theta_0 I - \frac{3(\beta M - \theta_0)\theta_0^2 + 2R\theta_0(2\beta M - \theta_0)}{4\beta(\beta M - \theta_0)}$. The condition $l_1\left(x^*\right) - l_2\left(x^*\right) \geq 2R$ must hold for all $u_3 \geq B_0 > u_2$. Using x^*_{3a} in $l_1\left(x^*\right)$ and $l_2\left(x^*\right)$ gives

$$\frac{\left(4MI\beta^2 + 3M\beta\theta_0 - 3\theta_0^2\right)\left(\beta M - \theta_0\right)}{\left(\beta M + 3\theta_0\right)\left(2\beta M - \theta_0\right)} - \frac{4M\beta^2 B_0\left(\beta M - \theta_0\right)}{\left(\beta M + 3\theta_0\right)\left(2\beta M - \theta_0\right)\theta_0} \geq 4R.$$

The left-hand side decreases with B_0. Therefore, we must check that the condition holds at the highest value of B_0, which is u_3, giving the requirement that

$$\theta_0^2 - (\beta M - 4R)\,\theta_0 + \frac{2}{3}R\beta M \leq 0.$$

When $2R > l_1(x^*) - l_2(x^*) \geq 0$, the boundaries are crossed in the order as follows: (i) $l_1 = -R$; (ii) $l_2 = -R$; (iii) $l_1 = R$; and (iv) $l_2 = R$. Solving $l_2(x^*) = -R$ for B_0 using x_2^* from (B.1.1) gives the cutoff debt level, $u_2^{'} = \theta_0 I + \frac{\theta_0 R}{2\beta}\left(\frac{5\beta M+19\theta_0}{\beta M-\theta_0}\right) - \frac{\theta_0}{\beta}\frac{\sqrt{6R(2R(\beta M+2\theta_0)+\theta_0(\beta M-\theta_0))(M\beta+3\theta_0)}}{(\beta M-\theta_0)}$. Similarly, solving $l_1(x^*) = R$ for B_0 using x_4^* from (B.1.4) yields the cutoff debt level, $u_3^{'} = \theta_0 I - \frac{\theta_0 R}{2\beta}\left(\frac{15\beta M+17\theta_0}{\beta M-\theta_0}\right) + \frac{(\beta M+\theta_0)}{\beta}\frac{\sqrt{2R\theta_0(2R(\beta M+15\theta_0)-3\theta_0(\beta M-\theta_0))}}{(\beta M-\theta_0)}$. We obtain $l_1(x^*) - l_2(x^*) = \frac{\left(I\beta\theta_0+\theta_0^2-\beta B_0-x^*\theta_0\right)(\beta M-\theta_0)}{(M\beta+3\theta_0)\theta_0}$, which gives $\frac{\partial(l_1(x^*)-l_2(x^*))}{\partial B_0} = -\frac{(\beta M-\theta_0)}{(\beta M+3\theta_0)\theta_0}\left(\beta + \theta_0\frac{\partial x^*}{\partial B_0}\right) < 0$ since $\frac{\partial x^*}{\partial B_0} = \frac{2\beta(l_1-l_2)}{((7R+l_1)(\beta M-\theta_0)+2\theta_0(l_1-l_2))} > 0$.

We need to check that $l_1(x^*) - l_2(x^*) < 2R$ at $l_2(x^*) = -R$ with $B_0 = u_2^{'}$, which produces the requirement that

$$\theta_0^2 - \theta_0(\beta M - 4R) + \frac{2}{3}R\beta M > 0.$$

We also need to check that $l_1(x^*) - l_2(x^*) \geq 0$ at $l_1(x^*) = R$ with $B_0 = u_3^{'}$ which holds if $\theta_0 > \frac{2}{3}R$.

If $R \geq \left(\frac{4-\sqrt{7}}{12}\right)\beta M$, then $2R \geq l_1 - l_2 > 0$ holds for all $\beta M > \theta_0 > \frac{2}{3}R$. For $\left(\frac{4-\sqrt{7}}{12}\right)\beta M > R$, $2R \geq l_1 - l_2 > 0$ holds when $\beta M > \theta_0 > \frac{\beta M}{2} - 2R + \sqrt{\left(\frac{\beta M}{2}\right)^2 - \frac{8}{3}R\beta M + 4R^2}$ and $\frac{\beta M}{2} - 2R + \sqrt{\left(\frac{\beta M}{2}\right)^2 - \frac{8}{3}R\beta M + 4R^2} \geq \theta_0 \geq \frac{2}{3}R$; while $l_1 - l_2 > 2R$ holds in case of $\frac{\beta M}{2} - 2R + \sqrt{\left(\frac{\beta M}{2}\right)^2 - \frac{8}{3}R\beta M + 4R^2} \geq \theta_0 \geq \frac{\beta M}{2} - 2R - \sqrt{\left(\frac{\beta M}{2}\right)^2 - \frac{8}{3}R\beta M + 4R^2}$. □

B.2: Proof of Corollary 1

We have $l_1 = 2\frac{(\beta M+\theta_0)(\theta_0-x)-2\beta(\theta_0 I-B_0)}{(\beta M+3\theta_0)}$ and $l_2 = \frac{\theta_0(\theta_0-x)-\beta(\theta_0 I-B_0)}{\theta_0}$. Partial differentiation with respect to B_0 yields $\frac{\partial l_1(x^*)}{\partial B_0} = -\frac{2(\beta M+\theta_0)}{(\beta M+3\theta_0)}\frac{\partial x^*}{\partial B_0} + \frac{4\beta}{(\beta M+3\theta_0)}$ and $\frac{\partial l_2(x^*)}{\partial B_0} = -\frac{\partial x^*}{\partial B_0} + \frac{\beta}{\theta_0}$.

(i) We have that x_2^* solves $\frac{(\beta M+3\theta_0)}{8R(\beta M-\theta_0)}(R + l_1(x^*))^2 - 3x^* = 0$. Differentiating with respect to B_0 gives $\frac{(\beta M+3\theta_0)}{8R(\beta M-\theta_0)}(R + l_1(x^*))\frac{\partial l_1(x^*)}{\partial B_0} - 3\frac{\partial x^*}{\partial B_0} = 0$. Substituting in $\frac{\partial l_1(x^*)}{\partial B_0}$, and rearranging terms, we get $\frac{\partial x^*}{\partial B_0} = \frac{2\beta(R+l_1(x^*))}{((\beta M+\theta_0)(R+l_1(x^*))+12R(\beta M-\theta_0))} \geq 0$.

(ii)(a) For $\theta_0^2 - \theta_0(\beta M - 4R) + \frac{2}{3}R\beta M \leq 0$, differentiating x_{3a}^* with respect to B_0 gives $\frac{\partial x^*}{\partial B_0} = \frac{\beta}{(2\beta M-\theta_0)} > 0$.

(ii)(b) For $\theta_0^2 - \theta_0(\beta M - 4R) + \frac{2}{3}R\beta M > 0$, we have $x^* = \frac{\theta_0}{4} - \frac{E[\theta_1]}{2}$ and $E[\theta_1] = \frac{1}{2R}\int_{l_2}^{l_1}\theta_1^C(\varepsilon)\,d\varepsilon + \frac{1}{2R}\int_{l_1}^{R}\theta_1^U(\varepsilon)\,d\varepsilon$. This gives $\frac{\partial x^*}{\partial B_0} = \frac{2\beta(l_1-l_2)}{((7R+l_1)(\beta M-\theta_0)+2\theta_0(l_1-l_2))} > 0$.

(iii) We have x_4^* solves $\theta_0 - 4x^* - \frac{\theta_0}{2R(\beta M-\theta_0)}(R - l_2(x^*))^2 = 0$. Differentiating with respect to B_0 gives $-4\frac{\partial x^*}{\partial B_0} + \frac{\theta_0(R-l_2(x^*))}{R(\beta M-\theta_0)}\frac{\partial l_2(x^*)}{\partial B_0} = 0$. Substituting in $\frac{\partial l_2(x^*)}{\partial B_0}$ and rearranging terms, we get $\frac{\partial x^*}{\partial B_0} = \frac{\beta(R-l_2(x^*))}{(4R(\beta M-\theta_0)+\theta_0(R-l_2(x^*)))} \geq 0$. □

B.3: Proof of Corollary 2

Complete forced liquidation occurs when $\varepsilon_1 < l_2$. When $l_2 \leq -R$, the probability of complete forced liquidation is zero, and when $l_2 \geq R$, the probability of complete forced liquidation is one. For $R > l_2 > -R$, the probability of complete forced liquidation is $\frac{1}{2}+\frac{l_2}{2R}$. Using $l_2 = \frac{\theta_0(\theta_0-x)-\beta(\theta_0 I-B_0)}{\theta_0}$, we have that the probability of complete forced liquidation is $\frac{1}{2}+\frac{\theta_0}{2R}-\frac{\beta(\theta_0 I-B_0)}{2R\theta_0}-\frac{x}{2R}$. In the absence of the strategic trader, we have $x = 0$. In the presence of the strategic trader, we have from Proposition 1 that $x^* \geq 0$ when $\beta M > \theta_0 > 0$. This implies for any B_0 that the probability of liquidation is lower in the presence of the strategic trader. This proves (i).

Over the range of ε_1 values for which the constraint is binding, we have on rearranging terms $\theta_1^* = \frac{\beta(\theta_0 I-B_0)-\theta_0(\theta_0-\varepsilon_1)}{(\beta M-\theta_0)}+\frac{\theta_0 x^*}{(\beta M-\theta_0)}$. If the strategic trader is present, $x^* \geq 0$ when $\beta M > \theta_0 > 0$. Thus, the strategic trader's presence loosens the constraint, allowing the arbitrageur to liquidate a smaller amount. This proves (ii).

For values of B_0 such that the constraint is never binding, the presence or absence of the strategic trader makes no difference, as the optimal strategic trader quantity is 0. For levels of B_0 such that the constraint is binding, we have above in (ii) that the presence of the strategic trader reduces the amount that the arbitrageur needs to sell to meet the constraint. Selling by the arbitrageur places downward pressure on prices, so the reduced amount that needs to be sold results in higher prices. This proves (iii). $\square$

B.4: Proof of Proposition 2

The proof follows the steps outlined in Appendix B.1. We repeat (18) here for convenience

$$\theta_1 = \begin{cases} \frac{1}{2}\left(\theta_0 - x - \frac{1}{2}\varepsilon_1\right) & \text{if} \quad \varepsilon_1 \geq L_1 \\ \frac{\theta_0(\theta_0-x-\varepsilon_1)-\beta(\theta_0 I-B_0)}{(\theta_0-\beta M)} & \text{if} \quad L_1 > \varepsilon_1 \geq L_2 \\ 0 & \text{if} \quad L_2 > \varepsilon_1 \end{cases}.$$

In computing $E[\theta_1]$ we need to consider five regions:

(i) Region 1: $-R > L_1$, where the arbitrageur is always unconstrained. In this case, $E[\theta_1] = \frac{1}{2}(\theta_0 - x)$ and $x^* = 0$;

(ii) Region 2: $R > L_1 > -R > L_2$, where the arbitrageur is sometimes constrained and otherwise unconstrained;

(iii) Region 3: (a) $L_1 > R > -R > L_2$, where the arbitrageur is always constrained; or (b) $R > L_1 > L_2 > -R$, where the arbitrageur is unconstrained, is constrained, or fully liquidates;

(iv) Region 4: $L_1 > R > L_2 > -R$, where the arbitrageur is sometimes constrained and otherwise fully liquidates; and

(v) Region 5: $L_2 > R$, where the arbitrageur always fully liquidates. In this case, $E[\theta_1] = 0$ and $x^* = \frac{\theta_0}{4}$.

For (ii), where $R > L_1 > -R > L_2$, we have that

$$E\left[\theta_1\right] = \frac{(R - L_1)}{4R}\left(\theta_0 - x - \frac{L_1}{4} - \frac{R}{4}\right) + \frac{R + L_1}{4R}\left((\theta_0 - x) + \frac{(\beta M + \theta_0) L_1 + 2\theta_0 R}{2(\theta_0 - \beta M)}\right)$$

which on substitution into $\theta_0 - 2E\left[\theta_1\left(x^*, \varepsilon_1\right)\right] - 4x^* = 0$ gives $\frac{(3\theta_0 + \beta M)}{4(\theta_0 - \beta M)}\frac{(R + L_1(x^*))^2}{2R} + 3x^* = 0$, which has the solution:

$$x_2^* = \frac{(\beta M + \theta_0)\theta_0 - 2\beta(\theta_0 I - B_0)}{(\beta M + \theta_0)} + \frac{R}{2}\frac{(7\beta M - 5\theta_0)(\beta M + 3\theta_0)}{(\beta M + \theta_0)^2}$$
$$+ \frac{\sqrt{12R(\beta M - \theta_0)(\beta M + 3\theta_0)(2((\beta M + \theta_0)\theta_0 - 2\beta(\theta_0 I - B_0))(\beta M + \theta_0) + 2R(2\beta M - \theta_0)(\beta M + 3\theta_0))}}{2(\beta M + \theta_0)^2}. \tag{B.4.1}$$

For (iii)(a), where $L_1 > R > -R > L_2$, we have that

$$E\left[\theta_1\right] = (\theta_0 - x) + \frac{(\theta_0 + \beta M)}{(\theta_0 - \beta M)}\frac{L_2}{2}$$

which on substitution into $\theta_0 - 2E\left[\theta_1\left(x^*, \varepsilon_1\right)\right] - 4x^* = 0$ gives $-\theta_0 - \frac{(\theta_0 + \beta M)}{(\theta_0 - \beta M)}L_2(x^*) - 2x^* = 0$, which has the solution:

$$x_{3a}^* = \frac{\theta_0(\theta_0 + \beta M) - 2\beta(\theta_0 I - B_0)}{2(2\beta M - \theta_0)}. \tag{B.4.2}$$

For (iii)(b), where $R > L_1 > L_2 > -R$, we have that

$$E\left[\theta_1\right] = \frac{(\theta_0 - x)(R - L_1)}{4R} - \frac{\left(R^2 - L_1^2\right)}{16R} + \frac{(\theta_0(\theta_0 - x) - \beta(\theta_0 I - B_0))(L_1 - L_2)}{2R(\theta_0 - \beta M)} - \frac{\theta_0\left(L_1^2 - L_2^2\right)}{4R(\theta_0 - \beta M)}.$$

which on substitution into $\theta_0 - 2E\left[\theta_1\left(x^*, \varepsilon_1\right)\right] - 4x^* = 0$ gives $\theta_0 - \frac{(\theta_0 - x)(R - L_1)}{2R} + \frac{\left(R^2 - L_1^2\right)}{8R} - \frac{(\theta_0(\theta_0 - x) - \beta(\theta_0 I - B_0))(L_1 - L_2)}{R(\theta_0 - \beta M)} + \frac{\theta_0\left(L_1^2 - L_2^2\right)}{2R(\theta_0 - \beta M)} - 4x^* = 0$, which has the solution:

$$x_{3b}^* = \theta_0 + \frac{7R}{2} + \frac{4\beta\theta_0^2(\theta_0 I - B_0) - 14R\theta_0^3}{\left(\theta_0^3 - 7M\beta\theta_0^2 - 5M^2\beta^2\theta_0 - M^3\beta^3\right)}$$
$$+ \frac{(M\beta + \theta_0)\sqrt{(M\beta + 3\theta_0)\begin{pmatrix} 4\theta_0\beta^2(I\theta_0 - B_0)^2 + 6\left(7M\beta\theta_0^2 - \theta_0^3 + M^3\beta^3 + 5M^2\beta^2\theta_0\right)\theta_0 R \\ -28(I\theta_0 - B_0)\beta\theta_0^2 R + \left(37\theta_0^3 + 84M\beta\theta_0^2 + 12M^3\beta^3 + 60M^2\beta^2\theta_0\right)R^2 \end{pmatrix}}}{\left(\theta_0^3 - 7M\beta\theta_0^2 - M^3\beta^3 - 5M^2\beta^2\theta_0\right)} \tag{B.4.3}$$

For (iv) where $L_1 > R > L_2 > -R$, we have that

$$E\left[\theta_1\right] = \frac{(R - L_2)}{2R}\left(\theta_0 - x - \frac{R}{2}\right) - \frac{\beta M}{(\theta_0 - \beta M)}\frac{(R - L_2)^2}{4R}$$

which on substitution into $\theta_0 - 2E\left[\theta_1\left(x^*, \varepsilon_1\right)\right] - 4x^* = 0$ gives $\frac{L_2}{R}\theta_0 - \left(\frac{3R+L_2}{R}\right)x^* + \frac{(R-L_2)}{2} + \frac{\beta M}{(\theta_0-\beta M)}\frac{(R-L_2)^2}{2R} = 0$, which has the solution:

$$
\begin{aligned}
x_4^* = \theta_0 &- \frac{R\left(4\beta M - 3\theta_0\right)\left(\beta M + \theta_0\right)^2}{4\theta_0^2\beta M} - \frac{2\beta\left(\theta_0^2 + \beta^2 M^2\right)\left(\theta_0 I - B_0\right)}{4\theta_0^2\beta M} \\
&- \frac{\sqrt{\left(\beta M - \theta_0\right)\left(\beta M + \theta_0\right)^2\begin{pmatrix} R^2\left(16\beta^3M^3 + 24\theta_0\beta^2M^2 + \theta_0^2\beta M - 9\theta_0^3\right) + 4\beta^2\left(\beta M - \theta_0\right)\left(\theta_0 I - B_0\right)^2 \\ +4\beta R\left(\left(4\beta^2M^2 + \beta M\theta_0 + 3\theta_0^2\right)\left(\theta_0 I - B_0\right) - 6\theta_0^3 M\right)\end{pmatrix}}}{4\theta_0^2\beta M}.
\end{aligned}
\tag{B.4.4}
$$

Solving $L_1 = -R$ for B_0 using, $x^* = 0$ gives $U_1 \equiv \theta_0\left(I - \frac{M}{2} - \frac{3R}{4\beta} - \frac{\theta_0}{2\beta}\right) - \frac{RM}{4}$, the cutoff debt level below which the arbitrageur is unconstrained. Solving $L_2 = R$ for B_0 using $x^* = \frac{\theta_0}{4}$ gives $U_4 \equiv \theta_0\left(I - \frac{3M}{4} + \frac{R}{2\beta}\right) + \frac{RM}{2}$, the cutoff debt level above which the arbitrageur always finds it optimal to liquidate.

When $L_1 - L_2 \geq 2R$, the boundaries are crossed in the order as follows: (i) $L_1 = -R$; (ii) $L_1 = R$; (iii) $L_2 = -R$; and (iv) $L_2 = R$. Solving $L_1 = R$ and $L_2 = -R$ for B_0 using x_{3a}^* from (B.4.2) gives cutoff debt levels $U_2 \equiv \theta_0 I - \frac{(\beta M+\theta_0)\theta_0}{2\beta} + \frac{(\theta_0-2\beta M)}{(\theta_0-\beta M)}\frac{(\beta M+3\theta_0)R}{6\beta}$ and $U_3 \equiv \theta_0 I - \frac{R}{2\beta}\frac{(\theta_0-2\beta M)(\theta_0+\beta M)}{(\theta_0-\beta M)} - \frac{3\theta_0\beta M}{2\beta}$. The condition $L_1 - L_2 \geq 2R$ must hold for all $U_3 \geq B_0 \geq U_2$.

Using x_{3a}^* in L_1 and L_2 gives

$$
\frac{4M\beta^2\left(\theta_0 - \beta M\right)}{\left(\theta_0 - 2\beta M\right)\left(\beta M + \theta_0\right)\left(M\beta + 3\theta_0\right)}B_0 - \frac{\theta_0\left(4MI\beta^2 + 3M\beta\theta_0 - 3\theta_0^2\right)\left(\theta_0 - \beta M\right)}{\left(\theta_0 - 2\beta M\right)\left(\beta M + \theta_0\right)\left(M\beta + 3\theta_0\right)} \geq 2R.
$$

For $\theta_0 > 2\beta M$, the left hand side increases with B_0, so we must check that the condition holds at the smallest value of B_0, which is U_2. For $2\beta M > \theta_0 > \beta M$, the left hand side decreases with B_0, so we must check that the condition holds at the largest value of B_0, which is U_3. Both of these yield the requirement that $\theta_0^2 - \theta_0\left(2R + \beta M\right) - \frac{4}{3}MR\beta \geq 0$. Also, we must have that $U_3 \geq U_2$, which requires that $\left(\theta_0^2 - \theta_0\left(2R + \beta M\right) - \frac{4}{3}MR\beta\right)\left(\theta_0 - 2M\beta\right) \geq 0$.

These conditions together give us the requirement that

$$
\theta_0 \geq \max\left(2M\beta, \frac{\beta M}{2} + R + \sqrt{\left(\frac{\beta M}{2}\right)^2 + \frac{7}{3}MR\beta + R^2}\right). \tag{B.4.5}
$$

Given R, β, and M such that $\frac{8}{3}R \geq \beta M$, we have that $\left(R + \frac{\beta M}{2}\right) + \sqrt{R^2 + \frac{7}{3}R\beta M + \left(\frac{\beta M}{2}\right)^2} \geq 2\beta M$, and (B.4.5) reduces to $\theta_0 \geq \frac{\beta M}{2} + R + \sqrt{\left(\frac{\beta M}{2}\right)^2 + \frac{7}{3}MR\beta + R^2}$, while for $\beta M > \frac{8}{3}R$ we must have that $\theta_0 > 2\beta M$.

When $2R > L_1 - L_2 \geq 0$, the boundaries are crossed in the order as follows: (i) $L_1 = -R$; (ii) $L_2 = -R$; (iii) $L_1 = R$; and (iv) $L_2 = R$. Solving $L_2 = -R$ for B_0 using x_2^* from (B.4.1) gives the cutoff debt level, $U_2' \equiv \theta_0 I - \frac{R\left(\theta_0^2+17M\beta\theta_0+6M^2\beta^2\right)}{2\beta(\theta_0-\beta M)} + \frac{\sqrt{6RM^2(M\beta+3\theta_0)\left(2MR\beta+(4R+M\beta)\theta_0-\theta_0^2\right)}}{(\theta_0-\beta M)}$.

Similarly, solving $L_1 = R$ for B_0 using x_4^* from (B.4.4) gives the cutoff debt level, $U_3' \equiv \theta_0 I - \frac{\left(2R\left(\theta_0^3+2\beta^3M^3\right)+13MR\beta\theta_0(\theta_0+\beta M)\right)}{2(\theta_0-M\beta)(M\beta+2\theta_0)\beta} + \frac{(M\beta+\theta_0)^2\sqrt{2R\left(14MR\beta\theta_0-6\theta_0^3+12R\theta_0^2+3M\beta\theta_0^2+6M^2R\beta^2+3M^2\beta^2\theta_0\right)}}{2(M\beta+2\theta_0)(\theta_0-M\beta)\beta}$. Using $L_1 - L_2 = 2\frac{(\theta_0-M\beta)}{(M\beta+\theta_0)(M\beta+3\theta_0)}\left(I\beta\theta_0 + \theta_0^2 - \beta B_0 - x\theta_0\right)$ we obtain the expression $\frac{\partial(L_1-L_2)}{\partial B_0} = -2\frac{(\theta_0-M\beta)}{(M\beta+\theta_0)(M\beta+3\theta_0)}\left(\beta + \theta_0\frac{\partial x}{\partial B_0}\right) < 0$ since $\frac{\partial x^*}{\partial B_0} = \frac{2\beta\theta_0(L_1-L_2)}{\left(\left(\theta_0^2-\beta^2M^2\right)\left(\frac{7}{2}R-\frac{1}{2}L_1+L_2\right)+2\theta_0\beta M(L_1-L_2)\right)} > 0$. This requires $L_1 - L_2 < 2R$ at U_2', giving that $\left(\theta_0^2 - (2R+\beta M)\,\theta_0 - \frac{4}{3}R\beta M\right) < 0$, and that $L_1 - L_2 > 0$ at U_3', giving $\theta_0 > \frac{2}{3}R$. □

B.5: Proof of Corollary 3

We have $L_1 = 2\frac{(\beta M+\theta_0)(\theta_0-x)-2\beta(\theta_0 I-B_0)}{(\beta M+3\theta_0)}$, $L_2 = 2\frac{\beta M(\theta_0-x)-\beta(\theta_0 I-B_0)}{\beta M+\theta_0}$, and $\theta_1^C = \frac{\theta_0(\theta_0-x-\varepsilon_1)-\beta(\theta_0 I-B_0)}{(\theta_0-\beta M)}$, which gives $\frac{\partial L_1(x^*)}{\partial B_0} = -\frac{2(\beta M+\theta_0)}{(\beta M+3\theta_0)}\frac{\partial x^*}{\partial B_0} + \frac{4\beta}{(\beta M+3\theta_0)}$, $\frac{\partial L_2(x^*)}{\partial B_0} = -\frac{2\beta M}{\beta M+\theta_0}\frac{\partial x^*}{\partial B_0} + \frac{2\beta}{\beta M+\theta_0}$ and $\frac{\partial\theta_1^C}{\partial B_0} = -\frac{\theta_0}{(\theta_0-\beta M)}\frac{\partial x^*}{\partial B_0} + \frac{\beta}{(\theta_0-\beta M)}$.

(i) We have that x_2^* solves $\frac{(3\theta_0+\beta M)}{4(\theta_0-\beta M)}\frac{(R+L_1(x^*))^2}{2R} + 3x^* = 0$. Differentiating with respect to B_0 gives $\frac{\partial x^*}{\partial B_0} = -\frac{(3\theta_0+\beta M)}{6(\theta_0-\beta M)}\frac{(R+L_1(x^*))}{2R}\frac{\partial L_1(x^*)}{\partial B_0}$. In this region we must have $\frac{\partial L_1(x^*)}{\partial B_0} > 0$, otherwise the arbitrageur would become less constrained as its debt level rises. This implies that $\frac{\partial x^*}{\partial B_0} < 0$.

(ii)(a) We have that $x_{3a}^* = \frac{2\beta(\theta_0 I-B_0)-\theta_0(\theta_0+\beta M)}{2(\theta_0-2\beta M)}$. Differentiating with respect to B_0 gives $\frac{\partial x^*}{\partial B_0} = -\frac{\beta}{(\theta_0-2\beta M)} < 0$.

(ii)(b) We have that $x^* = \frac{\theta_0}{4} - \frac{E[\theta_1]}{2}$ and $E[\theta_1] = \frac{1}{2R}\int_{L_2}^{L_1}\theta_1^C(\varepsilon)\,d\varepsilon + \frac{1}{2R}\int_{L_1}^{R}\theta_1^U(\varepsilon)\,d\varepsilon$. Differentiating with respect to B_0 and simplifying gives $\frac{\partial x^*}{\partial B_0} = \frac{2\beta\theta_0(L_1-L_2)}{\left(\left(\theta_0^2-\beta^2M^2\right)\left(\frac{7}{2}R-\frac{1}{2}L_1+L_2\right)+2\theta_0\beta M(L_1-L_2)\right)} > 0$; the numerator is greater than zero because $L_1 \geq L_2$ and the denominator is greater than zero because $R \geq L_1 \geq L_2 \geq -R$.

(iii) x_4^* solves $\frac{L_2}{R}\theta_0 - \left(\frac{3R+L_2}{R}\right)x^* + \frac{(R-L_2)}{2} + \frac{\beta M}{(\theta_0-\beta M)}\frac{(R-L_2)^2}{2R} = 0$. Differentiating with respect to B_0 gives $\left(2\theta_0 - 2x^* + 2\frac{\beta M}{(\theta_0-\beta M)}L_2 - \left(\frac{\theta_0+\beta M}{\theta_0-\beta M}\right)R\right)\frac{\partial L_2(x^*)}{\partial B_0} - (6R+2L_{L2})\frac{\partial x^*}{\partial B_0} = 0$. Substituting in $\frac{\partial L_2(x^*)}{\partial B_0}$ and rearranging terms, we get $\frac{\partial x^*}{\partial B_0} = \left(\frac{(6R+2L_2)(\theta_0-\beta M)(\theta_0+\beta M)}{2\beta(2(\theta_0-x^*)(\theta_0-\beta M)+2\beta ML_2-(\theta_0+\beta M)R)} + M\right)^{-1} > 0$ using the fact that $L_2(x^*) \geq -R$, $\theta_0 > 2\beta M$ and $\theta_0 > x^*$.

For $L_1 - L_2 \geq 2R$ we have that $x_4^* = -\frac{\theta_0}{2} + \frac{R(\theta_0+\beta M)}{2(\theta_0-\beta M)} < 0$ at $B_0 = U_3$ and $x_4^* = \frac{\theta_0}{4} > 0$ at $B_0 = U_4$. Continuity of x_4^* and the fact that $\frac{\partial x^*}{\partial B_0} > 0$ gives us that $x_4^* = 0$ for at least one $B_0 \equiv B_0(0) \in (U_3, U_4)$. Setting $x^* = 0$ in (B.4.4) we have that $B_0(0)$ solves

$$2\theta_0 L_2(0)(\theta_0 - \beta M) + (R\theta_0 - L_2(0))(R - \beta M L_2(0)) = 0. \tag{B.5.1}$$

For $L_1 - L_2 < 2R$ we have that $x_3^* \leq 0$ at $B_0 = U_2$ and $x_4^* = \frac{\theta_0}{4} > 0$ at $B_0 = U_4$. Continuity of x^* and the fact that $\frac{\partial x^*}{\partial B_0} > 0$ gives us that $x^* = 0$ for at least one $B_0 \equiv B_0(0) \in \left(U_2', U_4\right)$. If $x_3^* < 0$ at $B_0 = U_3'$, then $B_0(0) > \theta_0 I + \frac{(\beta M+3\theta_0)R-2(\beta M+\theta_0)\theta_0}{4\beta}$, and $B_0(0)$ solves (B.5.1). Otherwise $B_0(0)$ solves

$$R(4\theta_0 + R)(\theta_0 - \beta M) - (\beta M + 3\theta_0)L_1^2(0) + 4L_2(0)\left(2\theta_0^2 - 2\beta(\theta_0 I - B_0) - \theta_0 L_2(0)\right) = 0 \tag{34}$$

$\square$

B.6: Proof of Corollary 4

The probability of full liquidation is $\frac{L_2+R}{2R}$. In the absence of the strategic trader, this is $\frac{1}{2} + \frac{\beta M\theta_0-\beta(\theta_0 I-B_0)}{(\beta M+\theta_0)R}$. In the presence of the strategic trader, the probability is $\frac{1}{2} + \frac{\beta M\theta_0-\beta(\theta_0 I-B_0)}{(\beta M+\theta_0)R} - \frac{\beta M}{(\beta M+\theta_0)R}x$. When $x < 0$, the presence of the strategic trader causes an increased probability of full liquidation; while, for $x > 0$, the presence of the strategic trader causes a reduced probability of full liquidation. Thus, for $B_0 < B(0)$, the presence of the strategic trader causes an increase in the probability of full liquidation, while for $B_0 > B(0)$ the presence of the strategic trader causes a reduction in the probability of full liquidation. This proves (i).

Part (ii) follows directly from (i). $\square$

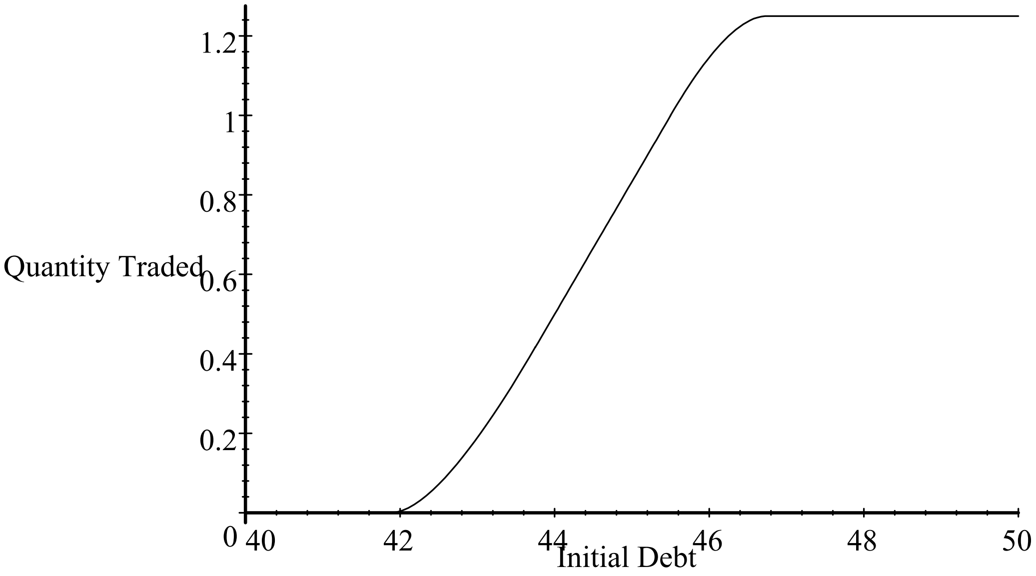

Figure 1: Strategic trader's optimal quantity for varying B_0; arbitrageur holds small position. Parameter values: $\theta_0 = 5$, $R = 0.5$, $\beta = 5$, $M = 2$, and $I = 10$. This implies $u_1 = 41.875$, $u_2 = 43.75$, $u_3 = 45.5$, and $u_4 = 46.75$.

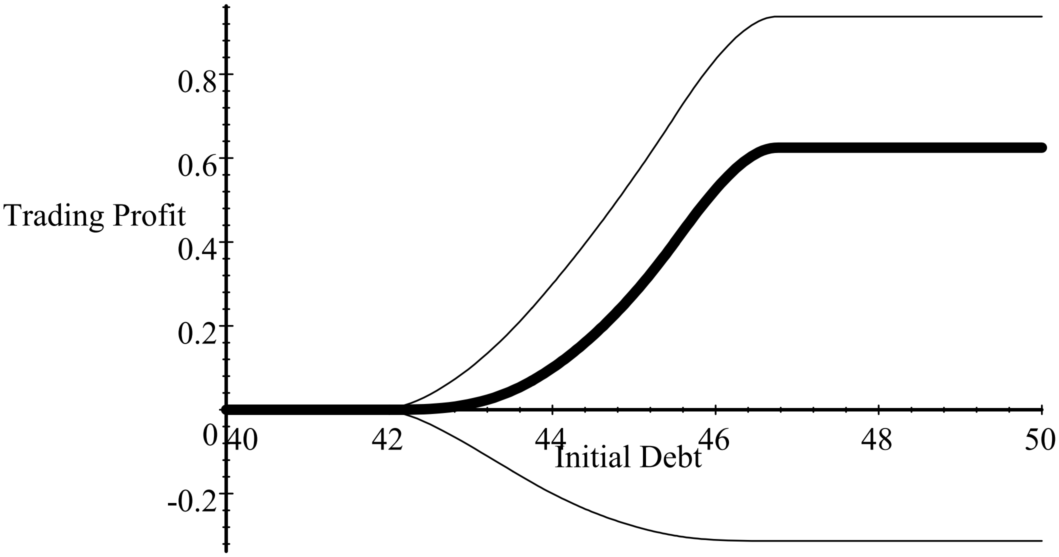

Figure 2: Strategic trader's trading profit for varying B_0; arbitrageur holds small position. The thick line represents the expected total trading profit computed as $xE[(P_2 - P_1)]$; the uppermost line is the expected first-period book profit computed as $x\,(I - E[P_1])$; and the lowermost line is the expected second-period book profit computed as $-x\,(I - E[P_2])$. Parameter values: $\theta_0 = 5$, $R = 0.5$, $\beta = 5$, $M = 2$, and $I = 10$. This implies $u_1 = 41.875$, $u_2 = 43.75$, $u_3 = 45.5$, and $u_4 = 46.75$.

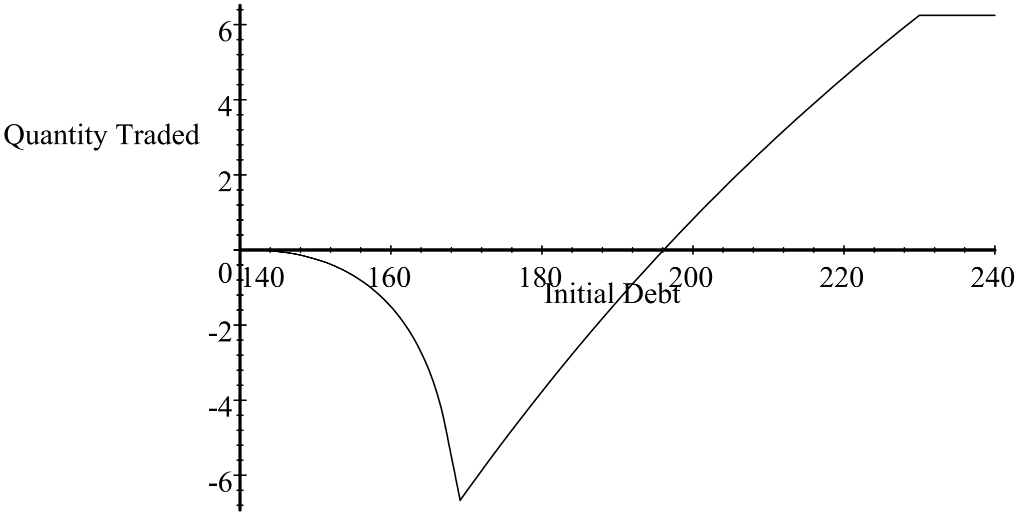

Figure 3: Strategic trader's optimal quantity for varying B_0; arbitrageur holds large position. Parameter values: $\theta_0 = 25$, $R = 5$, $\beta = 5$, $M = 2$, and $I = 10$. This implies $U_1 = 141.25$, $U_2 = 167.22$, $U_3 = 169.17$, and $U_4 = 230.0$.

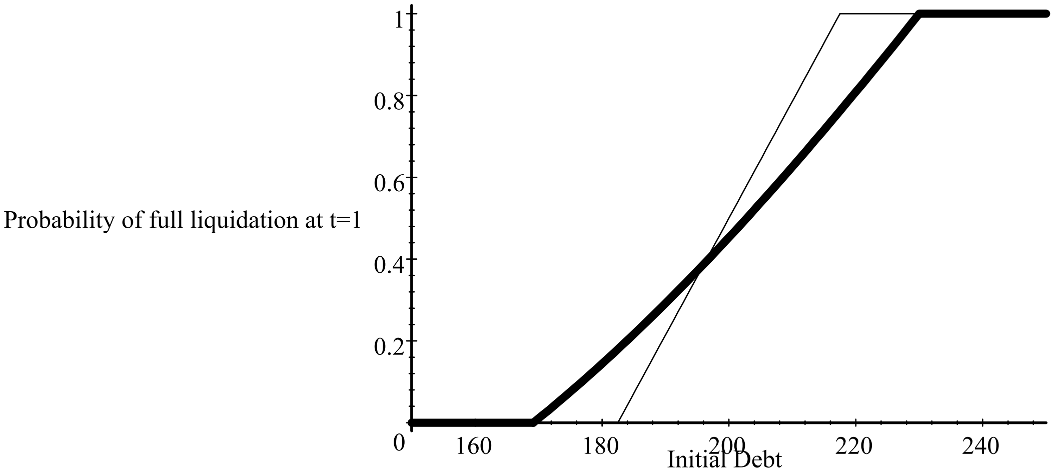

Figure 4: Probability of full arbitrageur liquidation at $t = 1$ without a strategic trader in the market (thin line) and with strategic trader in the market (thick line) for varying B_0; arbitrageur holds large position. Parameter values: $\theta_0 = 25$, $R = 5$, $\beta = 5$, $M = 2$, and $I = 10$. This implies $U_3 = 169.17$ and $U_4 = 230.0$.

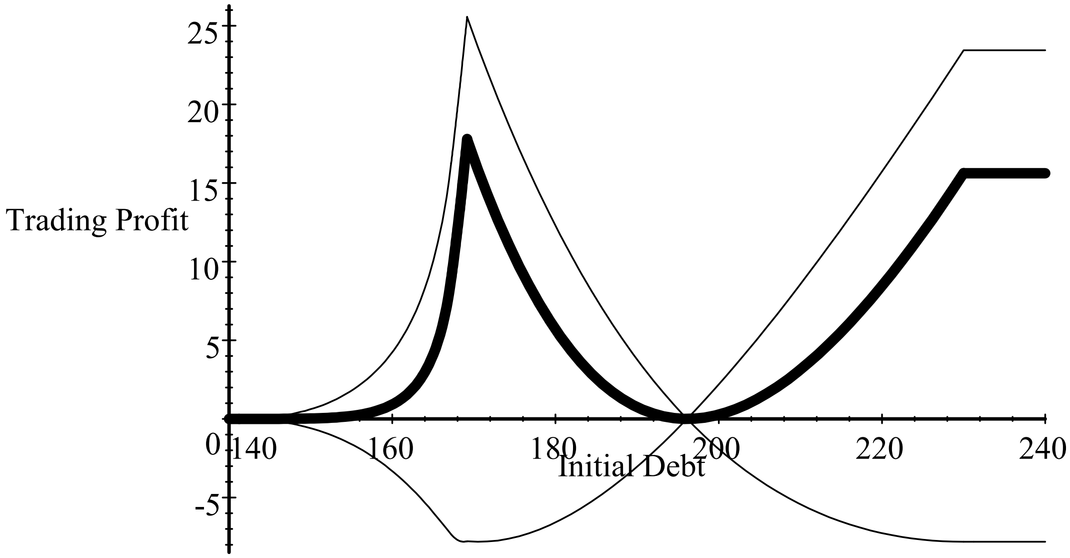

Figure 5: Strategic trader's expected profit for varying B_0; arbitrageur holds large position. The thick line plots the expected total profit $xE[(P_2 - P_1)]$. The line that is negative for low levels and positive for high levels of B_0 plots the expected first-period book profit $x\,(I - E[P_1])$, and the third line the expected second-period book profit $-x\,(I - E[P_2])$. Parameter values: $\theta_0 = 25$, $R = 5$, $\beta = 5$, $M = 2$, and $I = 10$. This implies $U_1 = 141.25$, $U_2 = 167.22$, $U_3 = 169.17$, and $U_4 = 230.0$.

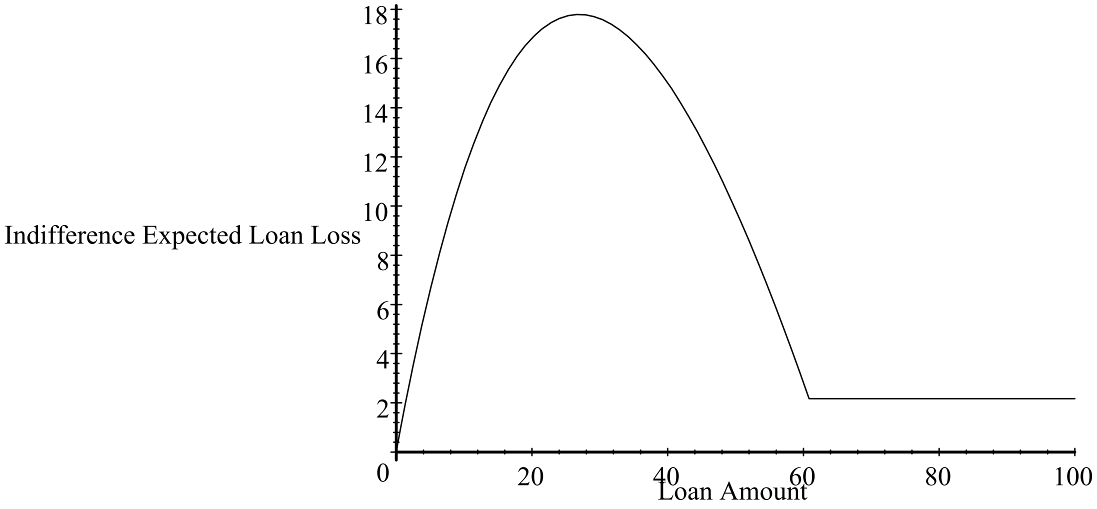

Figure 6: Level of expected loss on loan as a function of the amount lent that leaves the strategic trader indifferent between lending and trading and not lending for $B_0 > U_3$; arbitrageur holds large position. The strategic trader lends b_1 so that $U_3 = B_0 - b_1$. Parameter values used are $\theta_0 = 25$, $R = 5$, $\beta = 5$, $M = 2$, and $I = 10$. This implies $U_3 = 169.17$ and $U_4 = 230.0$.

References

Allen, F., and D. Gale (1992): Stock Price Manipulation. *Review of Financial Studies* 3, 503-529.

Allen, F., and G. Gorton (1992): Stock Price Manipulation, Market Microstructure, and Asymmetric Information. *European Economic Review* 36, 624-630.

Attari, M., and A.S. Mello (2001): Financially Constrained Arbitrage in Illiquid Markets. Working Paper, University of Wisconsin-Madison.

Bolton, P., and D.S. Scharfstein (1990): A Theory of Predation Based on Agency Problems in Financial Contracting. *American Economic Review* 80, 93-106.

Brunnermeier, M. K., and L. H. Pedersen (2002): Predatory Trading. Working Paper, Princeton University.

Cai, F. (2002): Does the Market Conspire Against the Weak? An Empirical Study of Front Running Behavior During the LTCM Crisis. Working Paper, University of Michigan.

Foster, F.D., and S. Viswanathan (1994): Strategic Trading with Asymmetrically Informed Traders and Long-Lived Information. *Journal of Financial and Quantitative Analysis* 29, 499-518.

Lowenstein, R. (2000): When Genius Failed: The Rise and Fall of Long-Term Capital Management, Random House, New York

Plender, J. (2002): Big Greedy Beasts Feed on Small Bank Prey, *Financial Times*, March 20, 2002.

Shleifer, A., and R.W. Vishny (1997): The Limits of Arbitrage. *Journal of Finance* 47, 35-55.

Xiong, W. (2001): Convergence Trading with Wealth Effects. *Journal of Financial Economics* 78, 247-292.